New Models for Predicting Diameter at Breast Height from Stump Dimensions

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ABSTRACT

Models to predict dbh from stump dimensions are presented for 18 species groups. Data used to fit the models were collected across thirteen states in the northeastern United States. Primarily because of the presence of multiple measurements from each tree, a mixed-effects modeling approach was used to account for the lack of independence among observations. The heterogeneous error variance was described as a function of stump diameter, which allowed for more accurate representation of prediction intervals. Application of the mean response model (fixed-effects parameters only) to independent data indicated an average absolute error between 0.2 and 0.7 in. for most groups. An additional advantage is that random-effect parameters allow the model to be calibrated to local conditions if some additional data are available. An example is provided that indicates the local calibration results in a mean residual value that is closer to zero compared with the mean response model. Efforts in other locales to use stump information to inform dbh predictions can obtain the same advancements by adopting a similar modeling methodology.

Keywords: mixed-effects model, stand reconstruction, heterogeneous variance, prediction interval

There are various reasons for reconstruction of sizes of removed trees, including reviewing harvesting practices, assessing damage due to catastrophic events, creating historical records of past management activities, and establishing loss due to timber theft (Wharton 1984, Corral-Rivas et al. 2007). The need to estimate the attributes of removed trees for which no readily usable information is available has led to numerous studies in which tree stump dimensions are used to predict tree characteristics. Most commonly, the relationships between stump characteristics and tree dbh are described via regression models. Most early works consisted of development of lookup tables or ordinary least-squares (OLS) linear models that were limited to species of high commercial importance (Cunningham et al. 1947, Hampf 1957, Bones 1960). Wider ranges of tree species were accommodated in later works. In addition to stump diameter, McClure (1968) used a log transformation of stump height to develop linear regression models for 53 species in the southern United States. A similar methodology was used by Alemdag and Honer (1977) to describe stump/dbh relationships for 11 species in Canada and by Raile (1978) to present linear models for dbh/stump diameter ratios for more than 20 species occurring in the Lake States (Minnesota, Wisconsin, and Michigan). Relationships between stump diameter and dbh for seventeen species in the northeastern United States were described via OLS linear regression models by Wharton (1984). Other research on prediction of dbh using stump information includes simple OLS linear regression models for lodgepole pine (Schlieter 1986), linear and geometric models for white and black oak in Michigan (Ojasvi et al. 1991), a linear model with a heteroscedastic variance function for baldcypress (Parresol 1993), and nonlinear models using stump diameter as a predictor for southern Indiana oaks (Weigel and Johnson 1997). Recently, Corral-Rivas et al. (2007) developed both linear and nonlinear models to predict dbh from stump diameter for five pine species in central Mexico. It is notable that relatively little work on prediction of dbh from stump dimensions has been done over the last two decades.

The most flexible prediction models were those that used both stump diameter and stump height (McClure 1968, Raile 1978). The models were calibrated using least-squares regression analysis, despite the violation of the assumption of independence of observations (multiple data points were taken from each stump or tree). The proper treatment of correlated observations is necessary to avoid bias in variance estimates (Swindel 1968, Sullivan and Reynolds 1976). Advances in statistical theory now allow for appropriate treatment of the data in the model fitting process. Particularly, correlations between observed data points can be taken into account such that unbiased model error estimates can be obtained (Gregoire et al. 1995). In this report, models that predict dbh from stump dimensions are presented. A mixed-effects modeling approach was taken to account for the within-tree correlations in the data (Garber and Maguire 2003, Trincado and Burkhart 2006). The mixed-model approach was further exploited to illustrate how the models may be refined for a particular area of interest.

Methods

Data

The data used in this study were collected by the Forest Inventory and Analysis (FIA) program of the US Forest Service as part of a tree taper study. The geographic range encompassed 13 states in the northeastern United States, including West Virginia, Maryland, Delaware, New Jersey, Pennsylvania, Ohio, New York, Massachusetts, Rhode Island, Connecticut, Vermont, New Hampshire, and Maine. Data were collected assuming species groupings described by Scott (1981), which are also used by FIA for tree volume estimation (implying similar tree form). The groupings represent a compromise between individual species and broad aggregations (e.g., hardwood and softwood). Tree species frequency and tree size information

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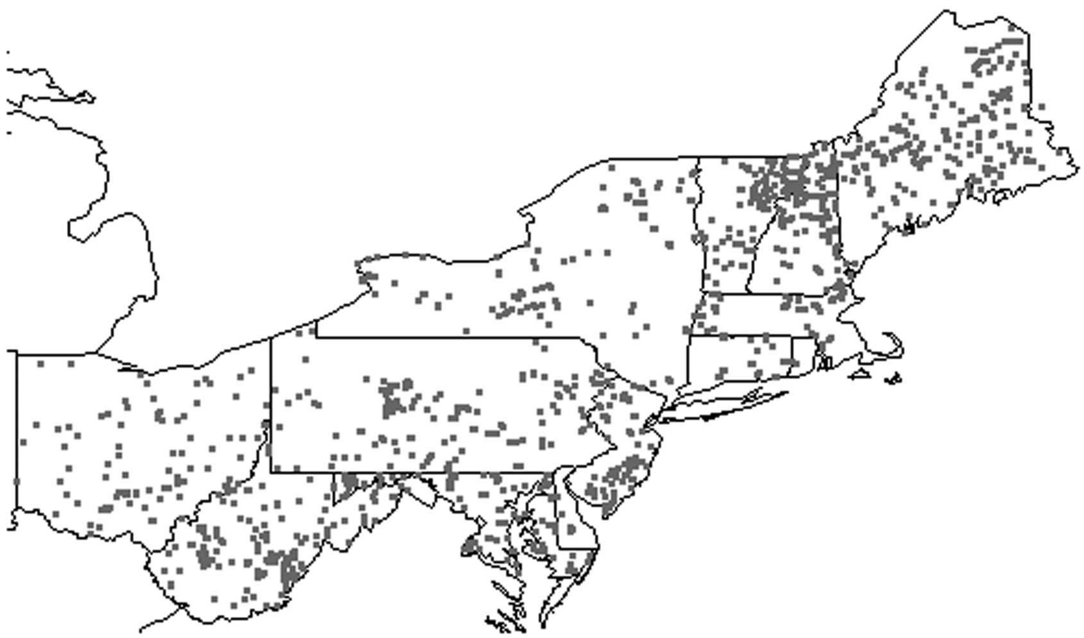

Figure 1. Spatial distribution of data collection locations across 13 states in the United States.

obtained from FIA inventory plots were used to allocate the sample. Geographic dispersion and elimination of the potential need to account for correlations between trees was accomplished by measuring only one tree per species group at a given sampling location. Figure 1 depicts the spatial distribution of sample locations.

Measurements were taken on a total of 2,464 trees over a range of tree species and sizes (Table 1). Height/diameter information was obtained using a Barr & Stroud dendrometer. Measurement points occurred at heights of approximately 1, 2, and 3 ft, where heights were recorded to nearest 0.1 ft and tree diameters were recorded to nearest 0.1 in.; dbh was measured at 4.5 ft of height. There were 7,371 height/diameter data pairs obtained.

Analysis

To predict dbh from stump measurements, there are three primary sources of information: (1) species, (2) the diameter of the stump (*d*), and (3) the height at which the diameter *d* occurs (*h*). The latter two pieces of information, along with the fact that dbh is measured at a height of 4.5 ft, can be used to describe the relationship between the stump dimensions and dbh for a given species or species group. For this study, two measures of distance between the stump height (*h*) and 4.5 ft were considered. First, the ratio of 4.5/*h* provides a relative metric of how far *h* is from the point where dbh is measured. Also, the difference $(4.5 - h)$ was calculated to represent the absolute distance between the two points of interest on the tree bole. These two variables, in addition to stump diameter, were used to specify the prediction model,

$$\widehat{\text{dbh}}_i = d_i \times (4.5/h_i)^{\hat{\beta}_0} + \hat{\beta}_1(4.5 - h_i) + \varepsilon_i, \qquad (1)$$

where dbh_i= estimated diameter breast height (in.) for tree *i*; d_i = stump diameter (in.) for tree *i*; h_i = stump height (ft) for tree *i*; $\hat{\beta}_0$, $\hat{\beta}_1$ = estimated fixed-effects parameters; and ε_i = random error for tree *i*.

Equation 1 is conditioned such that $\text{dbh}_i = d_i$ when $h_i = 4.5$ ft. To account for the multiple observations per tree, random-effects parameters were added to Equation 1 to indirectly estimate the within-tree correlations using the variance/covariance matrix of the model parameters.

During model development, it was also noted that the error variance increased with increasing stump diameter (surprisingly, the magnitude of error was unrelated to stump height). To ascertain the nature of this increase, the variances of the ε_i for each value of d_i were calculated and analyzed (observed diameters greater than 34 in. were grouped into 0.5-in. classes). The nonlinear trend was accounted for via the error variance formulation given in Equation 3.

$$\text{dbh}_i = d_i \times (4.5/h_i)^{\hat{\beta}_0 + \theta_{1i}} + (\hat{\beta}_1 + \theta_{2i})(4.5 - h_i) + \varepsilon_i \qquad (2)$$

$$\varepsilon_i \sim N(0,\ \hat{\sigma}^2 d_i^{\hat{\beta}_2}), \qquad (3)$$

where $\hat{\beta}_2$ = estimated fixed-effects parameter; $\hat{\sigma}^2$ = estimated model error variance; θ_{hi} = random-effects parameters for tree *i*; $\theta_h \sim N(0, \sigma^2{}_h)$, $h = 1, 2$; and other terms are as previously defined.

Results and Discussion

The regression analyses were conducted separately for each of the 18 species groups. Table 2 reports the estimated values for the fixed-effects parameters (β_{0-2}), model error variance (σ^2), variance of random-effects (σ^2_{1-2}), and covariance between random-effects (σ_{12}) for Equations 2 and 3. All fixed effects parameter estimates were statistically significant ($\alpha = 0.05$) for all groups except the β_1 parameter for three of the species groups (5, 10, and 14). The estimated parameters for β_2 in the error variance function 3 provided an accurate depiction of the relationship between residual variance and stump diameter. Figure 2 shows this relationship evaluated over the entire data set. Also shown is the constant error variance that would have been estimated in a traditional application of OLS regression. Figure 3 depicts the observed correlation between residual variance and stump height, as well as a linear regression through the data points. The slope of the regression line was not statistically different

Table 1. Tree frequency, tree size, and sample size information by species for 18 species groups.

Species group	Species name	No. of trees	dbh (in.)			n
			Minimum	Mean	Maximum	
1	Eastern white pine	102	3.2	12.6	36.7	304
1	Red pine	38	3.8	10.9	19.5	114
2	Black spruce	30	3.5	7.7	15.7	90
2	Red spruce	73	2.8	10.4	22.8	219
2	White spruce	45	3.1	10.0	19.4	135
3	Balsam fir	150	3.4	9.5	21.3	450
4	Eastern hemlock	133	3.4	13.6	33.5	399
5	Norway spruce	12	4.2	10.3	16.1	36
5	Scotch pine	8	3.7	7.8	13.3	24
5	Table Mountain pine	5	7.8	10.5	13.9	15
5	Virginia pine	26	3.1	9.7	16.8	78
5	Jack pine	3	6	8.4	10.3	9
5	Larch (introduced)	7	4.4	11.3	16	21
5	Loblolly pine	30	5.2	10.8	23.5	90
5	Pitch pine	30	3.4	9.8	15.3	89
5	Shortleaf pine	7	6.4	10.6	15.2	21
5	Tamarack (native)	18	4	9.1	14.2	54
6	Atlantic white-cedar	29	5	9.1	14.9	87
6	Eastern redcedar	35	3.5	10.3	21.8	105
6	Northern white-cedar	59	3.3	11.8	30.8	177
7	Sugar maple	138	3.1	13.0	31.4	412
8	Yellow-poplar	132	2.8	11.9	28.9	393
9	Balsam poplar	9	3.8	9.2	13.7	27
9	Bigtooth aspen	26	3.6	10.3	16.8	78
9	Black ash	12	3.9	8.3	15.2	36
9	Eastern cottonwood	7	7.5	15.6	28.9	21
9	Green ash	6	3.5	8.0	15.2	18
9	Quaking aspen	35	3.3	9.4	14.9	103
9	White ash	40	3.6	11.2	30.1	120
10	Black cherry	115	3	11.3	32.2	345
11	Gray birch	13	3.2	6.0	9.6	39
11	Paper birch	42	3.1	8.5	16.3	126
11	River birch	3	5.3	8.6	14	9
11	Sweet birch	41	3.2	9.6	27.3	123
11	Yellow birch	48	3.2	10.6	28.4	144
12	American beech	142	2.8	12.8	35.3	425
13	American basswood	48	3	10.4	28.2	144
13	Basswood	63	3	13.2	29.4	189
13	White basswood	4	5.2	9.6	14.6	12
14	Black oak	39	3.1	13.3	35.2	117
14	Blackgum	14	5.7	10.0	24.2	42
14	Northern red oak	46	5.2	13.9	44.3	136
14	Pin oak	7	6.8	15.0	30.3	21
14	Scarlet oak	23	4.8	12.4	27.3	69
14	Shingle oak	3	5.5	10.8	15.1	9
14	Southern red oak	7	7.8	13.0	17.5	21
14	Sweetgum	14	5.2	10.9	18.9	42
14	Willow oak	2	8	8.2	8.4	6
15	Chestnut oak	132	3.3	13.5	33.6	394
16	Bitternut hickory	19	3.7	9.5	15.3	57
16	Hickory	20	3	14.7	33	58
16	Mockernut hickory	29	3.4	8.6	17.9	87
16	Pignut hickory	36	4.1	9.1	14.4	106
16	Shagbark hickory	41	3.1	9.0	15.9	123
16	Shellbark hickory	1	3.2	3.2	3.2	3
17	American elm	9	3.3	8.1	15.6	27
17	American holly	5	5.3	6.2	7.2	15
17	Ohio buckeye	1	4.1	4.1	4.1	3
17	Black locust	20	3.9	12.8	24.2	60
17	Black walnut	11	9.6	12.5	17.2	32
17	Black willow	4	3.8	14.0	24.4	12
17	Buckeye	4	4.5	9.6	14	11
17	Bur oak	1	9.1	9.1	9.1	3
17	Butternut	1	11.4	11.4	11.4	3
17	Chinkapin oak	1	10.3	10.3	10.3	3
17	Cucumbertree	9	9.2	12.7	16.4	27
17	Elm	11	2.8	8.3	13.5	33
17	Hackberry	2	3	6.1	9.1	6
17	Honeylocust	2	9.4	12.0	14.6	6
17	Magnolia	6	10.8	13.0	15.2	18
17	Slippery elm	4	3.6	9.3	12.5	12
17	Swamp white oak	5	9.6	17.5	25.7	15
17	Sycamore	14	3.9	16.8	31.6	40
17	White oak	35	3.6	15.5	30.6	104
17	Yellow buckeye	1	9.2	9.2	9.2	3
18	Red maple	94	3	13.0	29.9	285
18	Silver maple	27	2.7	12.0	30.9	81
Total		2,464	2.7	11.5	44.3	7,371

Table 2. Estimates (and standard errors) for fixed-effects parameters (β_{0-2}), model error variance (σ^2), variance of random-effects (σ_{1-2}^2), and covariance between random-effects (σ_{12}) from Equations 2 and 3 for 18 species groups.

Species group	β_0	β_1	β_2	σ^2	σ_1^2	σ_2^2	σ_{12}
1	−0.1096 (0.0047)	0.0588 (0.0154)	1.4462 (0.1575)	0.0024 (0.0010)	0.0008 (0.0002)	0.0145 (0.0031)	−0.0003 (0.0005)
2	−0.1334 (0.0062)	0.0740 (0.0134)	2.1841 (0.1557)	0.0005 (0.0002)	0.0023 (0.0004)	0.0071 (0.0012)	−0.0021 (0.0006)
3	−0.1353 (0.0084)	0.1451 (0.0184)	2.4605 (0.1222)	0.0002 (0.0001)	0.0024 (0.0005)	0.0065 (0.0009)	−0.0022 (0.0007)
4	−0.1162 (0.0057)	0.0686 (0.0137)	2.5778 (0.1356)	0.0002 (0.0001)	0.0013 (0.0003)	0.0074 (0.0014)	0.0000 (0.0004)
5	−0.1117 (0.0030)		2.4009 (0.1874)	0.0004 (0.0002)	0.0012 (0.0001)		
6	−0.1631 (0.0103)	0.1517 (0.0305)	2.2783 (0.1585)	0.0005 (0.0002)	0.0040 (0.0009)	0.0157 (0.0034)	−0.0037 (0.0016)
7	−0.1323 (0.0072)	0.2442 (0.0231)	1.8478 (0.1376)	0.0011 (0.0004)	0.0014 (0.0003)	0.0132 (0.0020)	0.0006 (0.0004)
8	−0.0966 (0.0043)	0.0260 (0.0114)	1.7903 (0.1326)	0.0010 (0.0003)	0.0016 (0.0003)	0.0074 (0.0014)	−0.0018 (0.0008)
9	−0.1074 (0.0064)	0.0685 (0.0148)	1.9470 (0.1657)	0.0008 (0.0003)	0.0024 (0.0004)	0.0091 (0.0016)	−0.0007 (0.0005)
10	−0.0720 (0.0039)		2.1169 (0.1677)	0.0008 (0.0003)	0.0024 (0.0004)		
11	−0.1743 (0.0075)	0.1376 (0.0170)	1.7486 (0.1622)	0.0022 (0.0008)	0.0024 (0.0006)	0.0060 (0.0020)	−0.0002 (0.0009)
12	−0.1171 (0.0055)	0.0714 (0.0143)	2.1741 (0.1358)	0.0006 (0.0002)	0.0015 (0.0003)	0.0097 (0.0019)	−0.0006 (0.0005)
13	−0.1193 (0.0078)	0.1009 (0.0261)	1.7150 (0.1319)	0.0013 (0.0004)	0.0014 (0.0003)	0.0076 (0.0015)	0.0000 (0.0005)
14	−0.1651 (0.0038)		2.4990 (0.1610)	0.0004 (0.0002)	0.0021 (0.0002)		
15	−0.1329 (0.0082)	0.0659 (0.0224)	1.6448 (0.1551)	0.0022 (0.0008)	0.0026 (0.0005)	0.0133 (0.0032)	−0.0023 (0.0011)
16	−0.1578 (0.0055)	0.0615 (0.0136)	1.8147 (0.1755)	0.0009 (0.0004)	0.0007 (0.0003)	0.0087 (0.0029)	0.0018 (0.0003)
17	−0.1662 (0.0075)	0.1258 (0.0206)	1.4486 (0.1383)	0.0034 (0.0012)	0.0024 (0.0004)	0.0173 (0.0030)	−0.0008 (0.0008)
18	−0.1382 (0.0104)	0.1010 (0.0278)	1.8050 (0.1674)	0.0019 (0.0008)	0.0015 (0.0004)	0.0151 (0.0036)	0.0002 (0.0006)

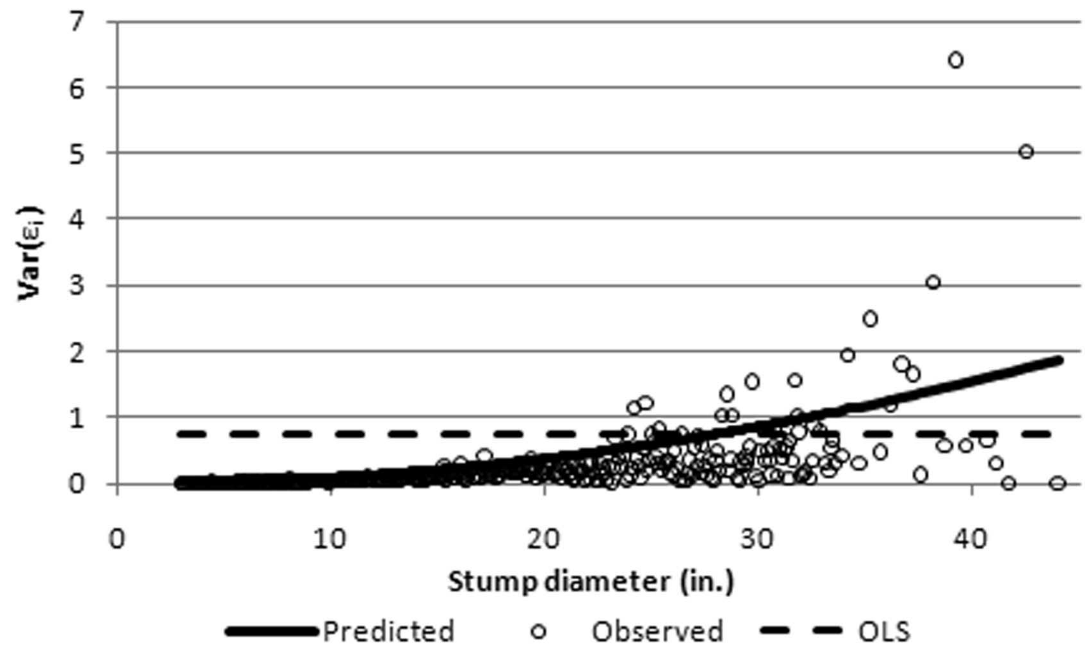

Figure 2. Observed and modeled error variance versus stump diameter across all species groups. Note: Observed diameters greater than 34 in. were grouped into 0.5-in. classes. OLS, ordinary least-squares.

from zero ($P = 0.71$). The variances of the random effects were also significant across all groups, indicating that there were inherent between-tree differences in stump dimension/dbh relationships within species groups. Examination of residuals versus predicted values indicated no systematic trends that would indicate model mis-specification.

Given that application of Equation 2 will be to predict dbh for new observations, prediction error intervals are of primary interest. Quantifying model error can be accomplished through Equation 3. In this setting, random-effects parameters are assigned their expected value of zero. Prediction intervals were computed from (Draper and Smith 1981)

$$\widehat{\text{dbh}}_i \pm t_{n-p,0.025}\sqrt{\sigma^2 \hat{\ell}_i^{\hat{\beta}_2}\,[1 + x_i'(X'X)^{-1}x_i]}, \quad (4)$$

where n = sample size; p = number of estimated parameters; $t_{n-p,\,0.025}$ = t statistic ($n - p$ degrees of freedom, probability $\alpha/2 = 0.025$); X = regression design matrix for fixed-effects parameters; and x_i = vector from design matrix associated with tree i.

As an example, the results for group 16 (hickory) were used to illustrate how the prediction error changes with stump diameter (Figure 4). The model predictions assumed a stump height of 1 ft, and the interval width represents the 95% confidence level. It is shown that the interval is relatively small when tree size is small, e.g., the interval is roughly ± 0.4 in. when dbh is estimated to be 9 in. In contrast, the interval increases to ± 1.9 in. when dbh is estimated to be 30 in. Note that this differs from the classical prediction intervals, where the interval is smallest at the mean of the predictor variable(s) and widens elsewhere (Figure 4).

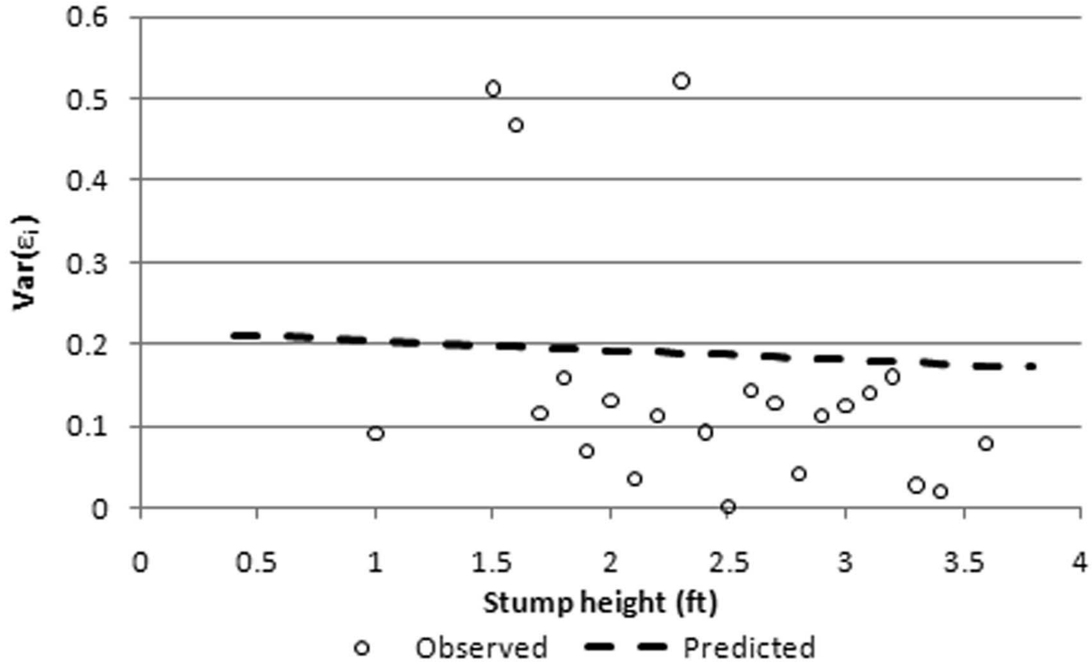

Figure 3. Observed error variance versus stump height and linear trend line across all species groups. Note: The slope of the trend line was not different from zero (P = 0.71).

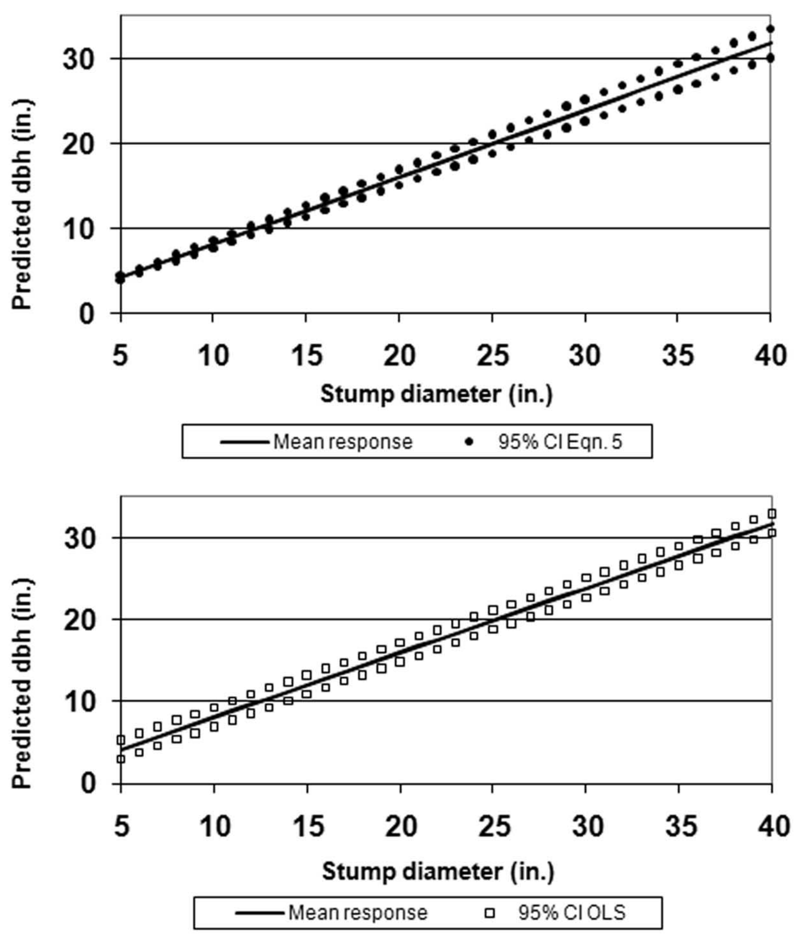

Figure 4. Mean response and 95% confidence intervals for prediction of new observations using variance from Equation 5 and using variance from ordinary least-squares (OLS) (hickory; species group 16).

Table 3. Means (and standard deviations) for raw residual, mean absolute residual, and mean relative residual from mean response model applied to independent data from seven national forests.

Species group	n	Mean residual	Mean absolute residual	Mean relative residual
1	160	0.077 (0.460)	0.319 (0.338)	0.006 (0.038)
2	19	0.057 (0.279)	0.236 (0.149)	0.008 (0.033)
3	12	−0.212 (0.518)	0.310 (0.462)	−0.020 (0.047)
4	8	−0.751 (1.012)	0.897 (0.866)	−0.062 (0.074)
5	112	0.300 (0.353)	0.355 (0.298)	0.035 (0.039)
6	53	−0.463 (0.878)	0.518 (0.846)	−0.060 (0.107)
7	213	−0.393 (1.148)	0.648 (1.025)	−0.036 (0.087)
8	46	−0.025 (0.946)	0.646 (0.685)	−0.002 (0.063)
9	63	−0.393 (0.951)	0.588 (0.842)	−0.034 (0.077)
10	46	0.017 (0.850)	0.577 (0.619)	−0.002 (0.046)
11	68	0.162 (0.555)	0.426 (0.387)	0.012 (0.046)
12	47	−0.186 (1.106)	0.657 (0.903)	−0.014 (0.072)
13	25	−0.528 (1.707)	1.160 (1.343)	−0.032 (0.113)
14	99	0.483 (1.224)	0.917 (0.940)	0.025 (0.064)
15	21	−0.009 (0.672)	0.513 (0.420)	0.000 (0.040)
16	26	0.177 (0.778)	0.567 (0.550)	0.011 (0.053)
17	41	−0.065 (1.034)	0.646 (0.803)	−0.003 (0.066)
18	88	0.355 (0.572)	0.497 (0.452)	0.028 (0.045)

Most commonly, the model will be applied by setting the random-effects parameters to zero (mean response model). To evaluate model performance under this scenario, independent data from studies conducted on seven national forests were used. Forests within the region where the model fitting data were collected included the Monongahela, Green Mountain, White Mountain, and Allegheny national forests. To obtain more data for some of the species groups, some forests outside the region were also included (Hiawatha, Hoosier, and Chequamegon-Nicolet national forests). Table 3 provides the means and standard deviations for raw residual, mean absolute residual, and mean relative residual (residual/dbh) by species group.

These results show the mean residual is generally within ± 0.5 in., although eastern hemlock (group 4) is somewhat higher. Mean absolute residuals ranged between approximately 0.2 to 0.7 in. for all groups except hemlock (group 4), basswood (group 13), and oak (group 14). These results compare favorably with those for the same region reported by Wharton (1984), in which mean error often exceeded 1.0 in. The mean relative residuals indicate that the average amount of error should be less than 4% of the true dbh for most groups. The percentage errors were somewhat higher for eastern hemlock (group 4) and cedar (group 6), having average values of −6.2 and −6.0%, respectively. Overall, eastern hemlock appeared to have the poorest predictions; however, there were only eight observations available for testing. Additional data are needed to better ascertain the quality of model predictions for this species.

At this point, it should be noted that similar predictive ability would be obtained from a model development strategy that ignored the correlated observations and heterogeneous variance (e.g., OLS). Estimates of fixed-effect model coefficients remain unbiased even when these factors are unaccounted for. However, the estimates of model error will be biased and associated statistics, such as significance tests for parameter estimates and confidence/prediction intervals, will be unreliable.

An alternative approach to implementation is to obtain predictions of random-effects for new observations using additional information collected where the model will be applied. For instance, Lappi (1991) calibrated a height/diameter model from local height and diameter measurements. Trincado and Burkhart (2006) describe how to localize a taper model by using upper-stem measurements for the trees of interest. The most likely approach to local calibration is measurement of a "stump" diameter/height and dbh from several nearby trees for each species group present. In this case, a set of random coefficients applicable at the stand level can be obtained via (Vonesh and Chinchilli 1997)

$$\theta = DZ'(ZDZ' + R)^{-1}(y - Xb), \qquad (5)$$

where θ = vector of predicted random-effect parameters; B = regression design matrix for random-effects parameters; F = matrix of partial derivatives of Equation 2 with respect to each fixed parameter (β_{0-1}) evaluated at d_i, h_i for each calibration tree; $Z = FB$; R = predicted variance/covariance matrix of residual errors; D = predicted variance/covariance matrix of random-effects (from Table 2); y = vector of observed tree dbh values; X = regression design matrix for fixed-effects parameters; and b = vector of fixed-effects parameters (from Table 2).

Independent data from 31 sugar maple trees (species group 7) were used to illustrate the process. For each tree, stump diameter at stump height = 1 ft and dbh information were available. Sixteen trees were randomly chosen to represent the harvested trees for which predictions of dbh are desired. Fifteen trees were used to estimate the random-effects parameters via Equation 5. The results were $\theta_{71} = 0.0100$ and $\theta_{72} = -0.0687$. Thus, the localized prediction model for sugar maple is given as

$$\mathrm{dbh}_i = d_i \times (4.5/h_i)^{(-0.1158+0.0100)} + (0.1047 + (-0.0687))(4.5 - h_i) + \varepsilon_i. \qquad (6)$$

Applying Equation 6 to the 16 harvest trees shows that the distribution of the residuals is shifted to be better centered about zero, compared with the residuals resulting from applying the nonlocalized mean model (Figure 5). The local calibration does result in poorer predictions for some trees, but better estimates of dbh for the entire sample are gained; the mean residual for the mean model (fixed-effects only) was −0.16 in., whereas the mean residual for the locally calibrated model was −0.04 in.

Conclusion

The models presented differ from previous efforts to estimate dbh from stump dimensions in three ways. First, the inclusion of random-effects parameters allows for unbiased estimates of error variance, which directly affect inferences regarding estimated parameters and estimation of confidence/prediction intervals. Second, the ability to locally calibrate the model provides an alternative to using the mean response over an often large geographic area. Third, the heterogeneous error variance was described as a function of stump diameter, which allows for more realistic prediction intervals than those based on an (often invalid) assumption of homogeneous error. This represents marked improvements over earlier efforts, in which such features were lacking.

The models are applicable to many species occurring in the northeastern United States. Given that many of these species also occur outside the area used in this study, the model may be used elsewhere. However, it is recommended that local calibration be performed if possible, and if not, considerable caution should be exercised, as unknown biases may produce inaccurate predictions. Ultimately, the most attractive option for other geographic areas would be to collect data and adopt a similar modeling methodology.

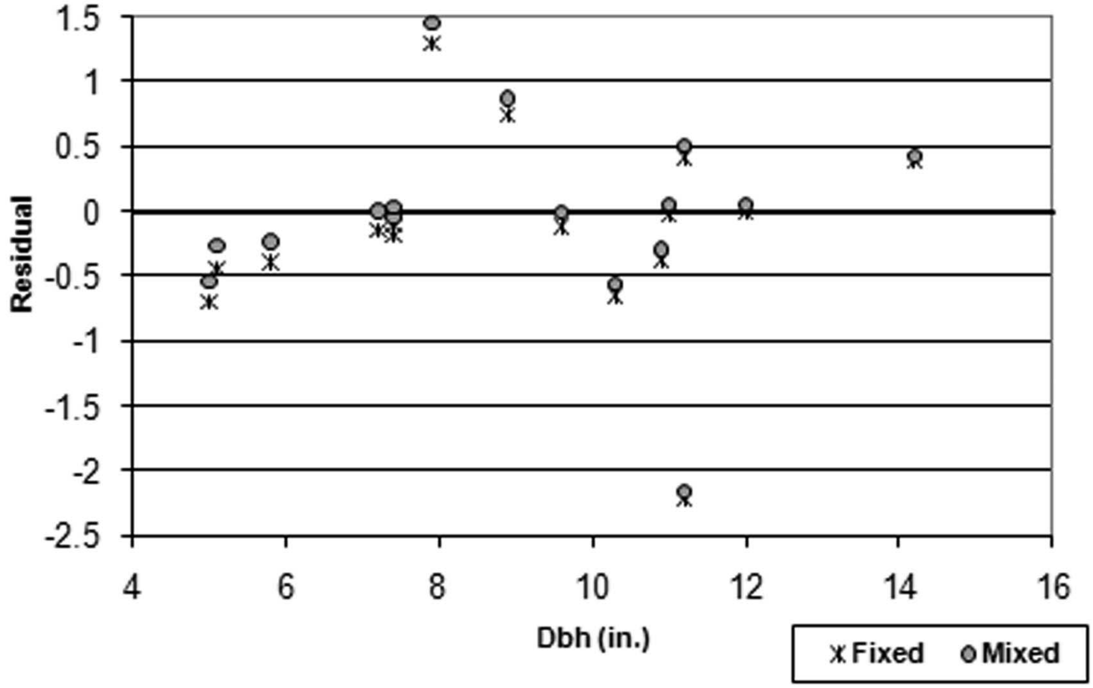

Figure 5. Comparison between residuals from mean response model (fixed-effects only) and locally calibrated mixed-effects model for 16 sugar maple trees (group 7).

The predicted values may be used to further construct the missing trees (e.g., via height-diameter models) or as input into volume or biomass models that the user may wish to use.

Literature Cited

ALEMDAG, I.S., AND T.G. HONER. 1977. *Metric relationships between breast-height and stump diameters for eleven tree species from eastern and central Canada.* Can. For. Serv., Forest Management Institute Inf. Rep. FMR-X49M. 62 p.

BONES, J.T. 1960. *Estimating d.b.h. from stump diameter in the pacific northwest.* US For. Serv. Res. Note PNW-186. 2 p.

CORRAL-RIVAS, J.J., M. BARRIO-ANTA, O.A. AGUIRRE-CALDERON, AND U. DIEGUEZ-ARANDA. 2007. Use of stump diameter to estimate diameter at breast height and tree volume for major pine species in El Salto Durango (Mexico). *Forestry* 80:29–40.

CUNNINGHAM, F.E., S.M. FILIP, AND M.J. FERREE. 1947. *Relation of tree-stump diameter to diameter breast height.* US For. Serv. Stat. Note No. 1. 3 p.

DRAPER, N.R., AND H. SMITH. 1981. *Applied regression analysis*, 2nd ed. John Wiley & Sons, New York. 709 p.

GARBER, S.M., AND D.A. MAGUIRE. 2003. Modeling stem taper of three central Oregon species using nonlinear mixed effects models and autoregressive error structures. *For. Ecol. Manag.* 179:507–522.

GREGOIRE, T.G., O. SCHABENBERGER, AND J.P. BARRETT. 1995. Linear modelling of irregularly spaced, unbalanced, longitudinal data from permanent-plot measurements. *Can. J. For. Res.* 25:137–156.

HAMPF, F.E. 1957. *Relationship of stump diameter to d.b.h. for white oak in the Northeast.* US For. Serv. Res. Note No. 66. 3 p.

LAPPI, J. 1991. Calibration of height and volume equations with random parameters. *For. Sci.* 37:781–801.

MCCLURE, J.P. 1968. *Predicting tree d.b.h. from stump measurements in the Southeast.* US For. Serv. Res. Note SE-99. 4 p.

OJASVI, P.R., C.W. RAMM, D.O. LANTAGNE, AND J. BRUGGINK. 1991. Stump diameter and DBH relationships for white oak and black oak in the Lower Peninsula of Michigan. *Can. J. For. Res.* 21:1596–1600.

PARRESOL, B.R. 1993. Modeling multiplicative error variance: An example predicting tree diameter from stump dimensions in baldcypress. *For. Sci.* 39:670–679.

RAILE, G. 1978. Estimating d.b.h. from stump dimensions. P. 30–33 in *Proc. of 1977 Midwest Forest Mensurationists Meeting.* US For. Serv. Gen. Tech. Rep. NC-46.

SCOTT, C.T. 1981. *Northeastern forest survey revised cubic-foot volume models.* US For. Serv. Res. Note NE-304. 3 p.

SULLIVAN, A.D., AND M.R. REYNOLDS, JR. 1976. Regression problems from repeated measurements. *For. Sci.* 22:382–385.

SCHLIETER, J.A. 1986. *Estimation of diameter at breast height from stump diameter for lodgepole pine.* US For. Serv. Res. Note INT-359. 4 p.

SWINDEL, B.F. 1968. On the bias of some least-squares estimators of variance in a general linear model. *Biometrika* 55:313–316.

TRINCADO, G., AND H.E. BURKHART. 2006. A generalized approach for modeling and localizing stem profile curves. *For. Sci.* 52:670–682.

VONESH, E.F., AND V.M. CHINCHILLI. 1997. *Linear and nonlinear models for the analysis of repeated measurements.* Marcel Dekker, New York. 560 p.

WEIGEL, D.R., AND P.S. JOHNSON. 1997. *Estimating dbh of southern Indiana oaks from stump diameter.* US For. Serv. Tech. Brief TB-NC-4. 4 p.

WHARTON, E.H. 1984. *Predicting diameter at breast height from stump diameters for northeastern tree species.* US For. Serv. Res. Note NE-322. 4 p.