Height-Diameter Equations for Thirteen Midwestern Bottomland Hardwood Species

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ABSTRACT: *Height-diameter equations are often used to predict the mean total tree height for trees when only diameter at breast height (dbh) is measured. Measuring dbh is much easier and is subject to less measurement error than total tree height. However, predicted heights only reflect the average height for trees of a particular diameter. In this study, we present a set of height-diameter equations for 13 riparian tree species using data obtained from bottomland hardwood forests along the Mississippi, Missouri, Illinois, and Des Moines rivers. Nonlinear regression techniques were used to develop the equations. The resulting equations provide a reasonable means of predicting unknown tree heights, given dbh, for these species. North. J. Appl. For. 19(4):171–176.*

Key Words: Bottomland hardwoods, height-diameter equations, allometric equations, and Midwest riparian forests.

Bottomland hardwood forests are a highly productive, yet underutilized resource in the Midwest United States. They represent a significant percentage of the forested land base. For example, in Iowa, bottomland forest types account for more than one-quarter of the total forested land (Hansen et al. 1992). In Illinois, they represent one-fifth of all forested land. Despite their prevalence on the landscape, there are few quantitative tools available for land managers working in this ecosystem.

Diameter is one of the most commonly measured mensurational parameters in forestry. This is principally because it is relatively easy to measure accurately. Conversely, total tree height is not as commonly measured for several reasons, which include: (1) time required to complete measurements; (2) chance of observer error; and (3) visual obstructions. Consequently, many foresters only subsample total tree heights or do not measure heights at all. If forest resource inventories are used in situations where total tree heights are required, a reasonable approach is to use average height-diameter equations to predict unknown tree heights. Specifically, these equations predict mean total tree height for a given diameter at breast height and species. Furthermore, height-diameter equations are used for estimating vertical forest structure and predicting heights in numerous forest growth simulators (e.g., Wykoff et al. 1982, Van Deusen and Biging 1985, Larsen and Hann 1987, Ritchie and Hann 1986, Larsen 1994).

A number of model forms have been used to predict tree height from diameter by species (e.g., Curtis 1967, Monserud 1975, Ek et al. 1984, Larsen and Hann 1987, Parresol 1992, Flewelling and de Jong 1994). Monserud's (1975) equation is a flexible form that readily fits many height-diameter datasets (e.g., Larsen and Hann 1987). Specifically, Monserud's model form is:

$$ht = bh + e^{(b_0 + b_1 D b_2)} \tag{1}$$

where *ht* is total tree height (ft), *bh* is breast height (4.5 ft for English units), and *D* is diameter at breast height (in.). This equation enforces the constraint that as *D* approaches zero, *ht* approaches *bh* (4.5 ft) given that b_1 and b_2 are negative (Larsen and Hann 1987).

The objective of this study was to develop height-diameter equations for 13 Midwest riparian tree species employing Monserud's model form.

Methods

The data used in this study were part of a larger study of riparian forests along major rivers in Missouri, Illinois, and Iowa. These riparian forest sites lay along sections of the Missouri, Platte, Illinois, Iowa, Des Moines, Cedar, and Mississippi Rivers, which were flooded in 1993. Collaborat-

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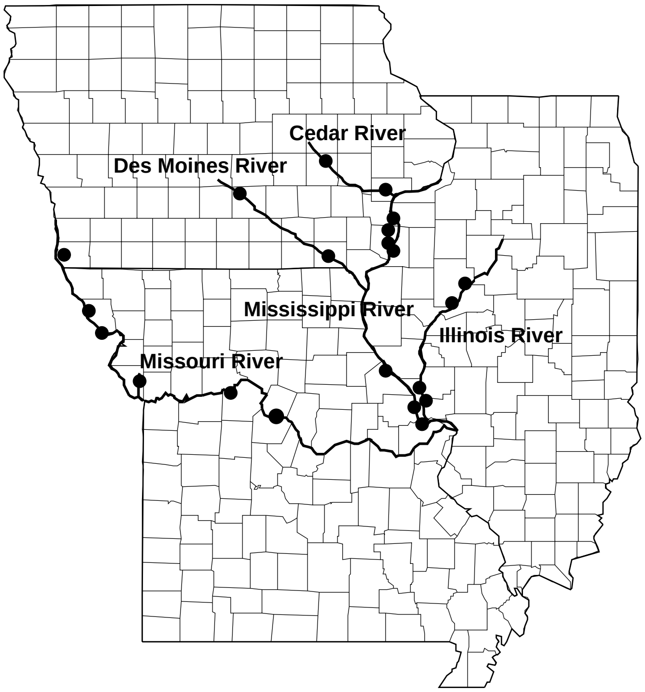

Figure 1. Map of sample locations along the Mississippi, Missouri, Illinois, and Des Moines rivers in Missouri, Iowa, and Illinois.

ing agencies that assisted with the data collection included the Missouri Department of Conservation, the Illinois Department of Natural Resources, the Iowa Department of Natural Resources, the USDA Forest Service-State and Private, and the Amana Society Forestry. The three state agencies identified forested sites on publicly accessible lands, which were flooded in 1993. Eight sites in Missouri, six in Illinois, and seven sites in Iowa were found to be acceptable, yielding a total of 21 sites sampled (Figure 1). The samples were designed to take into account spatial variation within riparian forests. Each plot was a cluster of 11 1/20th ac circular subplots arranged in a half circle with "spokes" every 45˚ (Figure 2).

Thirteen species had sufficient numbers of observations to analyze their height-diameter relationships. These included boxelder (*Acer negundo* L.); silver maple (*Acer saccharinum* L.); sycamore (*Plantanus occidentalis* L.); eastern cottonwood (*Populus deltoidies* Bartr. Ex. Marsh); pin oak (*Quercus palustris* Muenchh); black willow (*Salix nigra* Marsh.); American elm (*Ulmus americana* L.); hackberry (*Celtis occidentalis* L.); sugarberry (*Celtis laevigata* Willd.); green ash (*Fraxinus pennsylvatica* Marsh.); white ash (*Fraxinus*

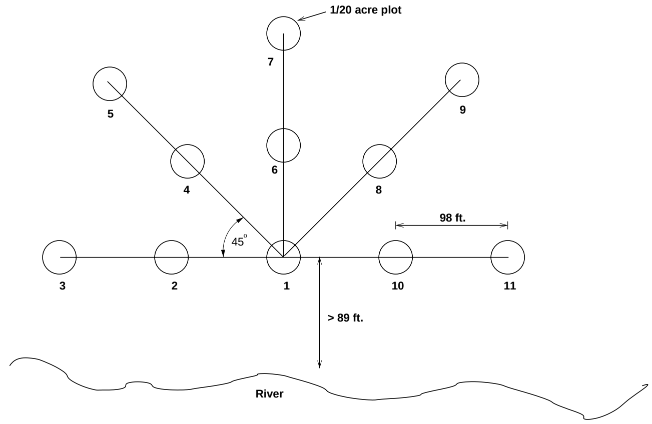

Figure 2. Illustration of the subplot layout used in this study.

americana L.); red mulberry (*Morus rubra* L.); and white mulberry (*Morus alba* L.). For analysis, we grouped members of the following genera: *Celtis*, *Fraxinus*, and *Morus* due to the similarity of species and the small number of observations in some species. Data are summarized by mean, standard deviation, minimum, and maximum for each species group in Table 1.

Initially, Equation (1) was transformed and fit to the data using linear regression analysis. Coefficient b_2 was fixed at –0.2 using the equation:

$$\ln(ht - bh) = b_o + b_1 D^{-0.2} \qquad (2)$$

The resulting values for b_0 and b_1 were used as starting points for a nonlinear fit of Equation (1). All equations were evaluated using Residual standard error and pseudo-coefficient of multiple determination (R^2). Because nonlinear fitting methods do not produce sum of squares, the following procedures were used. To compute the sum of squares for a fit, a residual was calculated for each observation in the dataset. From these residuals, the regression and total sum of squares were calculated, and then the R^2 calculated as in Equation (3).

$$R^2 = 1 - \left(\frac{SSR}{SSTO} \right) \qquad (3)$$

where *SSR* is the regression sum of squares and *SSTO* is the sum of squares total.

This procedure is labeled *pseudo* to indicate that the sum of squares is from a post-fitting ad hoc procedure, not the

Table 1. Summary of dbh (in.) and height (ft) statistics for each species group.

		Dbh (in.)				Height (ft)			
Species group	*N*	Mean	SE	Min	Max	Mean	SE	Min	Max
Boxelder	146	4.3	0.0308	0.1	22.0	25.2	0.1273	6.5	75.4
Silver maple	823	10.9	0.0103	0.3	38.7	58.5	0.0394	6.5	141.1
Sycamore	18	11.1	0.3722	2.6	23.3	63.6	1.4277	19.9	101.7
Eastern cottonwood	224	17.2	0.0277	1.9	41.1	97.8	0.1254	7.0	147.6
Pin oak	122	10.5	0.0557	0.6	30.3	60.7	0.2434	6.5	114.8
Black willow	66	6.3	0.1061	0.4	24.3	35.3	0.4939	6.5	105.0
American elm	222	4.9	0.0176	0.3	25.2	31.8	0.0874	6.5	98.4
Hackberry	310	2.8	0.0096	0.1	19.5	20.3	0.0480	6.5	85.3
Ash	110	9.4	0.0527	0.3	25.4	53.8	0.2518	6.5	144.4
Mulberry	171	3.1	0.0164	0.2	12.4	17.8	0.0678	6.5	72.2

Table 2. Coefficients for the fitted Equation (1) to predict height (ft) from diameter at breast height (in.) for each species group. *RSE* is the residual standard error (ft) and the R^2 is a puedo-R^2 as described in the text.

Species group	N	b_0	b_1	b_2	RSE	R^2
Boxelder	146	5.1328	–3.5461	–0.4298	6.559	0.88
Silver maple	823	5.0704	–3.1207	–0.5272	13.02	0.84
Sycamore	18	4.6653	–5.2094	–1.0108	15.63	0.67
Eastern cottonwood	224	6.4501	–5.5*	–0.7402	16.91	0.63
Pin oak	122	5.6812	–3.9049	–0.3965	10.42	0.88
Black willow	66	4.5535	–3.7529	–0.9168	13.00	0.84
American elm	222	5.7940	–4.1352	–0.3485	8.00	0.83
Hackberry	310	6.2865	–4.4757	–0.2702	5.62	0.85
Ash	110	5.2309	–3.7257	–0.5013	12.71	0.79
Mulberry	171	45.2985	–43.6193	–0.0199	5.24	0.80

* This parameter was constrained to improve the general behavior of the equation.

original fitting procedure. For consistency, all reported fit statistics are generated using this procedure (Larsen and Hann 1987).

Next, all species were fit using nonlinear regression techniques. The values obtained from Equation (2) for b_0 and b_1 were used as starting values to obtain b_0, b_1, and b_2 for each species.

Results

Nonlinear equations of the form of Equation (1) were fit to each of the species. Two species (eastern cottonwood and sycamore) produced an S-shaped (sigmoidal form) height curve. In sycamore equations, the curvature was minimal, and we considered these equations satisfactory for use. Eastern cottonwood equations produced a substantial curve at small heights and diameters. The predicted heights did not increase until diameters were larger than 3 in. This behavior is due to the particular characteristics of this dataset. Because of this, we constrained the equation to predict logical values by fixing the b_1 parameter to –5.5. This produced coefficients, which only slightly reduced the goodness-of-fit, and produced a model with more logical behavior for smaller diameters. For example, using the preferred constrained equation, we would predict a height of 5.3 ft for a 1 in. tree, and 21.2 ft for a 3 in. tree. The unconstrained equation predicted 4.5 and 8.0 ft, respectively, for these two diameters.

The coefficients and fit statistics for the 10 species groups are reported in Table 2. The equations are plotted for each species with the prediction equation as a solid line. Ninety-five percent confidence bounds are plotted as dashed lines along with the data observations (see Figures 3–12). The length of each height-diameter curve indicates the range of the fitted data set (see Table 1). Equations coefficients are suitable for the prediction of tree height within the range of diameters found in the modeling data set. Users should be cautious if they apply the height model outside that range of diameters.

Silver maple is the most common tree species in this dataset and has one of the best regression fits, an R^2 of 0.84. Other equations with R^2 of 0.80 or greater include boxelder, pin oak, black willow, American elm, hackberry, and mulberry. The regression coefficients for mulberry are different relative to the other species because of the limited diameter

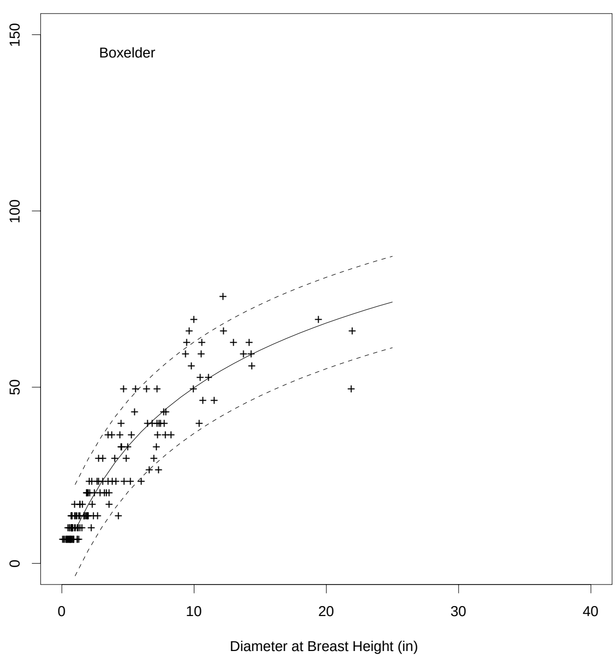

Figure 3. Height-diameter curve for boxelder. The prediction equations are drawn as a solid line, the 95% confidence limit is drawn as a dashed line and on the observed data.

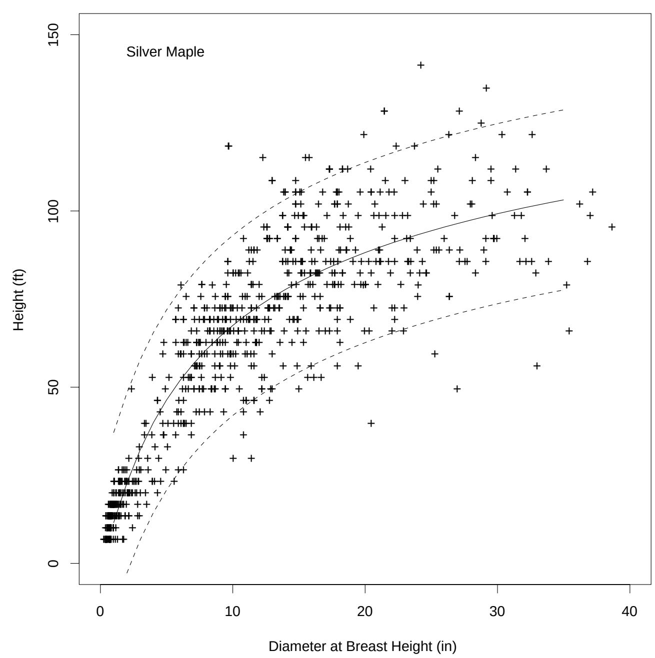

Figure 4. Height-diameter curve for silver maple. The prediction equations are drawn as a solid line, the 95% confidence limit is drawn as a dashed line and on the observed data.

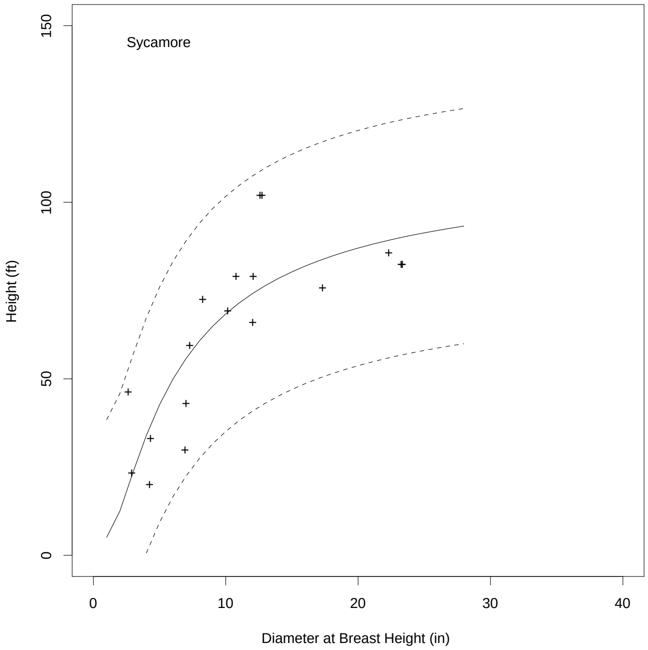

Figure 5. Height-diameter curve for sycamore. The prediction equations are drawn as a solid line, the 95% confidence limit is drawn as a dashed line and on the observed data.

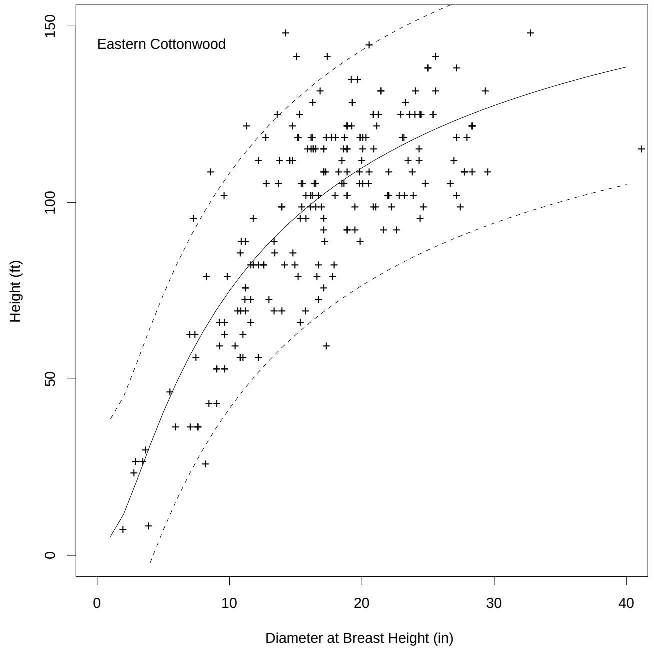

Figure 6. Height-diameter curve for eastern cottonwood. The prediction equations are drawn as a solid line, the 95% confidence limit is drawn as a dashed line and on the observed data.

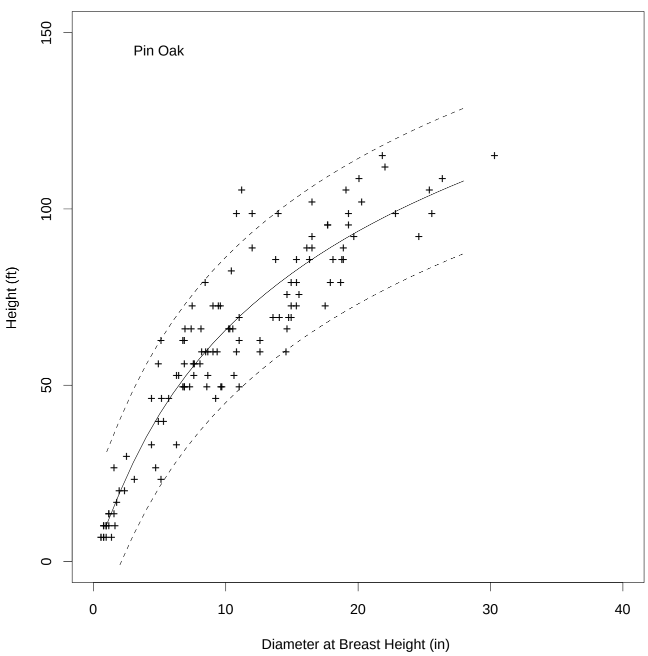

Figure 7. Height-diameter curve for pin oak. The prediction equations is drawn as a solid line, the 95% confidence limit is drawn as a dashed line and on the observed data.

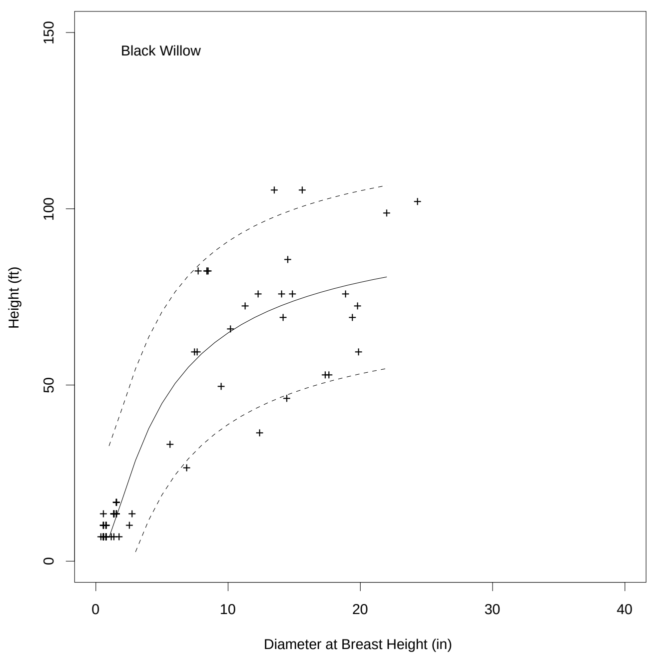

Figure 8. Height-diameter curve for black willow. The prediction equations are drawn as a solid line, the 95% confidence limit is drawn as a dashed line and on the observed data.

range of observed values, making the relationship between height and diameter very nearly linear over the observed diameter range. Sycamore and eastern cottonwood both have R^2 values less than 0.70. Note that sycamore is the smallest dataset with only 18 observations, a very small dataset on which to base an equation. The form of the equation is logical, and it is included for use at the readers' discretion. Eastern cottonwood, as described earlier, had some parts of the range of data with few or no observations and other parts with abundant observations. The structure forced the constraint of the b_1 parameter to yield coefficients that are consistent with the other species.

Conclusions

Allometric equations are rare for bottomland hardwood species. The described procedure produced height-diam-

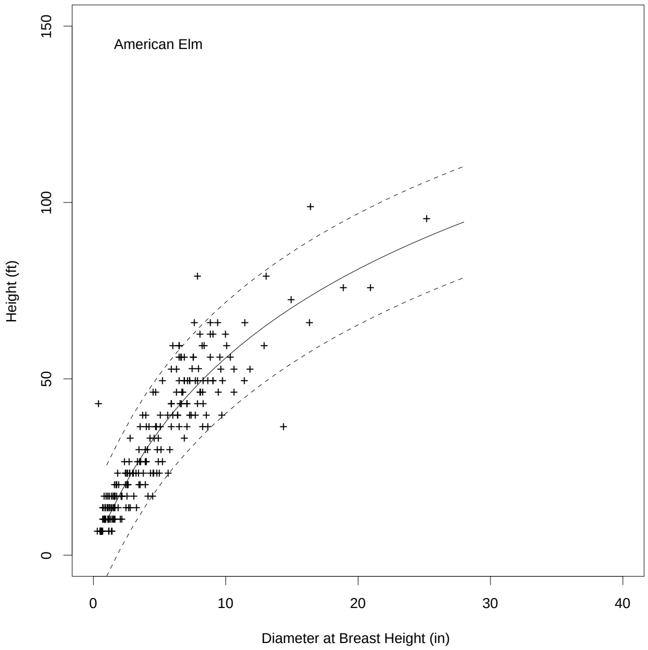

Figure 9. Height-diameter curve for American elm. The prediction equations are drawn as a solid line, the 95% confidence limit is drawn as a dashed line and on the observed data.

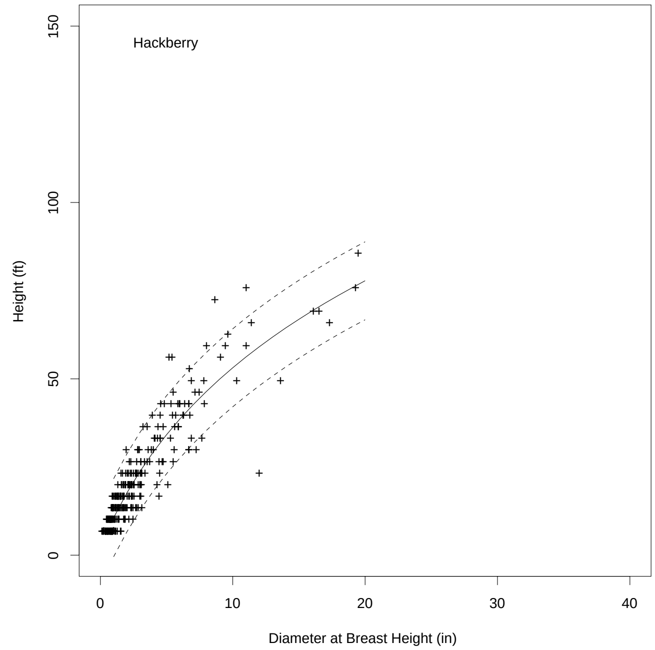

Figure 10. Height-diameter curve for hackberry. The prediction equations are drawn as a solid line, the 95% confidence limit is drawn as a dashed line and on the observed data.

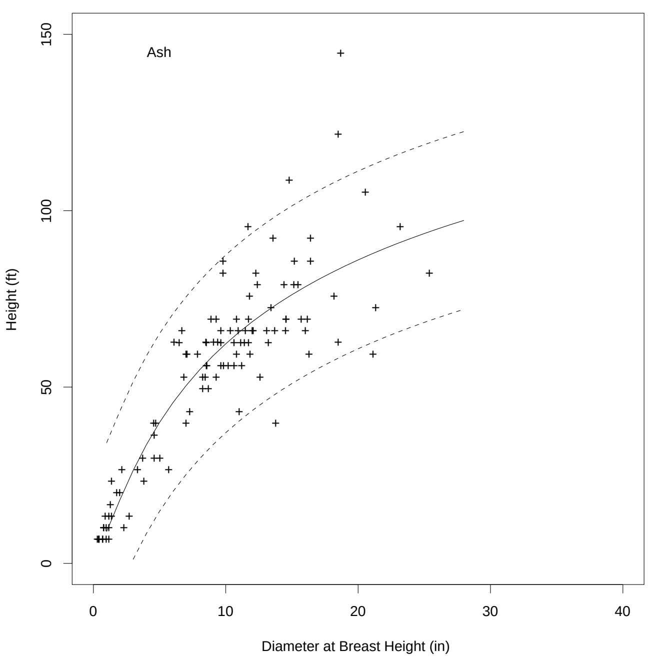

Figure 11. Height-diameter curve for ash. The prediction equations are drawn as a solid line, the 95% confidence limit is drawn as a dashed line and on the observed data.

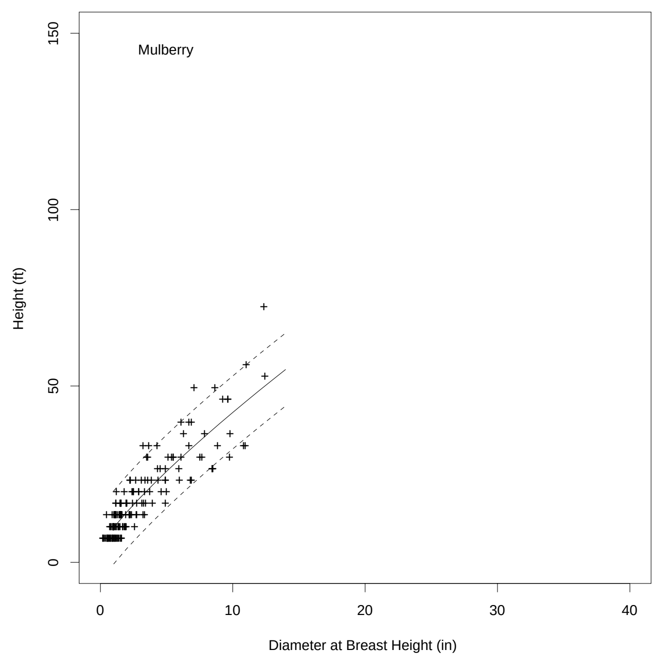

Figure 12. Height-diameter curve for mulberry. The prediction equations are drawn as a solid line, the 95% confidence limit is drawn as a dashed line and on the observed data.

eter equations that are consistent with biological expectations for each species group. The estimated coefficients were consistent in sign and magnitude with results reported previously for other species (Larsen and Hann 1987). This study used a model form that is easy to apply, is consistent among species in form, and is generally reasonable for extrapolation. These are important features in equations that predict average relationships, such as height-diameter equations. The species presented here are from riparian forests associated with major rivers in the Midwestern United States. They represent the flood plain forests of the Missouri, Mississippi, Illinois, and Des Moines rivers in Missouri, Iowa, and Illinois. Forest managers should find these relationships useful and informative in describing this underutilized, but highly productive forest type.

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