MENSURATIONAL AND BIOMASS RELATIONS FOR POPULUS 'TRISTIS #1' UNDER INTENSIVE CULTURE

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ABSTRACT.—Tree measurement data from plantations established in 1970 and 1973 and grown under intensive culture were used to establish various dimensional relations and biomass equations for Populus 'Tristis #1'. These equations subsequently have been used to estimate yields on study plots and for projections of future yields. They are presented here for others working with this hybrid and as a guide to model forms which might be utilized for other species.

KEY WORDS: tree measurement, plantations, hybrids, yield analysis.

Short rotation intensive culture (SRIC) studies begun in 1970 at the Hugo Sauer Nursery in Rhinelander, Wisconsin, have involved considerable growth and yield analysis. The principal variety used for studies of intensive cultural practices (fertilization, irrigation, etc.) to date has been Populus 'Tristis #1'. Dawson (1976) reviewed previous work on this variety. Ek and Dawson (1976) described earlier tree biomass equations developed for plantings established in 1970. The intent of this paper is to update that work by presenting equations developed from incorporating more recent data. Some of these data and the equations described here were used in yield analyses by Meldahl (1979). The new data are from the same nursery site and fertilization and irrigation treatments described in these earlier papers.

DESCRIPTION OF YIELD DATA

As the SRIC plantations have grown in number and tree sizes have increased, stem and branch analyses have evolved to consider more detailed stand and tree sampling techniques in developing biomass data. The earlier Ek and Dawson paper developed individual tree dry weight yield equations for stem wood, stem bark, branch wood, branch bark, branch tips, and leaves. The equations developed in that study were based on essentially complete analysis of small sample stems from three plots with square spacings of 0.2286, 0.3048, and 0.6096 m. Plots had sides measuring 5.0, 5.2, and 5.5 m. Sample trees were drawn from the interior of each plot each year. This led to 235 observations of tree dimensions and associated stem wood and bark weight. (There were fewer observations for other tree weight components.) Over 80 percent of these data were obtained at ages 1 and 2 years. The remaining 20 percent came from trees at ages 3 and 4 years. In addition to recording total tree height and basal diameter (2.5 cm above the base of the root collar), the trees were separated into the six above mentioned components. Harvesting was done in late summer at the time of maximal leaf biomass. These components were then oven dried at 70 C and weighed to the nearest 0.01 g.

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A small but undetermined number of leaves were lost prior to harvest.
These data subsequently are referred to as those from the 1970 plantings.

Additional larger plantings were established in June 1973 with square spacings of 0.3048, 0.6096, 1.2192, and 2.4384 m. These plots were 8 to 16 rows wide and had 24 to 36 trees per row. As in 1970, 20-cm cuttings were planted. Trees subsequently were selected and harvested or otherwise observed on these plots at ages 3 and 4 years to augment the data from the 1970 plantings. Additionally, two of the larger trees from the 1970 plantings were harvested in the fall of 1976 to augment information on larger stems. Together, these data subsequently are referred to as those from 1973 plantings. This data set included 20 to 34 trees, depending on the variable of interest.

The 1973 data were developed by selecting and harvesting three trees from the 0.3048, 0.6096, and 1.2192 m spacings near the peak of the third growing season (9 trees), and one more stem from each spacing at the end of that season (3 trees). In addition, two trees were drawn from each spacing at the end of the fourth growing season (6 trees). Also, two trees drawn from the 1970 plantings at the end of the seventh growing season were included in this data set. Except for the two trees from the 1970 plantings, the selected stems were a stratified sample, with stratification based on diameter outside bark at breast height (D). Equal numbers of stems were selected randomly from each of three D classes within each spacing. To minimize the influence on remaining stems, however, this harvesting was concentrated at one end of each plot, excluding border trees. The two trees from the 1970 plantings were purposively selected from among the largest stems from the plots established in that year. These stems were obtained from the interior of the 0.3048 and 0.6096 m spacings. These 20 stems are grouped together here because of their similar analysis as described below.

Laboratory analysis of the stems then was conducted using four or more systematically located 150-mm sections along the stem, beginning at 25 mm above the ground. Section measurements included green diameters inside and outside bark at each end, oven dry (70°C) weight of wood and bark, and the height of each section base above ground. Total stem height (H), D, and diameters outside bark every 30 cm up the stem also were determined. Section volumes for specific gravity determinations were developed using Smalians formula for frustrums of paraboloids.

Branch observations included measurement of the diameter (at 25 mm from stem base) of all branches and the height of the branch base above ground. Long and short branches then were selected randomly from each of the above stem sections for length, specific gravity, and dry weight of wood and bark determinations. Total branch wood plus bark dry weights were observed directly, but dry weight of wood and bark components were estimated, using the total branch weights multiplied by wood and bark specific gravities determined on sections. Section size and location procedure was similar to those used for the main stem. Leaf dry weight determinations were made only for the nine trees measured at the peak of the third growing season and only for the selected long and short branches on those stems.

An additional 14 trees were purposively selected from the 1973 plantings to cover the range of tree sizes present. These were observed only for height and basal diameter (2.5 cm above ground) and inside and outside bark diameters at breast height at the end of the third growing season.

Another data set, referred to as 1974 observations, involved determining diameters outside bark at six to eight systematically located positions along the stems of 30 trees. These trees were selected at random in the spring of 1974 from the 1970 plantings. Observations on these stems included basal diameter (2.5 cm above ground), diameter at 0.15 m, 0.30 m, 1.37 m, and at approximately 1.0 m intervals to the tip, and total height. Diameters at 0.15 m from the tip also were recorded. These data originally were intended for development of a stem taper equation, but even when combined with the 1973 data, extrapolation to larger tree sizes via the various equations tested was tenuous. Consequently, the development of taper equations was postponed and the 1974 data were used only to aid quantification of stem biomass.

**ANALYSIS**

The above yield data were analyzed by nonlinear regression analysis to develop two types of equations. The first set, primarily mensurational, was used to convert the 1970 data to a form compatible with the 1973 data (this involved estimating breast height diameter from basal diameter), and to develop tree component weights for the 1973 and 1974 data sets from the various subsample information on these trees. The second type of equations given describe tree dry weight or biomass components as a function of tree D and total height H. These equations are described in tables 1 and 2.

Computing stem weight for wood and bark components for the 1973 and 1974 data involved applying
the diameter inside bark and specific gravity equations (table 1, equations b, c, and d) to the periodic diameter outside bark data available for these stems. The section volumes were obtained using Smalian's formula.

Tree branch wood, branch bark, and leaf weights were obtained using equations developed from the branch data to express weight as a function of branch and tree characteristics. The model form used was:

\[ w = b_1 D^{b_2} (H-h)^{b_3} (H/D)^{b_4} + b_5 \]

where:
- \( w \) = branch component weight,
- \( d \) = branch diameter outside bark at 25 mm from base,
- \( h \) = height to base of branch and the \( b_i \) are constants

Summing predictions over all branches on the stem provided the total branch component weight used for these trees. Details of developing these equations are given by Ek (1979). Only average specific gravities of branch wood and bark are given in table 1, as these characteristics were not well correlated with with branch and/or tree dimensions.

The use of the biomass equations given in table 2 is recommended for trees 0.5 to 9.1 cm D. For smaller trees, the equations based on basal diameter and total height given by Ek and Dawson (1976) are recommended. With those equations, branch wood plus tip weight is analogous to the branch wood term estimated in this report. For extrapolation to trees larger than 9.1 cm, the equations given here based on D should provide useful approximations. Use of table 2 equation (a) for stem wood weight together with the stem specific gravity equation (g) for larger tree sizes leads to total stem volume values comparable to those given in table 3 of Gevorkiantz and Olsen (1955). The fact that table 3 is known to fit a wide range of species suggests the feasibility of extrapolation via equations given in table 2.

The model form used for tree biomass in table 2 is:

\[ w = b_1 D^{b_2} H^{b_3} \]

(1)

This model is an extension of the common allometric model. It has a long history of use in forest yield analyses (Husch, et al. 1972). The use of weighting procedures here produced a slight improvement in fits for the smaller tree sizes. Other models were tested but this form consistently performed best. An attempt to introduce a spacing term via the model form:

\[ w = b_1 D^{b_2} H^{b_3} S^{b_4} \]

where \( S \) was the initial stand spacing in m produced negligible improvement in fits.

From a physical standpoint, model (1) also may be rewritten as:

\[ w = b_1 D^{2.7} H^{3.7} (D^2H) \]

(2)

In this form, the first term on the right hand side \( F \) of (2) may be viewed as a combination of basic constants, stem form, and specific gravity factors applied to the dimensions of a cylinder of height = \( H \) and diameter = \( D \). Barring sharp changes in stem form and specific gravity not associated with tree size, this formulation suggests considerable extrapolative potential. Extrapolation is not encouraged, but may be necessary for some yield projections until more data are available. Of the equations in table 2, the one for leaf weight, because of its limited data base in terms of stand age, is perhaps least suitable for extrapolation.

The equations in table 2 also may be used with slight adjustment, to approximate yields above a 0.15 m stump. The total stem biomass for the higher stump for 20 trees from the 1973 plantings ranged from 90 to 98 percent of that for the 0.03 m stump. Percentage differences in yields for the two stump heights also decreased with increasing tree size. Percentage differences in biomass for branch components between the two stump heights were negligible.

**LITERATURE CITED**


Table 1.—Mensurational relations used to develop tree biomass information for Populus ‘Tristis #1’

<table>
<thead>
<tr>
<th>Equation</th>
<th>Dependent variable</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>Diameter outside bark at breast height (1.37 m) ( D = 0.2309 + 0.3221 D_{sb}^{0.72609} H^{0.63856} )</td>
<td>SE 0.203 0.98 3.13 0.82-9.20 53 1970, 1973</td>
</tr>
<tr>
<td>(b)</td>
<td>Stem diameter inside bark ( dib = -0.5247 (-0.5247 + dob^{0.20305}) + 0.9488 dob )</td>
<td>SE 0.760 0.99 26.59 1.75-112.95 349 1970, 1973 (29 trees)</td>
</tr>
<tr>
<td>(c)</td>
<td>Specific gravity of stem wood ( S_{gw} = 0.3944 (1.0 - 0.7438e^{-1.97469 dib}) )</td>
<td>SE 0.032 0.68 0.36 0.12-0.45 156 1973</td>
</tr>
<tr>
<td>(d)</td>
<td>Specific gravity of stem bark ( S_{gb} = 0.3621 (1.0 - 0.2711e^{-0.8581 dib}) )</td>
<td>SE 0.045 0.25 0.33 0.15-0.53 156 1973 (20 trees)</td>
</tr>
<tr>
<td>(e)</td>
<td>Specific gravity of branch wood ( S_{bw} = 0.4088 )</td>
<td>SE 0.060 — 93 1973 (20 trees)</td>
</tr>
<tr>
<td>(f)</td>
<td>Specific gravity of branch bark ( S_{bb} = 0.3298 )</td>
<td>SE 0.046 — 93 1973 (20 trees)</td>
</tr>
</tbody>
</table>

1Definition of terms: \( D_{sb} = \) stem basal diameter (2.5 cm above base) in cm; \( H = \) total tree height (m); \( dob = \) stem diameter outside bark (cm); \( dib = \) stem diameter inside bark (cm)
2Uncorrected \( R^2 \) values were all higher than those given and in no cases less than 0.98
3Diameters in this equation are in mm

Table 2.—Biomass Relations for Populus ‘Tristis #1’

<table>
<thead>
<tr>
<th>Equation</th>
<th>Standard error ( \text{SE} )</th>
<th>( R^2 )</th>
<th>Basis</th>
<th>No. of observations</th>
<th>Dependent variable ( \text{mean} ^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>Stem wood weight ( w_1 = 43.1256 D^{2.23921} H^{0.29812} )</td>
<td>10.80</td>
<td>0.98</td>
<td>247</td>
<td>471.60</td>
</tr>
<tr>
<td>(b)</td>
<td>Stem bark weight ( w_2 = 21.6690 D^{1.76075} H^{0.19632} )</td>
<td>5.38</td>
<td>0.92</td>
<td>217</td>
<td>94.81</td>
</tr>
<tr>
<td>(c)</td>
<td>Branch wood weight ( w_3 = 91.7349 D^{4.18066} H^{-2.37815} )</td>
<td>17.97</td>
<td>0.65</td>
<td>215</td>
<td>118.49</td>
</tr>
<tr>
<td>(d)</td>
<td>Branch bark weight ( w_4 = 38.4169 D^{3.35658} H^{-1.51204} )</td>
<td>8.53</td>
<td>0.74</td>
<td>215</td>
<td>71.04</td>
</tr>
<tr>
<td>(e)</td>
<td>Leaf weight ( w_5 = 205.7909 D^{3.55347} H^{-2.48585} )</td>
<td>13.48</td>
<td>0.71</td>
<td>217</td>
<td>96.67</td>
</tr>
<tr>
<td>(f)</td>
<td>Total tree weight ( w_6 = \sum_{i=1}^{5} w_i )</td>
<td>37.16</td>
<td>0.94</td>
<td>215</td>
<td>793.94</td>
</tr>
<tr>
<td>(g)</td>
<td>Stem wood specific gravity ( S_g = 0.3928 (1.0 - 0.5909e^{-1.65546 D}) )</td>
<td>0.0021</td>
<td>0.99</td>
<td>40 (1973 data)</td>
<td>0.3813</td>
</tr>
</tbody>
</table>

1Definition of terms: \( w_i = \) component dry weight in grams above a 0.03 m stump, \( D = \) tree diameter at breast height (1.37 m) in cm, \( H = \) total tree height in m
2Fit statistics for equations (a)-(f) were based on weighted nonlinear regression with weights = 1/(D \( H^2 \))
3Corrected \( R^2 \) values
4Range in tree \( D \) was 0.5-9.1 cm