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Optimal control of an invasive species with imperfect information about the level of infestation

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ABSTRACT

The presence of invasive species is often not realized until well after the species becomes established. Discovering the location and extent of infestation before the invasive species causes widespread damage typically requires intensive monitoring efforts. In this paper, we analyze the problem of controlling an invasive species when there is imperfect information about the degree of infestation. We model the problem as a partially observable Markov decision process in which the decision-maker receives an imperfect signal about the level of infestation. The decision-maker then chooses a management action to minimize expected costs based on beliefs about the level of infestation. We apply this model to a simple application with three possible levels of infestation where the decision-maker can choose to take no action, only monitor, only treat, or do both monitoring and treatment jointly. We solve for optimal management as a function of beliefs about the level of infestation. For a case with positive monitoring and treatment costs, we find that the optimal policy involves choosing no action when there is a sufficiently large probability of no infestation, monitoring alone with intermediate probability values and treatment alone when the probability of moderate or high infestation is large. We also show how optimal management and expected costs change as the cost or quality of information from monitoring changes. With costless and perfect monitoring, expected costs are 20–30% lower across the range of belief states relative to the expected costs without monitoring.

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1. Introduction

The movement of people and goods around the globe has increased the movement of species into novel environments often far removed from their place of origin. Some newly introduced species become invasive species that establish and spread because they lack effective competitors, pathogens, or predators to keep the population in check, and cause ecological or economic harm. Harm can occur because an invasive species reduces populations of native species, damages crops or forage (e.g., cheatgrass, *Bromus tectorum*), damages infrastructure (e.g., zebra mussels, *Dreissena polymorpha*, and their impacts on water intake pipes) or causes other forms of damage. Measures of the economic cost of invasive species are imprecise but these costs may be quite large. Some estimates of damage are in the billions of dollars annually (e.g., Lovell et al., 2006; OTA, 1993; Pimentel et al., 2000, 2005). The US government spends over $1 billion dollars annually on invasive species control efforts (US National Invasive Species Council, 2006). Allocating control efforts so that costs associated with invasive species are minimized is an important issue.

In this paper, we analyze the problem of controlling an invasive species when there is imperfect information about the degree of infestation. The presence of invasive species is often not realized until well after the species becomes established (Costello and Solow, 2003). Infestations often start with small populations that may go unnoticed for some time. Detection may not occur until the population has grown larger when it is easier to detect the species through search efforts or because there are observable damages that can be linked to the presence of the species. Even when the presence of a species is detected, the actual extent of the infestation may still be unknown. Optimally controlling an invasive species whose presence or degree of infestation is not known involves choosing a level of monitoring effort to learn about the degree of infestation as well as choosing a level of treatment to reduce or eliminate the invasive species in the environment. Because damage grows as the population of the invasive species grows, there is value to monitoring to be able to spot an invasion early. Because treatment is costly, there is value to monitoring to gain information about the scale of the problem to know when and where to apply treatment. However, because monitoring itself is costly, monitoring should only be done when monitoring costs are low relative to the probability of infestation and expected damages from infestation, and there is effective (but costly) treatment.

We model the problem of controlling an invasive species with imperfect information about the level of infestation as a partially observable Markov decision process (Cassandra, 1994). The process is partially observable because the decision-maker receives a signal about the degree of infestation but this information is imperfect. The signal (information) may be from reported observations of the species, but these reports may contain both false positive and false negative signals. There might also be observed damage to vegetation associated with the presence of an invasive species. However, damages may go unobserved for some time and some observed damage may be due to other causes. Higher infestation levels make it more likely that the decision-maker will get a correct signal of infestation but the signal is still not likely to be perfect, especially with respect to the degree of infestation. The decision problem of controlling an invasive species in this context is a Markov process because the degree of infestation in the next period is a function of the degree of infestation and management actions taken in the current period. The objective of the decision-maker is to try to minimize the sum of discounted costs associated with management (monitoring and treatment), and damages caused by infestation of the invasive species in the environment. Based on the beliefs about the probability distribution about the level of infestation, the decision-maker chooses whether to do nothing (no action), only monitor, only treat, or do both monitoring and treatment jointly. Monitoring improves the information about the degree of infestation while treatment reduces the level of infestation.

The introduction of imperfect information about the degree of infestation increases the complexity of solving for optimal management, as compared to the case with complete information where optimal policy only involves comparisons of benefits and costs of treatment. With incomplete information, the set of decisions includes monitoring and treatment. But optimal monitoring and treatment decisions require an assessment about the likely degree of infestation. Probability distributions of the degree of infestation should incorporate information on management action and probability distributions of the degree of infestation in the prior period, plus any new information

received, to update probabilities. The combination of imperfect information and a dynamic model, in which past states and actions influence the likelihood of current states, requires Bayesian updating (learning) and greatly increases model complexity.

In this paper we show how to apply the partially observable Markov decision process and to solve for an optimal management strategy involving both monitoring and treatment. We describe a partially observable Markov decision process for controlling invasive species in Section 2 and apply it to a specific example in Sections 3 and 4. The optimal management action depends upon the beliefs about the likely state of infestation. For the base case in the application in Sections 3 and 4, we find that the optimal policy involves choosing no action when there is a sufficiently large probability of no infestation, monitoring alone with intermediate probability values and treatment alone when the probability of moderate or high infestation is large. We also show how optimal management and expected costs change as the cost or quality of information from monitoring changes. We calculate the value of information from monitoring by comparing expected costs in a case with perfect and costless monitoring versus a case without monitoring. Expected costs are 20–30% lower across the range of belief states relative to the expected costs obtained without monitoring. The value of monitoring quickly declines as the costs of monitoring rises or imperfection in the information from monitoring increases.

There is a rapidly expanding literature on the economics of invasive species (see Perrings et al., 2000; Lovell et al., 2006; Olson, 2006 for reviews). Prior research on invasive species management includes work on preventing introductions (e.g., Costello and McAusland, 2003; Horan et al., 2002; McAusland and Costello, 2004; Sumner et al., 2005), the optimal control of an existing population of an invasive species (e.g., Eiswerth and Johnson, 2002; Eiswerth and van Kooten, 2002; Olson and Roy, 2002), spatial aspects of controlling the spread of an invasive species (e.g., Brown et al., 2002; Heikkila and Peltola, 2004; Sharov, 2004; Sharov and Liebhold, 1998), and models that combine prevention and control in a unified model (Finnoff and Tschirhart, 2005; Finnoff et al., 2007; Leung et al., 2002, 2005; Olson and Roy, 2005, Polasky, 2010).

Only a few prior papers analyze imperfect information about the status of an invasive species. Costello and Solow (2003) analyze a model in which the probability of observing an invasive species depends on population size, where population grows over time from the date of introduction. Their focus is on describing the pattern of discoveries of invasive species for a constant search rate rather than characterizing optimal management. They show that the observed increased in the number of invasive species may not be the result of increasing rate of invasions but rather a reflection of delayed discovery of past invasions. Mehta et al. (2007) and Polasky (2010) analyze optimal management in models with detection and control that are similar to the present paper. However, both papers make simplifying assumptions and do not use partially observable Markov decision processes or Bayesian updating. Mehta et al. (2007) characterize optimal intensity of search for a single invasive species assuming a known date of introduction and deterministic population growth. Polasky (2010) simplifies the information structure so that at the beginning of each period the manager knows whether the species was present or absent in the prior time period. Monitoring information is valuable only in that it allows observation of an invasion prior to widespread establishment so that treatment costs are lowered. There is also a related literature on decision-making under uncertainty in pest management with several papers that analyze the value of information about pest infestation (Feder, 1979; Swinton and King, 1994, and see King et al., 1998 for a review of empirical papers on value of information in weed management). These models are typically simpler than the context considered here and do not involve learning through Bayesian updating.

2. The model

In this section, we describe the partially observable Markov decision process (POMDP) for controlling an invasive species with imperfect information about the level of infestation. The model described here is quite general including potentially many management actions and levels of infestation. In Section 3, we apply the POMDP to a specific case with four mutually exclusive management actions, including monitoring and treatment, and three possible levels of infestation.

Suppose there are n possible states of the ecosystem representing different levels of infestation, indexed by i, $i=1,\ldots,n$. At the beginning of each period, $t=1,\ldots,T$, the decision-maker does not know for sure the current state of the ecosystem and instead has a set of beliefs represented by a probability distribution $(\pi_{1t},\ldots,\pi_{nt})$, where π_{it} equals the probability of being in state i at the start of period t, $\pi_{it}\geq 0$, $\sum_i \pi_{it}=1$. Each period t, the manager chooses a management action, a, from the feasible management set A, based on the current vector of belief probabilities $(\pi_{1t},\ldots,\pi_{nt})$. Following the management action, the state of the infestation may change, and we define an $n\times n$ transition matrix $\mathbf{P}^a$ where each element p^a_{ij} equals the probability of moving from state i in period t to state j in period $t+1$ after taking action a.

Recognizing that the manager may not observe the true state of the infestation, we define an index $\theta=1,\ldots,n$ for the states that the manager may observe in period $t+1$ after taking an action in period t. Further, we define an $n\times n$ matrix $\mathbf{R}^a$ with elements $r^a_{j\theta}$, $j=1,\ldots,n$ and $\theta=1,\ldots,n$ where each element $r^a_{j\theta}$ is the probability that the manager observes an infestation state θ in period $t+1$ given an actual infestation state j after taking action a in period t, with $r^a_{j\theta}\geq 0$ and $\sum_{\theta=1}^n r^a_{j\theta}=1$. The probability of correctly observing infestation state j is r^a_{jj}. For $\theta\neq j$, $r^a_{j\theta}$ is the probability of observing an incorrect infestation state. The matrix $\mathbf{R}^a$ represents the probability distributions of observations for all of the potential states $j=1,\ldots,n$.

We use the matrix $\mathbf{R}^a$ of probabilities of observing the infestation in different states to update the manager's belief probabilities from period t to period $t+1$. Given an observed state θ after taking action a in period t and the set of prior belief probabilities $(\pi_{1t},\ldots,\pi_{nt})$, Bayes' rule provides an estimate of the updated belief probability, π_{jt+1}, of being in state j period $t+1$:

$$\pi_{jt+1}|\theta,a=\frac{\sum_{i=1}^n \pi_{it}\,p^a_{ij}r^a_{j\theta}}{\sum_{i=1}^n\sum_{k=1}^n \pi_{it}\,p^a_{ik}r^a_{k\theta}} \tag{1}$$

The updated probability of being in state j in period $t+1$, $\pi_{jt+1}|\theta,a$, depends on the observed state θ after taking action a in period t. The updated belief probability is the ratio of the probability of observing state θ given the true state is j (numerator) and the probability of observing state θ over all possible states $k=1,\ldots,n$ (denominator). Without a perfect signal there will be some uncertainty about the true state. If the manager were to always observe the true infestation state, then the observation matrix $\mathbf{R}^a$ would be an identity matrix and there would be no uncertainty about the state.

In our model, the manager faces two types of costs: damage costs and management costs. Let d^a_i be the damage costs for a time period starting in state i and taking management action a. Let m^a_i be the management cost for a time period (e.g., the cost of monitoring and/or treatment) starting in state i and taking management action a.

At the beginning of each period $t=1,\ldots,T-1$, the manager's problem is to determine the best action given the set of beliefs about the infestation state. We define an optimal value function $V_t(\pi_{1t},\ldots,\pi_{nt})$ as the minimum discounted cost of optimal actions beginning period t with belief state $(\pi_{1t},\ldots,\pi_{nt})$ until the terminal period T. With this notation, the manager's problem can be formulated as a discrete-time dynamic program:

$$V_t(\pi_{1t},\ldots,\pi_{nt})=\min_{a\in A}\left[\sum_{i=1}^n \pi_{it}(d^a_i+m^a_i)+\delta\sum_{i=1}^n\sum_{j=1}^n\sum_{\theta=1}^n \pi_{it}\,p^a_{ij}r^a_{j\theta}V_{t+1}((\pi_{1t+1}|\theta,a),\ldots,(\pi_{nt+1}|\theta,a))\right]$$
$$t=1,\ldots,T-1 \tag{2}$$

with terminal condition,

$$V_T(\pi_{1T},\ldots,\pi_{nT})=\min_{a\in A}\left[\sum_i \pi_{iT}(d^a_i+m^a_i)\right] \tag{3}$$

The first term inside the brackets of Eq. (2) is the expected cost in period t of taking action a given the current belief probabilities $(\pi_{1t},\ldots,\pi_{nt})$. The second term inside the brackets of Eq. (2) is the expected discounted cost in period $t+1$ after taking action a in period t, where δ is the discount factor. Expected

discounted cost in period $t+1$ is the weighted sum of the optimal values associated with all possible sets of updated belief states where each weight, $\pi_{it}\, p^a_{ij} r^a_{j\theta}$, represents the probability of beginning in state i, moving to state j, and making observation θ, after taking action a.

We use a discrete, stochastic dynamic programming algorithm to find approximately optimal solutions to the POMDP problem represented by Eqs. (1)–(3). Although numerical procedures based on linear programming have been developed to find exact solutions to POMDP problems (see Monahan, 1982 and Cassandra, 1994 for reviews), these procedures are complicated and require custom software.

The stochastic dynamic programming algorithm works as follows. For computational reasons, we define discrete classes for the belief probability for each infestation state i. Each element π_{it} of the probability distribution of beliefs in period t ($\pi_{1t}, \ldots, \pi_{nt}$) is assigned to one of the discrete probability classes. Increasing the number of classes means we come closer to the probabilities calculated using Bayes' rule (Eq. (1)) and more closely approximate the optimal answer. The dynamic program (Eq. (2)) is solved backward starting from the terminal period T (Eq. (3)). In each period t, the optimal value function $V_t(\pi_{1t}, \ldots, \pi_{nt})$ is calculated for each belief state ($\pi_{1t}, \ldots, \pi_{nt}$) by choosing the management action with the minimum expected discounted cost. To calculate the expected discounted cost of a given management action a, we first calculate the expected cost of the action in period t (first summation inside the brackets in Eq. (2)). Then, we use Bayes' rule (Eq. (1)) to calculate $\pi_{jt+1}|\theta, a$, the updated belief probability of being in state j in period $t+1$ given observation θ after management action a was applied. The updated belief $\pi_{jt+1}|\theta, a$ is then assigned to one of the discrete probability classes. Repeating this procedure for all states $j=1, \ldots, n$, we obtain the updated belief state $((\pi_{1t+1}|\theta, a), \ldots, (\pi_{nt+1}|\theta, a))$. Finally, the optimal values associated with the updated belief states, $V_{t+1}((\pi_{1t+1}|\theta, a), \ldots, (\pi_{nt+1}|\theta, a))$, for all possible state transitions and observations are used to calculate the expected discounted cost starting in period $t+1$ from taking action a in period t (second summation inside the brackets in Eq. (2)).

The backward recursion is solved iteratively from period T to period 1. The result is an optimal policy that gives the best action to take in each period t for each belief state along with the optimal value of that action. Strictly speaking, our dynamic programming algorithm finds approximately optimal rather than exactly optimal solutions because the updated beliefs calculated using Bayes' rule (Eq. (1)) are classified into discrete probability classes. While simple in concept, this dynamic programming algorithm is very time consuming when applied to problems with many system states and actions. In the next section we solve for optimal solutions in the case where there are three possible states of the ecosystem and four mutually exclusive actions.

3. Application

We illustrate the partially observable Markov decision process for controlling an invasive species with imperfect information about level of infestation with a simple application that involves an ecosystem with three possible states: no infestation ($i=1$), moderate infestation ($i=2$), and high infestation ($i=3$). The set of actions, A, includes four mutually exclusive actions: no action ($a=1$), only monitoring ($a=2$), only treatment ($a=3$), and both monitoring and treatment ($a=4$). The state transition matrix depends on whether or not treatment occurs. Without treatment ($a=1$ or $a=2$), the state transition matrix is:

$$\mathbf{P}^1 = \mathbf{P}^2 = \begin{bmatrix} 0.8 & 0.2 & 0.0 \\ 0.0 & 0.8 & 0.2 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

With treatment ($a=3$ or $a=4$), the state transition matrix is:

$$\mathbf{P}^3 = \mathbf{P}^4 = \begin{bmatrix} 0.9 & 0.1 & 0.0 \\ 0.8 & 0.2 & 0.0 \\ 0.6 & 0.4 & 0.0 \end{bmatrix}$$

With choice of either no action or monitoring, there is a 20% chance of the system moving from no infestation to moderate infestation and from moderate to high infestation each year. The system will

remain at high infestation unless there is treatment. With treatment, there will either be no infestation or moderate infestation with probabilities of each state dependent on the initial state of the system. Treatment is not always fully effective as indicated by the fact that elements in the first column of $\mathbf{P}^3$ and $\mathbf{P}^4$ (no infestation) are not all equal to 1 and elements in the second column (moderate infestation) are not all equal to zero.

The observation matrix depends on whether or not monitoring occurs. Without monitoring ($a=1$ or $a=3$), the observation matrix is:

$$\mathbf{R}^1 = \mathbf{R}^3 = \begin{bmatrix} 0.5 & 0.5 & 0.0 \\ 0.3 & 0.4 & 0.3 \\ 0.1 & 0.4 & 0.5 \end{bmatrix}$$

With monitoring ($a=2$ or $a=4$), the observation matrix is:

$$\mathbf{R}^2 = \mathbf{R}^4 = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 1.0 & 0.0 \\ 0.0 & 0.0 & 1.0 \end{bmatrix}$$

With no monitoring, if there is actually no infestation, there is a 50% chance that the decision-maker correctly observes that there is no infestation, and a 50% chance that the decision-maker incorrectly observes a moderate infestation (false positive). For example, sometimes a forester observes dead leaves and incorrectly concludes that the forest is infected with a pest. If there is actually a moderate infestation, the decision-maker has a 30% chance of failing to observe the infestation, a 40% chance of correctly observing the moderate infestation, and a 30% change of incorrectly concluding that it is a high infestation. Finally, if there is actually a high infestation, the decision-maker has a 10% chance of incorrectly observing there is no infestation, a 40% chance of incorrectly observing there is a moderate infestation and a 50% chance of correctly observing a high infestation. With monitoring, we assume that observations return perfect information about the actual state of the ecosystem. We relax the assumption of perfect information from monitoring in Section 4.3 below.

In this application, we assume that damage cost depends on the level of infestation at the start of the period but not the action taken. We assume there are zero costs for no infestation, a cost of 10 units for moderate infestation and a cost of 20 units for high infestation:

$$d_i^1 = d_i^2 = d_i^3 = d_i^4 = \begin{bmatrix} 0 \\ 10 \\ 20 \end{bmatrix}$$

Management costs are the sum of monitoring costs and treatment costs. We assume that both monitoring and treatment costs are same regardless of the state of the ecosystem; they only depend on the action taken. We assume that monitoring costs in a time period are zero units with no monitoring and 4 units with monitoring. Treatment costs in a time period are zero units with no treatment and 20 units with treatment. In general, we could make monitoring or treatment costs a function of the level of infestation, but in this application we assume that there is a single level of monitoring and treatment costs. Adding together monitoring and treatment costs, management costs for this application are: $m_i^1 = 0, m_i^2 = 4, m_i^3 = 20, m_i^4 = 24$, for all i.

Finally, we assume a discount factor, δ, of 0.95 and a time horizon, T, of 20 years. We use a 20-year horizon because we found that the optimal policy was invariant for all but the final few years approaching year 20. We solve the model using a stochastic dynamic programming algorithm that we designed and coded in Fortran. The program is executed on a Lenovo T60 laptop computer with an Intel Core 2 central processing unit. We used 20 discrete probability classes. Solutions to this problem with three infestation states and four actions are obtained in seconds.

4. Results

4.1. Optimal policy in the base case

The optimal policy for the POMDP described above is a state-dependent feedback rule that gives the best action to take in each period t for each belief state $(\pi_{1t}, \pi_{2t}, \pi_{3t})$. We present the optimal policy for year one of a 20-year horizon. To illustrate the optimal policy, we present the belief state in two dimensions (Fig. 1). The x-axis represents the decision-maker's current belief about the probability of moderate infestation and the y-axis represents the decision-maker's current belief about the probability of no infestation. One minus the sum of these probabilities represents the decision-maker's current belief about the probability of high infestation. For example, if the manager's belief state includes probabilities of no infestation and moderate infestation of 0.20 and 0.40, respectively, the probability of high infestation is 0.40.

For the application described in Section 3, the optimal policy involves choosing no action when there is a sufficiently large probability of no infestation, monitoring in intermediate cases, and treatment when the probability of moderate or high infestation is large (Fig. 1). No action is optimal when the probability of no infestation is greater than 0.60 regardless of the probability of moderate or high infestation. Monitoring in these belief states doesnot add enough information to the system to justify its cost and the high probability that treatment is not necessary means that its cost is not justified. Monitoring enters the optimal policy when the probability of no infestation is between 0.50 and 0.60 regardless of the probability of moderate or high infestation. Monitoring is also optimal for combinations of probability of no infestation between 0.40 and 0.45 and moderate infestation between 0.15 and 0.60. Monitoring in these belief states resolves the uncertainty about the current state in the subsequent period so that treatment is avoided when there is no infestation but occurs if it is revealed that there is either a moderate or high infestation. Treatment is the optimal action whenever the probability of no infestation is less than 0.40, i.e., when the combined probabilities of having moderate or high infestation is sufficiently large (>0.60). In this case, the expected damages are high enough to justify the expense of treatment. For the set of parameter values set out in Section 3, there is no combination of beliefs for which it is optimal to choose both treatment and monitoring in the same period (i.e., action 4). With treatment, the probability of no infestation at the beginning of the next period will be at least 0.60 so that the benefits of combining monitoring with treatment to learn

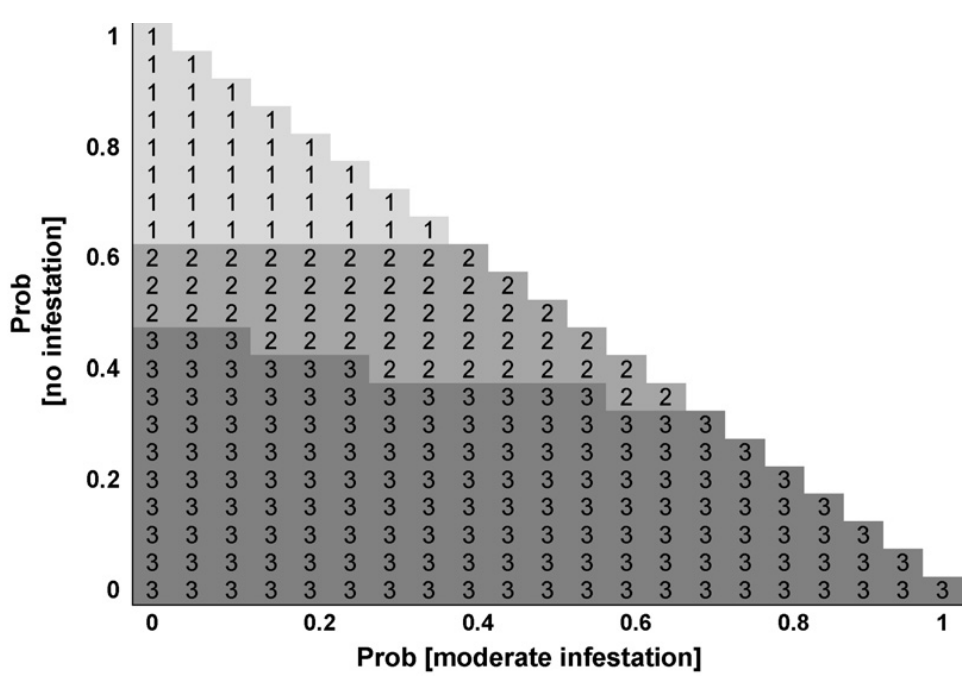

Fig. 1. Optimal policy in year one as a function of the belief state. Monitoring costs 4 units per application. 1 = no action; 2 = monitor; 3 = treat.

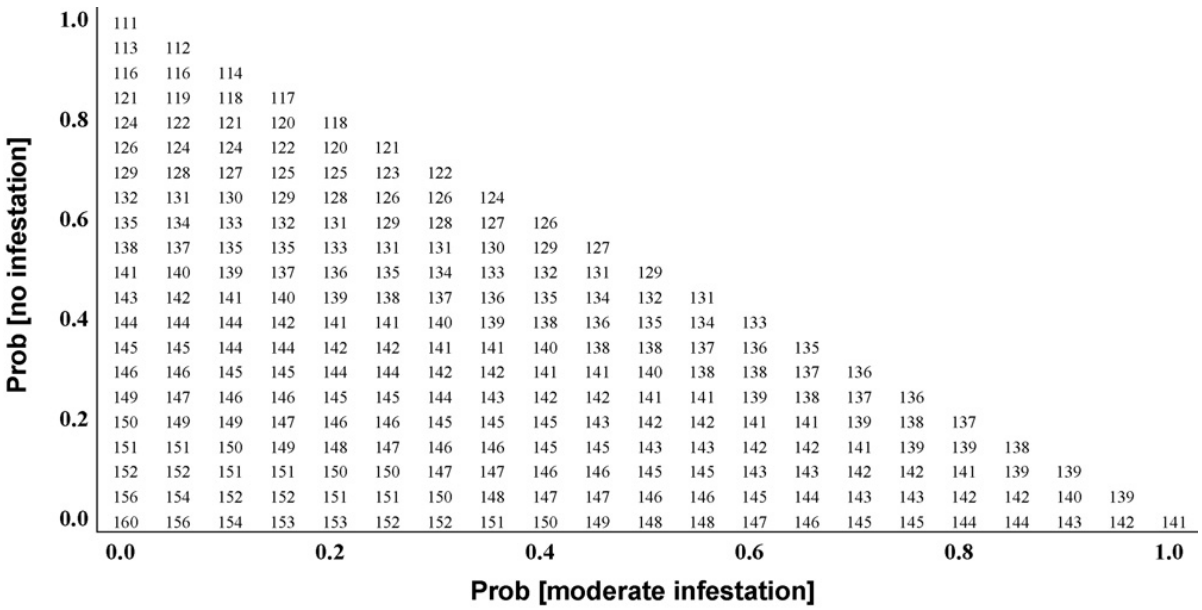

Fig. 2. The expected discounted cost over 20 years of the optimal action taken in year one for each belief state when monitoring costs 4 units per application.

the treatment outcome are not sufficient to justify the additional cost of monitoring. For some combinations of parameter values and beliefs it will be optimal to do both treatment and monitoring in the same period as we will see in Section 4.2 below.

The optimal value function associated with the policy in Fig. 1 reports the expected discounted cost over 20 years for each belief state in period one. Similar to Fig. 1, we illustrate the discounted costs of optimal actions in two dimensions (Fig. 2) where the x-axis represents the decision-maker's current belief about the probability of moderate infestation and the y-axis represents the decision-maker's current belief about the probability of no infestation. The highest discounted cost (160 units) occurs in the lower left hand cell where the likelihoods of no and moderate infestation states are low and the likelihood of high infestation is large. For a given probability of moderate infestation, discounted cost decreases as the probability of no infestation increases (going up any column in Fig. 2). Similarly, for a given probability of no infestation, discounted cost decreases as the probability of moderate infestation increases (moving from left to right in any row of Fig. 2). Note that the difference in cost from lowest to highest expected cost in column one is higher than the one-time treatment cost (20 units). If treatment was always perfectly effective in removing all infestation then this difference would decline.

4.2. The value of a perfect monitoring system

To estimate the value of a monitoring system that provides perfect information about the state of the infestation, we compare the optimal value function for the case with two potential actions (no action and treatment) versus the case where monitoring is also possible. We assume that monitoring (when it can be done) is costless and allows the manager to observe the true infestation state (i.e., the observation matrix $\mathbf{R}^a$ is an identity matrix). By estimating the optimal value function for the case where monitoring is costless and returns perfect information and comparing it to the case where monitoring is not allowed, we obtain the value of information, the difference between being fully informed about the infestation state versus not, which is an upper bound on the value of the monitoring system.

The optimal policy for the case without monitoring is to take no action when the probability of no infestation is greater than 0.45 and to treat when the probability of no infestation is less than 0.40 regardless of the probability of moderate infestation (Fig. 3). Between 0.40 and 0.45 probability of no infestation, no action is optimal when the sums of the probabilities of no and moderate infestations are

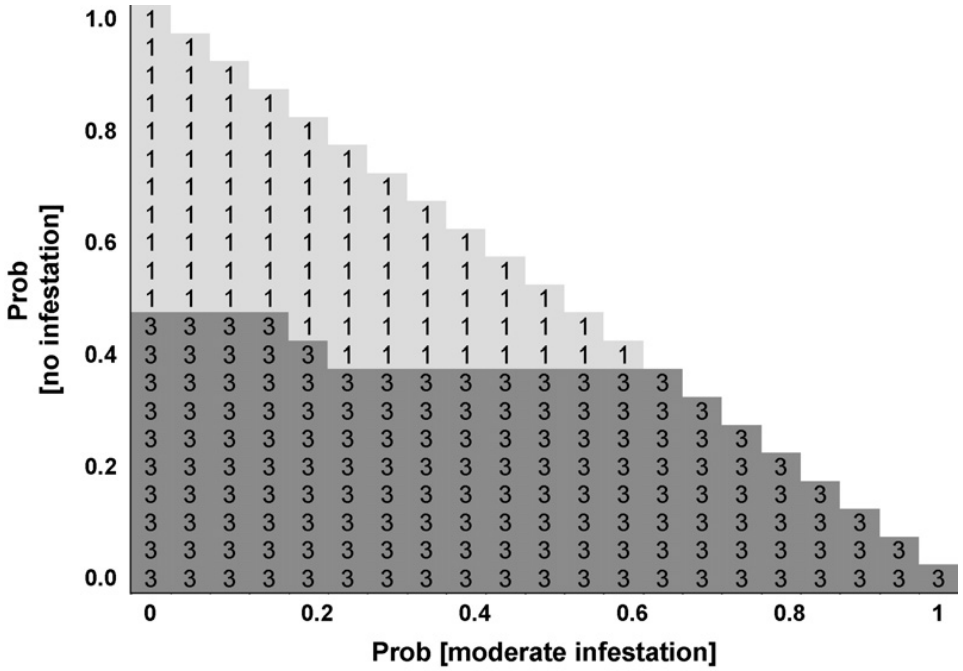

Fig. 3. Optimal policy in year one as a function of the belief state when monitoring is not allowed. 1 = no action; 3 = treat.

greater than 0.65 (i.e., the probability of high infestation is less than 0.35). The expected discounted costs of the optimal actions as a function of the belief state in year one are shown in Fig. 4. The highest expected cost (162 units) occurs if the manager is sure the infestation state is high, and expected costs decrease as the belief probabilities of no or moderate infestation increase.

When monitoring without cost is possible, it is always optimal to monitor as there may be some benefit from improved information and there is no cost (Fig. 5). The optimal policy is to monitor and treat when the probability of no infestation is less than 0.30 and only monitor when the probability of no infestation is higher than 0.35 regardless of the probability of moderate infestation. Between these probabilities, monitoring but not treating is optimal when the sums of the probabilities of no and

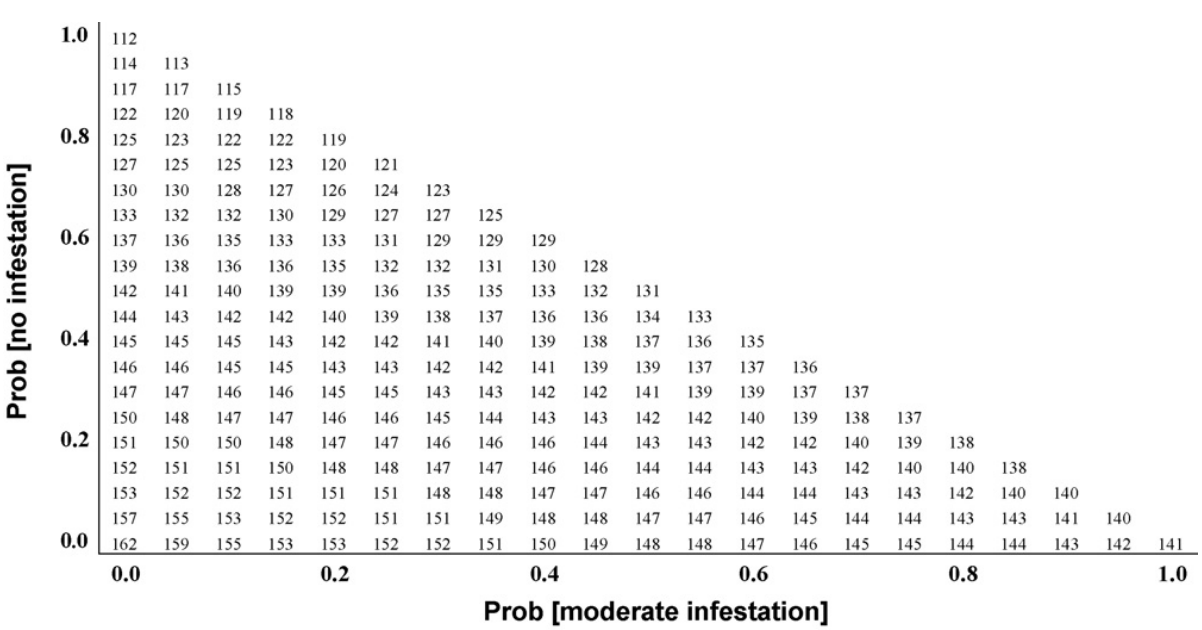

Fig. 4. The expected discounted cost over 20 years of the optimal action taken in year one for each belief state when monitoring is not allowed.

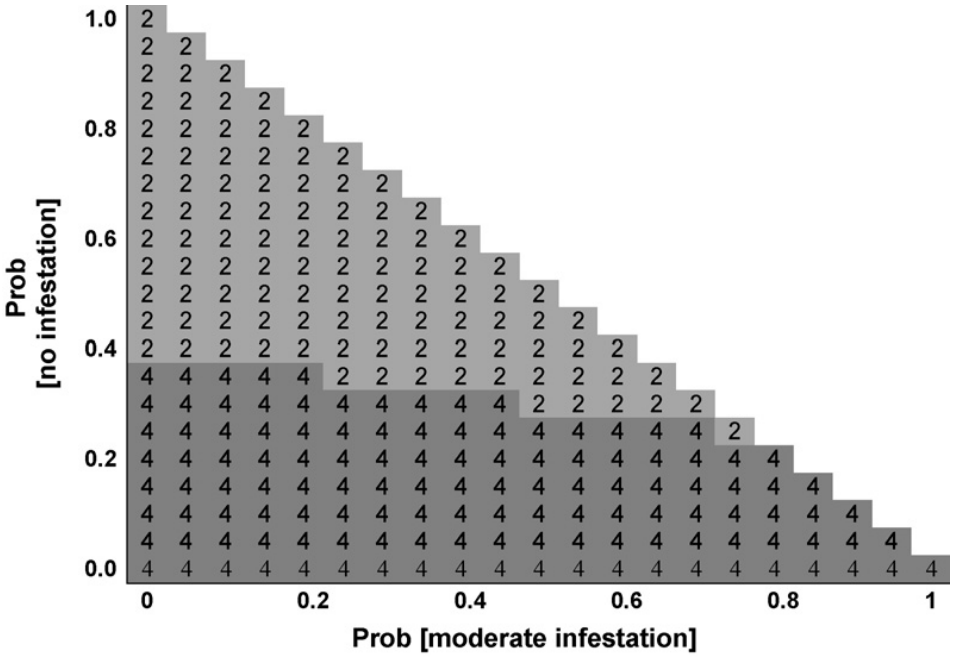

Fig. 5. Optimal policy in year one as a function of the belief state. Monitoring is costless and provides perfect information. 2 = monitor; 4 = monitor and treat.

moderate infestation are greater than approximately 0.60. Note that the range of beliefs over which it is optimal to treat is smaller when monitoring is costless compared to the case of costly monitoring (Fig. 1). In the case when monitoring is free, it makes sense to reduce the range of belief states in which treatment is prescribed because costless monitoring will be done and may reveal that costly treatment is not needed. Note that although monitoring is undertaken each period and reveals perfect information about the state of the system, the timing of decisions and information revelation means that the manager will have some uncertainty about the level of infestation when deciding on treatment. Monitoring in period t reveals the state of infestation in period t. However, there is a probabilistic state transition to period $t+1$ so that at the time the manager chooses treatment in period $t+1$ there is some uncertainty about the level of infestation.

Allowing the manager to employ monitoring without cost greatly reduces the expected costs of management (Fig. 6). In this case, costs are reduced not only because monitoring costs fall to zero but also because treatment is more finely targeted to cases when it is needed, thereby reducing treatment costs as well as damages from high infestation. Expected costs are 20–30% lower across the range of belief states relative to the expected costs obtained without monitoring (Fig. 4). For example, when the manager believes there is 0.40 chance of no infestation and 0.20 chance of moderate infestation, the expected discounted cost associated with the optimal policy that includes monitoring (108 units) is 24% less than the expected discounted cost of the optimal policy that does not allow monitoring (142 units). These cost reductions represent the value of the option to employ a monitoring system that provides perfect information at no cost.

The value of a monitoring system, and the cases in which it would be applied, are reduced as the cost of monitoring increases. The impacts of increasing the cost of monitoring from 0 to 4 units are seen by comparing Figs. 1 and 5. When a monitoring action costs 4 units, monitoring is the optimal action in a smaller range of belief states, and no action is preferred when the probability of no infestation is relatively high. Increasing the cost of applying the monitoring system from 0 to 4 units increases the expected discounted cost of management (compare Figs. 2 and 6). With the higher cost of monitoring, the expected discounted cost of management is almost as high as the cost of management when monitoring is not allowed (Fig. 4). When monitoring costs more that 5 units per application, monitoring does not enter the optimal policy. In this case, the value of information obtained from monitoring is less than the cost of monitoring.

4.3. The value of an imperfect monitoring system

The value of a monitoring system also depends on the accuracy of the information it provides. In the cases examined above, we assumed that monitoring provides perfect information about the state of the system (i.e., the observation matrix is an identity matrix). Here, we assume that a monitoring activity costs nothing and provides imperfect information (i.e., the diagonal elements of the observation matrix are less than 1.0). We determined optimal policies using the following observation matrices:

$$\mathbf{R}^1 = \mathbf{R}^3 = \begin{bmatrix} 0.5 & 0.5 & 0.0 \\ 0.3 & 0.4 & 0.3 \\ 0.1 & 0.4 & 0.5 \end{bmatrix} \mathbf{R}^2 = \mathbf{R}^4 = \begin{bmatrix} 0.75 & 0.25 & 0.00 \\ 0.15 & 0.70 & 0.15 \\ 0.05 & 0.20 & 0.75 \end{bmatrix}$$

Whereas the observation matrices for no action ($\mathbf{R}^1$) and treatment alone ($\mathbf{R}^3$) are the same as before, the observation matrix for monitoring alone ($\mathbf{R}^2$) and monitoring and treatment ($\mathbf{R}^4$) now provides imperfect information. The observation matrix associated with monitoring indicates that a manager is more likely to observe the correct infestation state than without monitoring but that the signal received with monitoring is no longer perfect.

The optimal policy for this case in which monitoring provides imperfect information at no cost is to monitor and treat when the probability of no infestation is less than 0.35 and monitor alone when the probability of no infestation is higher than 0.40 regardless of the probability of moderate infestation (Fig. 7). Between these probabilities, monitoring alone is optimal when the sums of the probabilities of no and moderate infestation are more than about 0.60 (i.e., the probability of high infestation is less than 0.40). Comparing the outcomes with imperfect monitoring versus perfect monitoring, both at no cost, shows that the range of beliefs over which it is optimal to treat are greater with imperfect monitoring (Fig. 5 versus Fig. 7). The range of beliefs over which it is optimal to treat, however, is not as large as the case when monitoring is not possible (Fig. 4). Imperfect monitoring provides an intermediate case between perfect monitoring (Fig. 5) and no monitoring (Fig. 4). Compared to no monitoring, it makes sense to reduce the range of belief states in which treatment is prescribed because even imperfect monitoring reveals some information about whether costly treatment is needed. Imperfect monitoring does not provide as good of a signal as when monitoring generates perfect information so treatment occurs over a wider range of parameter values with imperfect as compared to perfect information.

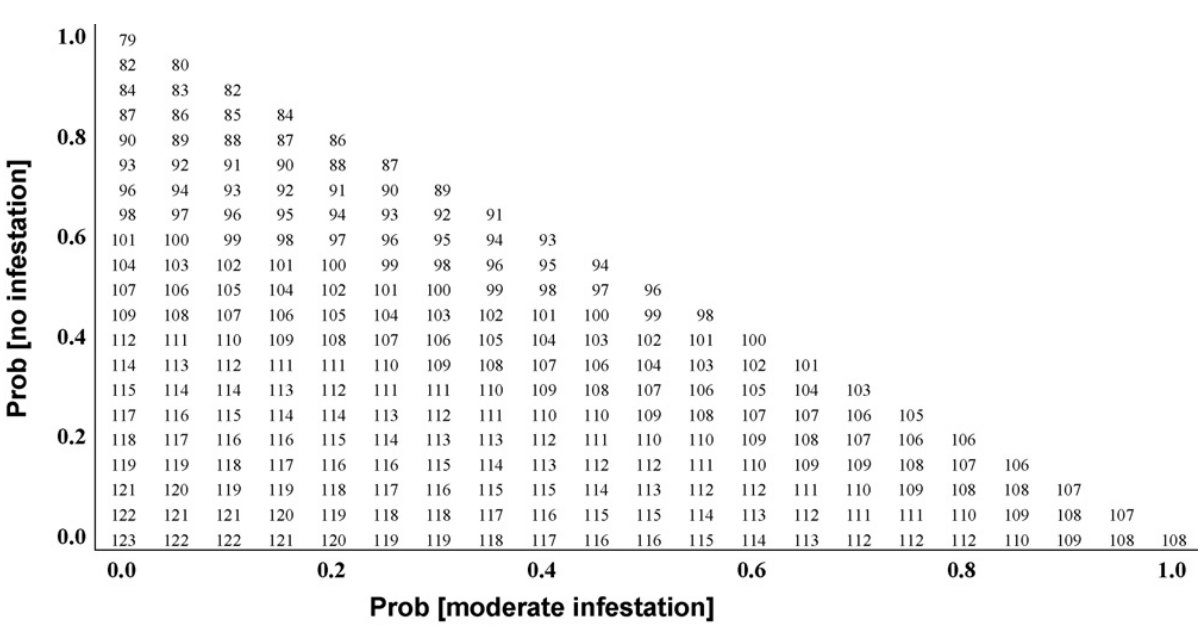

Fig. 6. The expected discounted cost over 20 years of the optimal action taken in year one for each belief state when monitoring is costless and provides perfect information.

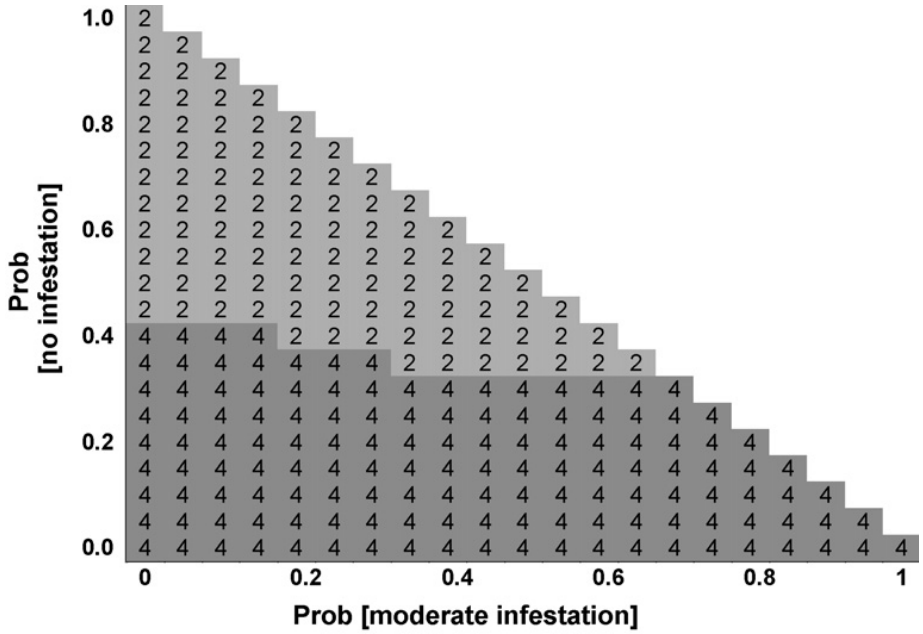

Fig. 7. Optimal policy in year one as a function of the belief state. Monitoring is costless and provides imperfect information. 2 = monitor; 4 = monitor and treat.

The expected discounted costs for the case where monitoring provides imperfect information at no cost (Fig. 8) are 3–6% less than the expected costs obtained without monitoring (Fig. 4), indicating that monitoring is still an efficient activity even though it does not always provide accurate information. However, when the cost of applying the monitoring action increases from 0 to 4 units, it is no longer optimal to use the imperfect monitoring action in any belief state. The expected discounted costs of management when monitoring provides imperfect information (Fig. 8) are 25–35% higher across the range of belief states relative to the expected costs obtained when monitoring provides perfect information (Fig. 6). For example, when the manager believes there is 40% chance of no infestation and 20% chance of moderate infestation, the expected discounted cost associated with the optimal policy that includes the option to monitor and obtain imperfect information (135 units) is 25% more than the

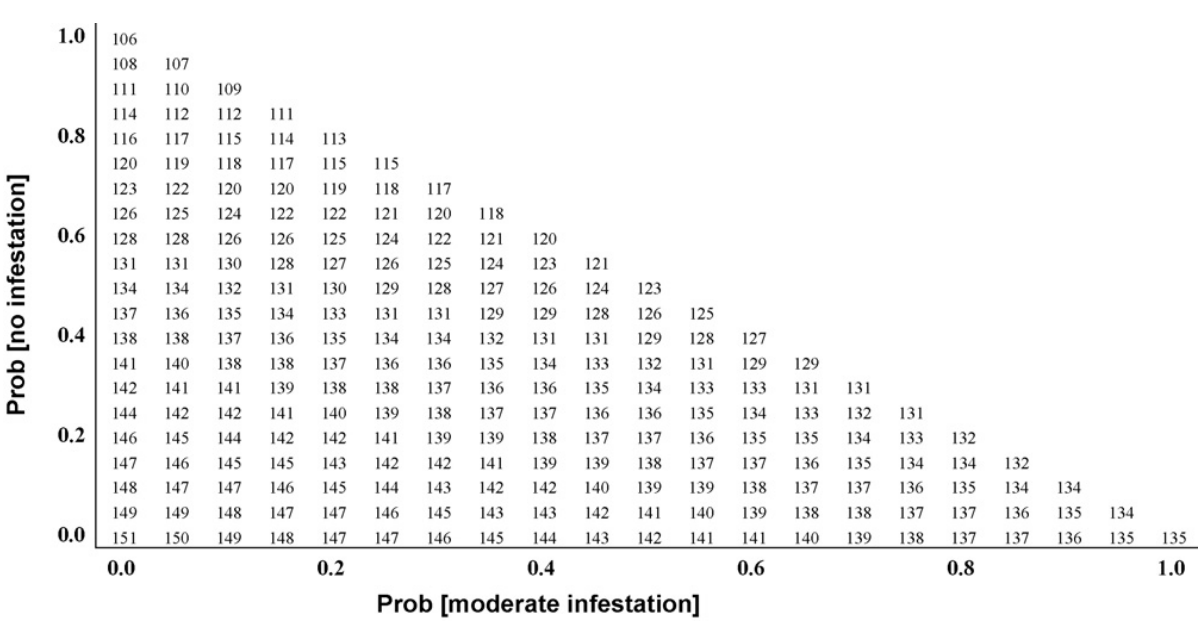

Fig. 8. The expected discounted cost over 20 years of the optimal action taken in year one for each belief state when monitoring is costless and provides imperfect information.

expected discounted cost of the optimal policy when monitoring provides perfect information (108 units). This difference in expected cost is an estimate of the value of improving the accuracy of the monitoring system.

5. Conclusions

In this paper, we modeled the problem of controlling an invasive species with imperfect information about the level of infestation as a partially observable Markov decision process. This approach allowed us to find optimal management solutions when information about the degree of infestation is imperfect and allowed us to address issues of updating beliefs about the state of the infestation and the value of improved information through monitoring. We solved for the optimal management strategy in a simple application with three states (no infestation, moderate infestation, and high infestation) and four potential management choices (no action, monitoring alone, treatment alone, and joint monitoring and treatment).

We showed how the choice of optimal management action depends on beliefs about the likely degree of infestation and how this choice is affected by the cost and accuracy of monitoring. For the base case in the application in Sections 3 and 4, we found that the optimal policy involves choosing no action when there is a sufficiently large probability of no infestation, monitoring alone with intermediate probability values and treatment alone when the probability of moderate or high infestation is large. We also show how optimal management and expected costs change as the cost or quality of information from monitoring changes. For low costs of monitoring and large probabilities of moderate or high infestation it is optimal to both monitor and treat. We calculate the value of information from monitoring by comparing expected costs in a case with perfect and costless monitoring versus a case without monitoring. Expected costs are 20–30% lower across the range of belief states relative to the expected costs obtained without monitoring.

Besides shedding light on optimal policy for controlling invasive species, our results also fit into a larger literature on optimal management and investment under uncertainty (e.g., Dixit and Pindyck, 1994), adaptive management (e.g., Walters, 1986) and the value of information (e.g., Hanemann, 1989). Other studies have solved for the value of information to improve management in a variety of other resource management contexts such as fisheries management (e.g. Costello et al., 1998), climate change and agriculture (e.g., Adams et al., 1995; Solow et al., 1998), exhaustible resource extraction (e.g. Polasky, 1992), and pest management (e.g., Feder, 1979; King et al., 1998; Swinton and King, 1994), among other applications. Information is valuable when it improves management decisions. In our case, improved information about the degree of infestation allows application of treatment to reduce damages when infestation levels are high and avoids costly treatment when there is no infestation. We also showed that the value of monitoring quickly declines as the costs of monitoring rises or imperfection in the information from monitoring increases.

A promising direction for future research is to combine data from specific invasive species with a POMDP model to provide insights into optimal management for these cases. A potential application is the management of emerald ash borer infestations of urban forests. Emerald ash borer (*Agrilus planipennis* Fairmaire), a phloem-feeding beetle native to Asia, was discovered near Detroit, MI in 2002. As of October 2009, isolated populations of emerald ash borer (EAB) have been detected in 12 additional states. EAB is a highly invasive forest pest that has the potential to spread and kill native ash trees (*Fraxinus* sp.) throughout the United States and cost homeowners and local governments billions of dollars for tree removal and replacement (Kovacs et al., 2010). The presence of EAB is usually not realized until well after the species is established and discovering the location and extent of infestation requires intensive monitoring. Urban forest managers rarely have perfect information about the state of an EAB infestation and must decide how much, if any, resources should be allocated to monitoring and treatment. While monitoring with insect traps is relatively inexpensive, it does not provide accurate information about the infestation state. Treatments such as insecticide application and tree removal are expensive and may not eradicate the infestation. An application of the POMDP model to an EAB infestation could provide useful information about the value of monitoring and guidance for treatment application.

The partially observable Markov decision process model described in Section 2 is quite general and can be used to provide insights into how well various management approaches might work in more realistic and complex situations. For example, the model could be used to solve for optimal management with more levels of infestation, and with differing levels of intensity of monitoring or treatments, all of which would lead to more sophisticated Bayesian updating of beliefs. Other additional aspects that would be useful to add would include strategies to reduce the probability of introduction and the spatial pattern of infestation. However, adding more complexity increases the difficulty of finding solutions. Another necessary direction for future research is to find heuristic methods that find good, but not necessarily optimal, solutions for complex models.

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