Second-Order Polynomial Model to Solve the Least-Cost Lumber Grade Mix Problem

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Abstract

Material costs when cutting solid wood parts from hardwood lumber for secondary wood products manufacturing account for 20 to 50 percent of final product cost. These costs can be minimized by proper selection of the lumber quality used. The lumber quality selection problem is referred to as the least-cost lumber grade mix problem in the industry. The objective of this study was to create a least-cost optimization model using a design that incorporates a statistical approach to address shortcomings of existing models using linear optimization methods. The results of this study showed that optimal solutions tend to use as much low-quality lumber as possible to minimize costs. Comparison of results from this new least-cost grade mix model with other existing least-cost lumber grade mix models has shown that the new model results in lower-cost solutions.

In rough mills of the secondary wood products industry, secondary hardwood manufacturers cut hardwood boards into parts of specified sizes, qualities, and quantities according to customer orders, called cutting bills (Buehlmann et al. 1999). This is an economically important process since raw material costs contribute up to 70 percent of total product cost (Carino and Foronda 1990, Wengert and Lamb 1994, Mitchell et al. 2005).

Cutting lumber into smaller components is a typical cutting stock problem (Dyckhoff 1990). The problem exists in many industries, such as in the paper, glass, or car manufacturing industries. In the 1960s, Gilmore and Gomory (1961, 1963, 1965, 1966) proposed a series of solutions to the cutting stock problem using linear and dynamic programming. Since then, numerous additional research efforts have been made to address the problem in different numbers of dimensions, such as in one, two, three, and more dimensions (Haessler 1975, Ferreira et al. 1990, Goulimis 1990, Sweeney and Haessler 1990, Fayard et al. 1998, Harjunkoski et al. 1998, Wagner 1999, Belov and Scheithauer 2002, Liang et al. 2002, Umetani et al. 2003). Furthermore, Hinxman (1979) and Haessler and Sweeney (1991) studied the problem in 1.5 dimensions. Generally speaking, solving the cutting stock problem has involved three different methods: algorithm method (Harjunkoski et al. 1998, 1999; Westerlund et al. 1998), heuristic method (Eisemann 1967, Haessler 1975, Coverdale and Wharton 1976, Goulimis 1990, Haessler and Sweeney 1991), and metaheuristic method (Glover 1986, Feo and Resende 1995, Fayard et al. 1998, Liang et al. 2002). Besides theoretical models, many empirical models have been developed to resolve the problem for specific industries, such as the steel industry (Karelahti 2002), the glass industry (Arbib and Marinelli 2004), the paper industry (Goulimis 1990, Harjunkoski et al. 1996, Ostermark 1999), and the wood products industry (Ronnqvist and Astrand 1998, Wagner 1999), to name a few.

In the wood products industry, the cutting stock problem exists throughout the whole supply chain from felling trees, cutting logs, sawing lumber, and processing small parts of furniture or cabinets. To optimize the bucking and allocation of logs, Faaland and Briggs (1984) and Eng and Daellenbach (1985) made use of dynamic programming, while Brunner et al. (1989), Foronda and Carino (1991), Klinkhachorn et al. (1993), and Carnieri et al. (1993, 1994) used heuristic methods to determine the optimal lumber cut-up patterns for higher yield. Later, more complex models

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that include lumber defects in the cutting process were developed to provide more realistic solutions (Astrand and Ronnqvist 1994, Ronnqvist and Astrand 1998).

Wood products manufacturers are sensitive to changes in lumber quality (referred to as lumber grade in the industry; National Hardwood Lumber Association [NHLA] 1998) and price. Matching cutting bill requirements (e.g., the size of individual parts in a cutting bill, the distribution of these sizes, and the individual quantities of parts required; Buehlmann et al. 1999) with the most appropriate lumber grade or lumber grade combination minimizes material purchasing and processing costs. Finding the best match between cutting bill and lumber grade composition for least cost is usually referred to as the least-cost lumber grade mix problem in the industry. Using the least-cost lumber grade mix implies that a mill processes an optimal lumber grade combination minimizing costs while satisfying cutting bill requirements. Efforts to solve the least-cost lumber grade mix problem started in the late 1960s (Englerth and Schumann 1969). Since then, much research (Hanover et al. 1973, Martens and Nevel 1985, Carino and Foronda 1990, Steele et al. 1990, Timson and Martens 1990, Harding 1991, Fortney 1994, Suter and Calloway 1994, Lawson et al. 1996) has been conducted to find solutions for the problem. Hamilton et al. (2002) advanced the linear programming approach that addressed the nonlinearity issue. However, their research focused on filling a cutting order from a given lumber stock with a defined lumber grade mix. Most, if not all, solutions developed to date rely on linear programming methodology. Reasons for this focus on linear programming methodology may have been the seemingly linear relationship between lumber grade or grade combinations and yield as well as the speed of performing the necessary calculations on computers (Bethel and Harrell 1957). However, even if true optimal solutions were obtained from linear programming models, the solution may not always be applicable on the manufacturing floor because the actual constraints, such as limited storage space, random machine downtime, and random raw material inputs in the plants, could not be included in the linear programming model (Sampson 1979). Also, the yield nomograms used for yield estimation in most least-cost lumber grade mix optimization models may be inaccurate because of the advancement of processing technologies and changes in the wood resource (Hoff 2000).

The primary requirement for applying linear programming is that both the objective and the constraint functions be simple linear (Winston 1994). Thus, solving the least-cost lumber grade mix problem using linear programming technologies requires that the relationship between yield and lumber grade or grade combinations be simple linear. Zuo et al. (2004) found that the relationship between yield and two and/or three lumber grade combinations are not always linear. In fact, it was shown that the simple linear relationship between yield and lumber grade mix does not hold for an estimated 90 percent of the cutting bill–grade combinations tested. Also, the study by Zuo et al. (2004) did not find a simple rule to predict if a cutting bill–grade mix combination behaves linearly. Thus, linear programming models apply only in selected cases that require narrow and short parts with relatively even distribution.

The violation of the linearity assumption limits the validity of the solutions produced by all major least-cost lumber grade mix models relying on linear programming. The objective of this study then was to use a statistical method to search for the least-cost lumber grade or grade combination without violating the simple linearity assumption.

Materials and Methods

A major problem in finding the least-cost lumber grade mix is to estimate the expected yield of required parts obtained from the various lumber grades and grade mix combinations. For this purpose, existing simulation software (Thomas 1999) and digital representations of lumber boards (Gatchell et al. 1998) were used.

Lumber cut-up simulator

The USDA Forest Service's ROMI-RIP 2.0 (RR2) simulation software (Thomas 1999) was used to collect simulated lumber cut-up yields for this study. RR2 simulates the cut-up of lumber in rip-first rough mills (Thomas and Buehlmann 2002). In rip-first processing, narrow strips are cut from the board first. These are subsequently cut to the lengths required by the cutting bill (Buehlmann et al. 1999). The RR2 simulation software was set up to reflect real world conditions. In particular, the settings used are the following:

- All-blades—movable arbor type
- Salvage cut to primary lengths and widths
- No excess salvage yield included
- Complex dynamic exponential part prioritization
- No random-width or random-length parts
- Continuously updated part counts
- ¼-inch end and side trim

Cutting bill

Buehlmann's cutting bill (Buehlmann et al. 2008a, 2008b), a theoretical representation of the ''average'' industrial cutting bill with respect to size and quantity requirements, was used in this study. Proportional adjustments of part requirements were made to fit the lumber samples used for this study (Table 1).

Additionally, 10 industrial cutting bills were used (Wengert and Lamb 1994, Thomas 1996). Table 2 summarizes all 11 cutting bills and shows their relative difficulty ranking in respect of obtaining all parts from given lumber (Thomas 1996, Zuo et al. 2004).

Lumber data

Because of its economic importance to the hardwood lumber industry and the volume of red oak processed in secondary hardwood dimension mills, digital representations of red oak lumber boards were used for this study. The following lumber grades were used: FAS, SELECTS (SEL), 1 Common, 2A Common, and 3A Common (NHLA 1998). These five grades are listed in decreasing order of quality

Table 1.—Cutting bill requirements of the Buehlmann cutting bill.

Part width (in.)	Part length (in.)				
	10	17.5	27.5	47.5	72.5
1.50	136	297	433	243	103
2.50	152	298	480	262	98
3.50	46	102	146	88	57
4.25	49	99	158	85	40

Table 2.—Length and width summary and difficulty ranking for the 11 cutting bills used in this study.

Cutting bill	Difficulty ranking[a]	No. of parts	No. of widths	No. of lengths
A	1	5	3	4
B	2	10	4	9
C	3	25	7	16
D	4	5	3	5
E	5	4	4	4
F	6	12	4	6
Buehlmann	7	20	4	5
G	8	20	7	12
H	9	8	2	8
I	10	16	4	11
J	11	9	5	4

[a] The cutting bills were ranked from easiest to hardest as defined in Thomas's (1996) study; the rankings for the Wengert and Lamb (1994) and Buehlmann (1998) cutting bills were based on the same criteria as used in Thomas's study.

and price. Each lumber grade was used as a factor in the model. To control the model's computing time, lumber grade mix increments of 10 percent were used for creating the lumber grade combinations. To allow for three replicates of each lumber cut-up simulation to smooth out random variation, three lumber grade or grade combination samples with 1,000 board feet of lumber each were randomly selected from the 1998 Kiln Dried Red Oak Data Bank (Gatchell et al. 1998) using RR2′s MAKEFILE utility (Thomas 1999) for each lumber grade combination.

Experimental design

The volume of each lumber grade in any given lumber grade combination is between 0 and 100 percent of the total lumber volume used in each lumber set. The sum of all lumber grade proportions is 100 percent. A five-factor mixture design (Myer and Montgomery 2002) was applied with the five factors being the proportion of each grade utilized in a given lumber sample. An upper bound of 80 percent was placed on 3A Common lumber because of its lack of capacity for producing the large parts in cutting bills. In fact, during preliminary testing, it was found that cutting bills G, I, and J were extremely difficult to satisfy with the combination of 80 percent 3A Common and 20 percent 2A Common lumber (the lowest-quality grade combination permitted under the rules discussed previously). Therefore, the upper bound for 3A Common lumber was adjusted to 60 percent for these three bills. Tables 3 and 4 show the details of the lumber grade mixture design executed for the 11 cutting bills (Tables 3 and 4, first six columns).

Analysis

Cost calculations.—In order to determine the grade combination that satisfied each cutting bill at the lowest lumber cost, cost information for each grade combination was acquired to build a lumber grade–cost response surface. The simulated yields (Table 3, last eight columns, and Table 4, last three columns) from the 25 initial grade combination sample runs on RR2 were used to build a response surface. However, since the lumber grade combination with the least cost was the outcome of interest, Equation 1 had to be used for the transformation of yields to cost. Equation 1 correlates yield, the grade distribution, and the market price for each lumber grade with costs:

$$\mathrm{COST}_j = \frac{\sum_i^5 G_i \times M_i}{\mathrm{YIELD}_j} \quad (1)$$

where

G_i = the proportion of each lumber grade;

M_i = the market price per thousand board feet (MBF) of each lumber grade; $i = 1$ for FAS, 2 for SEL, 3 for 1 Common, 4 for 2A Common, and 5 for 3A Common; and

j = observation of a grade combination run.

To calculate total lumber costs per MBF for satisfying a given cutting bill, $^4/_4$-inch-thick, kiln-dried red oak lumber prices from January 2002 were used (*Weekly Hardwood Review* 2002). Data from 2002 were used to fit research described in this article with work done by Zuo et al. (2004) and by Buehlmann et al. (2008c). The price for SELECTS, which is not reported in *Weekly Hardwood Review,* was estimated through industry contacts. The prices used were as follows: FAS, \$1,570 per MBF; SEL, \$1,350 per MBF; 1 Common, \$1,000 per MBF; 2A Common, \$748 per MBF; and 3A Common, \$500 per MBF.

An optimal lumber grade combination (e.g., a minimum cost lumber grade combination) found using Equation 1 minimizes raw material cost without consideration of processing costs. Processing costs, in this study, were defined as all nonlumber (raw material) costs incurred when producing dimension parts in the production process. Thus, production costs, i.e., the sum of lumber and processing costs, must be optimized (minimized) to find the true lowest-cost lumber grade mix for a given cutting bill. Research conducted in a dimension mill has shown that adding \$200 per MBF of input lumber to lumber costs is a good approximation of true processing costs in a state-of-the-art dimension mill (Buehlmann and Zaech 2001). This approach assumes that the processing costs are related to yield. Thus, for lumber inputs that yield more usable parts per MBF of input lumber, processing costs are lower because less lumber is needed to fulfill the cutting bill. Although this finding may not be true for all dimension mills depending on operational setup and technology used, Buehlmann and Zaech's findings were used for the development of this model. Modifications to this cost assumption can easily be incorporated into the model should a particular rough mill have proprietary processing cost estimates. For a mill having more detailed cost information, Equation 2 can be used and the specific processing costs (P_i) be applied for each grade:

$$\mathrm{COST}_j = \frac{\sum_i^5 G_i \times (M_i + P_i)}{\mathrm{YIELD}_j} \quad (2)$$

where

G_i = the proportion of each lumber grade;

M_i = the market price per MBF of each lumber grade;

P_i = the processing cost per MBF of each lumber grade;

i = 1 for FAS, 2 for SEL, 3 for 1 Common, 4 for 2A Common, and 5 for 3A Common; and

j = observation of a grade combination run.

Table 3.—Design matrix for five-factor mixture design with 80 percent upper bound for 3A Common lumber and average yield response from three replicates for eight cutting bills tested.

	% for each grade					Average yield (%)							
Run no.	FAS	SEL	1Com	2ACom	3ACom	A	B	C	D	E	F	Buehlmann	H
1	0	0	0	20	80	29.31	37.39	44.17	37.16	15.59	8.92	27.03	19.65
2	0	0	0	60	40	36.71	47.70	49.38	43.02	24.88	18.29	37.06	35.52
3	0	0	0	100	0	42.99	54.98	54.76	48.36	30.95	26.67	47.93	46.29
4	0	0	20	0	80	32.59	44.65	46.37	39.72	21.41	16.45	36.02	31.18
5	0	0	50	50	0	50.09	60.74	60.76	54.14	43.90	43.62	57.68	54.57
6	0	0	50	50	0	49.54	60.36	60.38	53.96	44.37	41.57	57.21	54.68
7	0	0	60	0	40	45.30	56.49	56.68	50.10	38.62	40.79	52.46	50.24
8	0	0	100	0	0	56.43	65.61	65.50	60.08	53.60	54.03	63.83	60.79
9	0	20	0	0	80	33.80	46.40	45.80	40.50	24.37	23.76	39.02	34.97
10	0	50	0	50	0	49.23	62.47	60.17	55.46	48.32	52.93	59.78	56.53
11	0	50	0	50	0	48.30	63.12	60.76	55.88	49.03	52.66	59.09	57.35
12	0	50	50	0	0	55.39	67.37	66.28	60.75	59.95	59.78	65.83	63.34
13	0	50	50	0	0	55.38	67.32	66.03	60.93	58.25	59.30	65.11	62.57
14	0	60	0	0	40	44.00	58.96	54.83	51.17	44.79	48.21	53.98	52.49
15	0	100	0	0	0	54.59	69.56	64.01	61.78	63.34	61.13	66.16	64.95
16	50	0	0	50	0	56.50	66.74	67.00	61.24	60.03	58.78	65.28	60.93
17	50	0	0	50	0	56.24	66.75	66.69	61.14	60.51	59.70	64.70	61.08
18	50	0	50	0	0	63.39	71.01	71.42	66.81	66.33	67.60	70.92	66.49
19	50	0	50	0	0	63.79	70.99	71.36	67.02	66.41	67.30	70.66	67.04
20	50	50	0	0	0	62.86	72.93	72.26	67.81	68.02	68.72	72.45	68.57
21	50	50	0	0	0	62.13	71.97	71.33	67.33	66.78	68.19	71.83	67.67
22	60	0	0	0	40	53.63	64.35	63.92	59.06	58.50	55.49	61.31	58.75
23	60	0	0	0	40	53.16	64.36	63.98	58.73	57.92	55.65	61.55	58.60
24	100	0	0	0	0	70.11	76.08	76.05	73.21	72.64	75.50	76.68	71.80
25	100	0	0	0	0	69.85	76.21	76.61	73.55	72.43	75.61	76.96	71.92

Table 4.—Design matrix for five-factor mixture design with 60 percent upper bound for 3A Common lumber and average yield response from three replicates for three cutting bills tested.

	% for each grade					Average yield (%)		
Run no.	FAS	SEL	1Com	2ACom	3ACom	G	I	J
1	0	0	0	40	60	25.42	13.11	5.21
2	0	0	0	70	30	34.84	14.77	6.88
3	0	0	0	100	0	42.35	22.64	12.63
4	0	0	40	0	60	39.60	32.44	24.68
5	0	0	50	50	0	52.48	45.92	40.68
6	0	0	50	50	0	52.14	45.19	38.84
7	0	0	70	0	30	50.30	47.18	38.22
8	0	0	100	0	0	60.13	57.52	48.38
9	0	0	100	0	0	60.36	58.93	49.38
10	0	40	0	0	60	43.47	36.72	27.38
11	0	50	0	50	0	55.90	50.97	40.49
12	0	50	0	50	0	55.79	51.96	41.76
13	0	50	50	0	0	63.42	60.88	52.09
14	0	50	50	0	0	63.35	60.94	51.90
15	0	70	0	0	30	55.30	51.99	41.33
16	0	100	0	0	0	66.57	64.74	55.33
17	40	0	0	0	60	48.45	42.18	42.04
18	50	0	0	50	0	60.82	59.07	56.07
19	50	0	0	50	0	59.82	59.44	56.89
20	50	0	50	0	0	69.14	68.32	62.22
21	50	0	50	0	0	68.52	68.62	61.08
22	50	50	0	0	0	69.94	68.97	63.13
23	50	50	0	0	0	70.32	69.15	61.79
24	70	0	0	0	30	61.49	61.11	57.92
25	100	0	0	0	0	75.75	75.09	70.86

In this study, production costs thus become the sum of market price of lumber (M_i) plus \$200 processing costs (P_i), e.g., FAS, \$1,770 per MBF; SEL, \$1,550 per MBF; 1 Common, \$1,200 per MBF; 2A Common, \$948 per MBF; and 3A Common, \$700 per MBF. By adding a uniform dollar amount per unit of input as processing cost to each lumber grade, a penalty for processing lower-grade lumber is created since more input volume of lower-grade lumber is needed to meet a cutting bill.

Model generation.—The objective of this research was to find a global optimal solution for individual cutting bills with regard to minimum raw material costs or minimum production costs. To fit the lumber grade–cost response surface, a second-order polynomial model (Eq. 3) was generated for each cutting bill using SAS 8.2 (SAS Institute Inc. 2002):

$$u_y = \beta_0^* + \sum_{i=1}^{5} \beta_i^* x_i + \sum \sum_{i<j} \beta_{ij}^* x_i x_j \qquad (3)$$

where

u_y = the cost of satisfying a given cutting bill;
$x_i x_j$ = the proportions of each lumber grade;
β_0^* = the intercept;
β_i^* = the coefficients of linear terms; and
β_{ij}^* = the coefficients of the interaction terms; $ij = 1$ for FAS, 2 for SEL, 3 for 1 Common, 4 for 2A Common, and 5 for 3A Common.

Using this lumber grade–cost response surface, an exhaustive search using SAS was conducted to locate the lowest cost point of the surface within the experimental area (Zuo 2003). The lumber grade or grades corresponding with the minimum cost point provide the optimal grade combination associated with minimum costs to satisfy given cutting bill requirements.

Results and Discussion

First, results from testing Buehlmann's cutting bill (Buehlmann et al. 2008a, 2008b) are presented followed by results for the 10 industry cutting bills (Wengert and Lamb 1994, Thomas 1996).

Buehlmann's cutting bill

The full model for the lumber grade–raw material cost response surface for Buehlmann's cutting bill is shown in Equation 4:

$$\begin{aligned} \text{COST} = {} & 1{,}886 + 157.84 \times \text{FAS} + 171.86 \times \text{SEL} \\ & - 313.38 \times \text{1Com} - 348.02 \times \text{2ACom} \\ & - 107.21 \times \text{FAS} \times \text{SEL} + 29.14 \times \text{FAS} \times \text{1Com} \\ & - 29.09 \times \text{FAS} \times \text{2ACom} \\ & - 504.29 \times \text{FAS} \times \text{3ACom} \\ & - 81.18 \times \text{SEL} \times \text{1Com} \\ & - 130.95 \times \text{SEL} \times \text{2ACom} \\ & - 730.82 \times \text{SEL} \times \text{3ACom} \\ & - 134.07 \times \text{1Com} \times \text{2ACom} \\ & - 796.84 \times \text{1Com} \times \text{3ACom} \\ & + 639 \times \text{2ACom} \times \text{3ACom}. \end{aligned} \qquad (4)$$

Based on this fitted response surface, an iterative search for the minimum cost point was conducted using SAS 8.2. For each search step, 10 percent grade increments were applied. The lowest cost grade combination for the Buehlmann cutting bill to minimize raw material costs according to the statistical model shown in Equation 4 is a grade mix of 70 percent 1 Common and 30 percent 3A Common lumber. The same procedure was followed to obtain the least-cost grade combination that minimizes total production cost (e.g., lumber plus processing costs). The production cost response surface for the Buehlmann cutting bill is shown in Equation 5:

$$\begin{aligned} \text{COST} = {} & 2{,}607 - 302.50 \times \text{FAS} - 238.70 \times \text{SEL} \\ & - 718.00 \times \text{1Com} - 662.00 \times \text{2ACom} \\ & - 139.20 \times \text{FAS} \times \text{SEL} + 6.47 \times \text{FAS} \times \text{1Com} \\ & - 132.20 \times \text{FAS} \times \text{2ACom} \\ & - 999.30 \times \text{FAS} \times \text{3ACom} \\ & - 111.70 \times \text{SEL} \times \text{1Com} \\ & - 218.60 \times \text{SEL} \times \text{2ACom} \\ & - 1{,}272.70 \times \text{SEL} \times \text{3ACom} \\ & - 186.90 \times \text{1Com} \times \text{2ACom} \\ & - 1{,}236.90 \times \text{1Com} \times \text{3ACom} \\ & + 821.10 \times \text{2ACom} \times \text{3ACom}. \end{aligned} \qquad (5)$$

The optimal grade combination for minimum production costs for the Buehlmann cutting bill is 80 percent 1 Common and 20 percent 3A Common lumber. Because of the many large parts that are difficult to obtain from 3A Common lumber, a high proportion of 1 Common lumber was included in the minimum total cost grade mix. Including processing costs indicated that cost inefficiency was associated with cutting the 3A Common lumber; thus, 10 percent more 1 Common lumber was recommended to minimize overall production costs. This shift in lumber grade distribution is the result of the advantage of lower processing cost per unit output for higher-grade lumber.

Actual industrial cutting bills

Each cutting bill has specific individual part size and quantity distributions (Buehlmann 1998) resulting in differing yield results for different lumber grades. Therefore, there is no uniform model that can be used for all cutting bills. Hence, for each cutting bill, a specific lumber grade–yield cost response surface has to be created on the basis of estimated yields obtained from RR2 (Thomas 1999). Table 5 presents the parameters for each factor (grade) and each factor interaction for the 10 cutting bills studied for the case when no processing costs are included. When processing costs were included, the lumber grade–cost response surface, and thus the regression model parameters changed. Using these lumber grade–raw material or production cost response surface models, iterative searches were conducted for both minimum material costs and minimum material and production costs using SAS 8.2. Table 6 shows the optimal lumber grade combinations that minimize raw material costs for the 10 industrial cutting bills according to the statistical models. Results for Buehlmann's cutting bill are included in Table 6 to allow for comparisons. The cutting bills are listed by order of difficulty as ranked in a previous study (Zuo et al. 2004) and cited in Table 2. Table 6 shows that lower-grade lumber, 2A Common and 3A Common, are the only grades required to satisfy the first four cutting bills (A, D, C, and B) at minimum costs. These cutting bills were defined as easy-to-

Table 5.—Regression parameters for raw material cost surfaces of the 10 industrial cutting bills.

Factor	Cutting bill									
	A	B	C	D	E	F	G	H	I	J
Intercept	1,876.75	1,393.75	1,222.88	1,461.22	3,587.85	5,598.41	2,196.86	2,589.52	2,800.77	4,852.68
FAS	3.67	6.68	8.34	6.78	−14.23	−35.21	−1.01	−4.05	−7.07	−25.37
SEL	6.04	5.54	8.86	7.25	−14.23	−32.39	−1.55	−4.66	−7.14	−23.75
1Com	−1.06	1.36	3.03	2.04	−17.06	−37.10	−5.31	−9.19	−10.88	−27.95
2Acom	−1.44	−0.45	1.45	0.86	−12.21	−29.75	−4.78	−10.43	5.17	9.51
FAS × SEL	−0.01	0.00	−0.02	−0.00	0.00	−0.03	0.01	−0.00	0.01	−0.02
FAS × 1Com	0.02	0.01	0.00	0.01	−0.04	−0.03	−0.01	−0.00	−0.01	−0.04
FAS × 2ACom	0.03	0.01	0.01	0.02	−0.14	−0.16	0.01	0.01	−0.30	−0.80
FAS × 3ACom	0.02	−0.01	0.03	0.03	−0.32	−0.59	−0.08	−0.17	−0.13	−0.58
SEL × 1Com	−0.00	0.00	−0.02	0.00	−0.01	−0.06	0.00	−0.01	0.01	−0.00
SEL × 2ACom	0.02	0.01	−0.00	0.01	−0.04	−0.20	−0.00	0.00	−0.27	−0.64
SEL × 3ACom	0.01	−0.02	0.04	0.03	−0.25	−0.86	−0.07	−0.22	−0.09	−0.31
1Com × 2ACom	0.00	0.00	−0.00	0.01	−0.06	−0.08	−0.00	−0.00	−0.24	−0.69
1Com × 3ACom	−0.02	−0.03	0.00	0.01	−0.23	−0.64	−0.08	−0.22	−0.10	−0.35
2ACom × 3ACom	−0.00	0.01	−0.00	−0.00	−0.03	0.16	0.10	0.04	0.66	2.44

cut cutting bills in Zuo et al.'s (2004) study. With increasing levels of cutting bill difficulty, more higher-grade lumber is required for the least-cost solution. However, lower-grade lumber (e.g., low-cost lumber) is kept as much as possible in the grade mix.

The consistent inclusion of a proportion of lower-grade lumber (i.e., 2A and 3A Common) in the optimal lumber grade combinations derived by the model is explained by the variation in lumber cost between lumber grades. According to the market prices used in this study for red oak lumber in January 2002 (*Weekly Hardwood Review* 2002), the price gap between FAS and 2A common was \$822 per MBF (\$1,570 per MBF vs. \$748 per MBF) and \$1,070 per MBF for FAS and 3A Common (\$1,570 per MBF vs. \$500 per MBF). Thus, using 2,090 board feet (BF) of 2A Common or 3,140 BF of 3A Common lumber costs the same as using 1,000 BF of FAS lumber. Put another way, a cutting bill that needs 1,000 BF of FAS lumber to meet the requirements could be satisfied more cheaply if less than 2,090 BF of 2A Common or less than 3,140 BF of 3A Common lumber were used. This is the scenario observed for the easy-to-cut cutting bills (A, D, C, and B) that require smaller parts and that, therefore, can be satisfied efficiently using lower-grade lumber.

Table 6.—Optimal lumber grade mix to minimize raw material cost based on the five-factor statistical model with interactions (without consideration of processing costs).

Cutting bill	Difficulty ranking	% without processing cost				
		FAS	SEL	1Com	2ACom	3ACom
A	1				100	
D	2				20	80
C	3				20	80
B	4				100	
H	5			70		30
G	6			80		20
E	7	10		70		20
Buehlmann	8			70		30
I	9			80	20	
F	10		50	20		30
J	11	40		40	20	

However, when cutting bills are difficult to cut, i.e., when they either require large parts (cutting bills I, F, and J) or have unevenly distributed part sizes or quantity requirements, better lumber qualities are needed. For example, in cutting bill J, 70 percent of the part lengths are shorter than 41 inches, while 25 percent are longer than 71 inches. There are no part requirements for lengths between 41 inches and 70 inches in cutting bill J. Either such cutting bills cannot be satisfied by lower-grade lumber only, or they require large quantities of lower-grade lumber, thus putting the lower grades in an economically inferior position. Processing more of the higher-grade lumber is economically advantageous in such cases. For example, cutting bills F and J, the two most difficult-to-cut cutting bills, require 50 percent SEL and 40 percent FAS lumber, respectively, to minimize raw material costs.

Table 7 presents the solutions that minimize total production costs (processing cost plus raw material cost). The least-cost lumber grade mix solutions for cutting bills A, C, B, H, and I are unchanged from the raw material–based cost scenario (Table 6) despite the fact that processing costs of \$200 per 1,000 BF are now included in the cost calculation. It is likely that the optimum grade composition

Table 7.—Optimal lumber grade mix to minimize total production cost based on the five-factor statistical model with interactions (with processing costs included).

Cutting bill	Difficulty ranking	% with \$200/MBF processing cost				
		FAS	SEL	1Com	2ACom	3ACom
A	1				100	
D[a]	2				100	
C	3				20	80
B	4				100	
H	5			70		30
G[a]	6			90		10
E[a]	7	50		30		20
Buehlmann[a]	8			80		20
I	9			80	20	
F[a]	10		60	10		30
J[a]	11	60		10	30	

[a] Cutting bills for which the optimal least-cost lumber grade mix shifted when processing costs were included in the model.

for these cutting bills changed slightly, but since the search was done in 10 percent increments, the change was not detected. The least-cost lumber grade mix for the other six cutting bills shifted toward higher-quality lumber. Using the total cost model, cutting bill D now requires 100 percent 2A Common lumber to minimize production cost, thus eliminating 3A Common from the mix. Cutting bills G, E, and F and Buehlmann also require more higher-grade lumber to minimize overall production cost than when no processing costs were included (Table 6). The most significant change in grade combinations occurred for cutting bill E. When minimizing total production cost, it requires 50 percent FAS, 30 percent 1 Common, and 20 percent 3A Common lumber, while 10 percent FAS, 70 percent 1 Common, and 20 percent 3A Common lumber was required to minimize raw material cost only.

Cutting bill J's grade mix shifted toward higher-grade lumber (60 percent FAS when processing costs are included vs. 40 percent FAS when no processing costs are included), but at the same time, the amount of lower-grade material required also increased (30 percent 2A Common when processing costs are included vs. 20 percent 2A Common when no processing costs are included). This change occurred because relative production costs for the lower-grade portion of the lumber inputs decreases when the larger parts are obtained from higher-grade lumber (e.g., from the 60 percent FAS in the case of cutting bill J). When the large parts are readily obtained using higher-grade lumber, yield from lower grades increases since only smaller part sizes need to be cut from the lower grades.

The importance of placing a reasonable boundary on the maximum allowable use of low-quality lumber (e.g., 3A Common) is demonstrated by cutting bill F. When setting the upper boundary for 3A Common for cutting bill F at 80 percent, the least-cost lumber grade mix model returns a solution of 60 percent SEL, 10 percent 1 Common, and 30 percent 3A Common. However, iterative testing showed that a better solution is 90 percent 1 Common and 10 percent 3A Common (Buehlmann et al. 2008c). When large quantities of low-grade lumber can be used in the model's initial tests, the least-cost lumber grade mix generates low yield figures (8.92 percent yield when using 20 percent 2A Common and 80 percent 3A Common; Table 3), skewing the response surface of the model. Thus, the minimum cost solution cannot be found. However, the problem can be corrected by reducing the maximum amount of low-grade material permitted in any solution or by imposing a limit for the minimum yield obtained from any lumber grade or lumber grade combination. A more detailed discussion can be found in Buehlmann et al. (2008c).

The changes in grade composition observed when adding processing costs to lumber costs demonstrates the sensitivity of the model. When $200 per MBF processing costs are added to total lumber costs, a more serious ''penalty'' for processing lower-grade lumber is introduced. Under the total production cost scenario, the relative total cost of 3A Common lumber increases 40 percent and increases 27 percent per MBF for 2A Common lumber. However, the relative cost increase is only 12 percent for FAS lumber. The total production cost ratio between FAS and 2A Common is 1:1.87. The ratio between FAS and 3A Common lumber is 1:2.53. Thus, for a cutting bill that uses 1,000 BF of FAS, the volume of lower-grade lumber required has to be less than 1,870 and 2,530 BF for 2A Common and 3A Common lumber, respectively, to result in lower cost. Compared with the ratios based on lumber costs only (2.09 and 3.14), the lower-grade lumber in the total cost scenario has to produce higher yields to be cost competitive with higher-grade lumber. For the easy-to-cut cutting bills (A, D, C, and B), lower-grade lumber still achieves yields that are sufficiently high to be more economical than higher-grade lumber despite the less favorable ratios.

The least-cost lumber grade mix model discussed in this article is expected to become an important tool for rough mill managers to use in scrutinizing their lumber grade mix decisions to find lower-cost solutions. Therefore, the validation of the model and the development of software incorporating the model was a high priority. As extensive testing showed, the new methodology to solve the least-cost lumber grade mix problem yields better (e.g., cheaper and more realistic) lumber grade mix solutions than earlier models that were based on the assumption of a linear relationship between yield and lumber grade(s) (Buehlmann et al. 2008c).

Conclusions

A statistical model to find the least-cost lumber grade mix was developed. The new model uses a mixture design to establish a test protocol to obtain simulated yields from the USDA Forest Service's ROMI-RIP 2.0 rough mill simulator. These simulated yields are then used to build a polynomial cost response surface that allows for an exhaustive search for the lowest-cost grade mix. The model searches for the lowest material cost grade mix or the lowest total production cost (e.g., lumber plus processing costs) grade mix.

The optimal least-cost lumber grade mix solutions generated by the statistical model discussed in this article tend to use as much low-grade lumber (2A and 3A Common) as possible, as long as the cutting bill requirements (in particular, large parts) still can be satisfied. The model indicated that low-grade lumber was preferred to fulfill the part requirements for easy-to-cut cutting bills. Difficult-to-cut cutting bills' requirements (e.g., cutting bills that require significant amounts of large parts) were satisfied by combining higher-grade lumber (SEL and FAS) with lower-grade lumber (2A Common and 3A Common). When processing costs are included in addition to the lumber costs, lower-grade lumber becomes more expensive relative to higher grades. Therefore, the more economical solution is to process more higher lumber grades. However, all these observations apply only within certain price differentials between different lumber grades.

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