A Merchantable and Total Height Model for Tree Species in Maine

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ABSTRACT: *A model for predicting merchantable and total tree height for 18 species groups in Maine is presented. Only tree-level predictor variables are used, so stand-level attributes, such as age and site quality, are not required. A mixed-effects modeling approach accounts for the correlated within-tree measurements. Data-collection protocols encompass situations in which merchantability to a specified top diameter is not attained due to tree characteristics. The advantage of using the height prediction model over taper-derived estimates of merchantable height is demonstrated. North. J. Appl. For. 23(4):241–249.*

Key Words: Nonlinear mixed-effects model, tree volume, tree taper, tree biomass.

While forest inventories are conducted for numerous purposes, one attribute that is commonly estimated from the sample is tree volume. Determination of volume often depends on a measure of tree height (Burkhart 1977, Scott 1981). Total tree volume relies on a measure of total height, whereas merchantable volumes are derived from height measurements to a specified top diameter (e.g., 4 in.) or to where other merchantability constraints occur (e.g., forking or excessive branching). Often, these height data are troublesome to collect because of the difficulty encountered in obtaining an unobstructed view of the measurement point and, for merchantable heights, the need to determine the point at which the top-diameter limit occurs. We present a model for estimating both merchantable and total tree height for species in Maine. Our approach differs from traditional height-diameter curves (Curtis 1967, Colbert et al. 2002) in that tree-level variables other than dbh are used. Also, auxiliary information, such as age and site index, are not required (Ek et al. 1984, Carmean et al. 1989).

Data

The data used in this research were collected under a cooperative agreement by the Northeastern unit of the USDA Forest Service's Forest Inventory and Analysis (NE-FIA) program and the Maine Forest Service (MFS). In 1999, NE-FIA and the MFS began implementing an annual inventory system in Maine. Under this federally mandated program, sampling is based on an interpenetrating panel design at an intensity of 1 plot per approximately 6,000 ac (Reams et al. 2005). Data were collected from 1999 to 2003; this period represents the first full measurement cycle of all sample plots in Maine (all five panels).

On each sample plot, FIA collects a myriad of data at various levels of detail. Of particular interest for this study are measures of total and merchantable height for all trees 5.0 in. or larger in dbh. Total height is measured from ground level to tip. A pulpwood merchantable height is obtained at the first of the following: 1) the point at which no 4-ft section can be produced due to excessive limbs, forks, or crooks; 2) a 4-in. top diameter; or 3) the point at which the central stem terminates by branching before reaching 4 in. in diameter (height can extend up through the major subdivisions of the main stem). For trees of sufficient size (≥9.0-in. dbh for softwoods; ≥11.0-in. dbh for hardwoods), an additional merchantable height for sawtimber is obtained. These measurements are taken at a minimum 7.0-in. top diameter for softwoods and 9.0-in. top diameter for hardwoods unless there is a point lower on the bole above which no 8-ft log is present due to tree form (USDA 2004). Observed height measurements were ocularly estimated or obtained with an instrument such as a clinometer. Trees with broken tops were removed from the data. A total of 185,971 height observations were available for analysis. Twenty-five percent of the data were selected randomly for validation purposes. The data are summarized in Table 1.

Model Development

Most of the existing tree-height models (as opposed to height-diameter curves) use site index and/or age (Hilt 1985, Harrison et al. 1986) as predictor variables. Although

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Table 1. Summary of data (trees with dbh ≥5.0 in.) collected from 1999 to 2003 in Maine.

	Minimum	Mean	Maximum
dbh (in.)	5.0	8.2	46.7
Saw height (ft)	12	29.9	89
Bole height (ft)	4	29.4	99
Total height (ft)	10	46.1	124
Basal area (ft^2/ac)	1	102	328
Stand age (years)	3	58	190

the measures often are computed from sample data, these attributes make little sense for application in uneven-aged stands. Furthermore, site index often is species-specific. Thus, conversions (if they exist) are needed to properly apply the model to other species in the stand. This creates an additional difficulty in applying such models to mixed-species stands. To facilitate application across a wide range of species and forest conditions, our model includes only tree-level predictor variables. The model form is based on the Chapman-Richards growth equation (Richards 1959):

$$H_{ij} = (\beta_0 D_{ij} + \beta_1 CC_{1i} + \beta_2 CC_{2i} + \beta_3 CC_{3i}) \cdot (1 - \exp(-\beta_4 DBH_i)) \cdot \exp(\beta_5 CR_i + \beta_6 TC_i + ((D_{ij}/DBH_i) + 0.01)^{\beta_7})_{+\varepsilon_{ij}} \quad (1)$$

where H_{ij} = tree height (ft) of the ith tree at jth top diameter D; D_{ij} = jth top-diameter (in.) within tree i;

$$TC_i = \text{Tree class of tree } i = \begin{cases} 1 = \text{preferred} \\ 2 = \text{acceptable} \\ 3 = \text{rough/rotten cull, dead} \end{cases}$$

CC_{ki} = Crown class indicators of tree i

$$= \begin{cases} k = 1, & = 1 \text{ intermediate, dead; } 0 \text{ otherwise} \\ k = 2, & = 1 \text{ dominant, codominant, open grown; } 0 \text{ otherwise} \\ k = 3, & = 1 \text{ overtopped; } 0 \text{ otherwise} \end{cases}$$

DBH_i = diameter at breast height (in.) of tree i; CR_i = compacted crown ratio (%) of tree i[1]; β_0–β_6 = fixed-effects population parameters; ε_{ij} = random error for ith tree at jth top diameter, $\varepsilon_{ij} \sim N(0, \sigma^2)$. As is often the case with forestry sample data, there is a lack of independence among observations. In the context of this study, there are two or three height measurements for each tree (total height, pulpwood height, and possibly sawlog height). This violates the ordinary least-squares regression assumption of independent observations. Although unbiased estimates of model parameters still are obtained when these correlations are ignored, the estimate of model standard error is biased (Swindel 1968, Sullivan and Reynolds 1976). This is a cause for concern because this directly affects inferences on estimated model parameters. Correlations among observations can be accounted for in the model fitting process by modeling the covariance structure or incorporating random-effects parameters into the model (Gregoire and Schabenberger 1996). We used the mixed-effects modeling approach, which allows model parameters to vary from the population estimate on an individual-tree basis (Valentine and Gregoire 2001). Our model was specified to have random-effects parameters associated with D (top-diameter limit) and DBH (dbh). A number of alternative random-effects formulations were evaluated. The expression chosen provided the best results. Other configurations exhibited poorer fit statistics or had convergence difficulties:

$$H_{ij} = ((\beta_0 + \gamma_{1i}) D_{ij} + \beta_1 CC_{1i} + \beta_2 CC_{2i} + \beta_3 CC_{3i}) \cdot (1 - \exp((-\beta_4 + \gamma_{2i})) \cdot \exp(\beta_5 CR_i + \beta_6 TC_i + ((D_{ij}/DBH_i) + 0.01)^{\beta_7})_{+\varepsilon_{ij}}, \quad (2)$$

where γ_{mi}= random-effects parameters for tree i, $\gamma_{mi} \sim N(0, \sigma^2_m)$, m = 1,2 other variables as defined previously.

Results

Our desire to cover the range of forest tree species in Maine required that certain species be grouped to maintain an adequate sample size (Table 2). Table 3 includes information by species group on most of the independent variables used in the model. Model 2 was fitted to each species or species group using the SAS NLMIXED procedure (SAS Institute, Inc. 2003). Estimates of fixed-effects parameters and variance components associated with each species group are given in Table 4. All estimates are significant at a type 1 error rate of 0.05.

To ensure that the correlation structure was adequately addressed via the specification of random-effects parameters, autocorrelation plots were developed from model residuals (Shumway and Stoffer 2000). In this application, lag distance was defined as the distance (in feet) between measurements on an individual tree. It is expected that correlation decreases as distance between measurement points increases. Figure 1 shows the autocorrelation plot and 95% confidence interval for zero for species group 12 (poplars). These results indicate that there are no significant correlations among residuals.

In some of the autocorrelation plots we examined, there were a few values that were below the lower 95 percent confidence line (we would expect 1 out of 20 to be outside this range). This situation occurred relatively infrequently, and the values were only slightly outside the confidence interval. For all practical purposes, the correlations among within-tree measurements were adequately accounted for. It should also be noted that this assessment differs from the typical time-series autocorrelation analysis in that the number of observations does not decrease with increasing lag distance. Thus, the computed correlation values for the first few and last few lag distances may be unreliable due to the small number of observations.

Table 2. Species groupings and composition percentages used to fit model 2.

Species	Group no.	Species group	Percentage of group
Larch (introduced)	1	Miscellaneous softwood	6.02
Norway spruce	1	Miscellaneous softwood	11.68
Jack pine	1	Miscellaneous softwood	5.66
Red pine	1	Miscellaneous softwood	66.24
Pitch pine	1	Miscellaneous softwood	9.67
Pond pine	1	Miscellaneous softwood	0.55
Scotch pine	1	Miscellaneous softwood	0.18
Tamarack (native)	2	Tamarack (native)	100.00
Eastern white pine	3	Eastern white pine	100.00
White spruce	4	White spruce	100.00
Black spruce	5	Black spruce	100.00
Red spruce	6	Red spruce	100.00
Balsam fir	7	Balsam fir	100.00
Eastern hemlock	8	Eastern hemlock	100.00
Northern white-cedar	9	Northern white-cedar	100.00
Sugar maple	10	Sugar maple	100.00
White ash	11	Ash	75.76
Black ash	11	Ash	21.01
Green ash	11	Ash	3.23
Balsam poplar	12	Poplars	8.43
Eastern cottonwood	12	Poplars	0.30
Bigtooth aspen	12	Poplars	29.34
Swamp cottonwood	12	Poplars	0.03
Quaking aspen	12	Poplars	61.90
Shagbark hickory	13	Miscellaneous hardwood	0.10
Black cherry	13	Miscellaneous hardwood	9.87
Scarlet oak	13	Miscellaneous hardwood	0.05
Northern red oak	13	Miscellaneous hardwood	82.11
Black oak	13	Miscellaneous hardwood	4.15
Basswood	13	Miscellaneous hardwood	0.15
American basswood	13	Miscellaneous hardwood	3.57
Yellow birch	14	Yellow birch	100.00
Paper birch	15	Paper birch	100.00
Maple	16	Other hardwood	0.06
Striped maple	16	Other hardwood	15.93
Silver maple	16	Other hardwood	0.96
Mountain maple	16	Other hardwood	0.19
Norway maple	16	Other hardwood	0.13
Ohio buckeye	16	Other hardwood	0.06
Serviceberry	16	Other hardwood	0.90
Sweet birch	16	Other hardwood	3.39
Gray birch	16	Other hardwood	24.70
American hornbeam	16	Other hardwood	0.19
Butternut	16	Other hardwood	0.26
Osage-orange	16	Other hardwood	0.06
Apple	16	Other hardwood	5.31
Eastern hophornbeam	16	Other hardwood	24.18
Pin cherry	16	Other hardwood	8.51
Chokecherry	16	Other hardwood	0.26
White oak	16	Other hardwood	5.37
Swamp white oak	16	Other hardwood	0.06
Willow	16	Other hardwood	0.58
Black willow	16	Other hardwood	0.06
White willow	16	Other hardwood	0.06
American mountain-ash	16	Other hardwood	2.69
Elm	16	Other hardwood	0.26
American elm	16	Other hardwood	5.82
Red maple	17	Red maple	100.00
American beech	18	American beech	100.00

Application of the fitted model to the validation data is accomplished by setting the random-effects parameters equal to their expected value (0). Plots of residuals (observed minus predicted) versus predicted values and residuals versus predictor variables in the validation data indicated no systematic problems with the fitted models. The parameter estimates in Table 4 were obtained by fitting the models to all available data.

Examination of Table 4 reveals several interesting outcomes. The estimates of the β_0 parameter indicate that, on average, between 4 and 5 ft should be subtracted from total height for every 1-in. increase of top-diameter limit. Thus, for many species, the merchantable height associated with a 4-in. top diameter is often 16–20 ft below tip height. There are also some notable changes in magnitude for some parameter estimates when moving from softwood to hardwood

Table 3. Summary of data for independent variables used in model 2.

Group	*n*	DBH		No. of trees by tree class			No. of trees by crown class indicator			No. of trees by crown ratio				
		Minimum	Maximum	1	2	3	CC_1	CC_2	CC_3	0–20	21–40	41–60	61–80	81–100
1	547	5.0	27	55	462	30	80	450	17	45	240	100	100	62
2	565	5.0	23.4	45	462	58	51	512	2	73	259	170	53	10
3	4324	5.0	34.6	296	3654	374	746	3104	474	840	2630	648	173	33
4	1827	5.0	23.5	442	1276	109	300	1360	167	161	684	539	297	146
5	2620	5.0	17.9	391	2110	119	306	2253	61	479	1368	543	199	31
6	10671	5.0	46.7	2319	7937	415	1917	7724	1030	1511	6061	2404	602	93
7	12831	5.0	18.5	1273	10927	631	2868	8121	1842	1369	5240	3822	2026	374
8	4750	5.0	30.3	373	4003	374	1187	2389	1174	228	1795	1990	697	40
9	8821	5.0	33.1	504	6823	1492	2279	5378	1162	1133	3729	2886	966	107
10	4405	5.0	41.8	164	3968	273	583	3617	205	252	2680	1289	177	7
11	1671	5.0	27.2	79	1462	130	225	1401	45	355	1123	177	16	0
12	3299	5.0	26.5	105	3015	178	234	3035	29	808	2041	401	48	1
13	2046	5.0	31.5	93	1829	124	247	1720	79	179	1523	321	23	0
14	3838	5.0	34.5	99	3237	502	631	3043	164	199	1987	1346	282	23
15	5155	5.0	25.4	113	4721	321	633	4414	108	746	3582	739	85	3
16	1564	5.0	24.9	4	1098	462	502	900	162	318	900	302	39	5
17	10866	5.0	30.5	86	9640	1140	1493	9015	358	1293	7934	1532	102	5
18	3776	5.0	21.7	23	3028	725	1111	2297	368	575	1851	1087	243	20

species groups. The estimates for β_7 are reduced by roughly a factor of 10 for the hardwood groups compared with softwood groups. The estimates for parameters β_5 and β_6 also exhibit changes in magnitude between softwoods and hardwoods. These results indicate that the shape of the curve for softwood species is mostly influenced by crown ratio and tree class. For hardwood species, the ratio of top diameter to dbh is the primary driver of curve shape. Overall, model error (denoted by σ^2) was smaller for softwoods than hardwoods. It is proposed that these differences arise out the deliquescent and excurrent tree forms that characterize hardwood and softwood species, respectively.

The parameter relationships outlined above indicate that the response curves for softwoods and hardwoods may exhibit different behaviors. Figure 2 shows height trends by dbh for three selected top diameters for species group 1 (miscellaneous softwood) and species group 13 (miscellaneous hardwood). For predictions of total height, the softwood group has a smaller height for the same diameter when trees are relatively small. However, the total heights are asymptotically similar between hardwoods and softwoods, which confirms the visual assessment that the softwood curve has a greater slope. For merchantable height associated with 4-in. top diameter, the softwood group again has a lower predicted height at the smaller diameters. However, the asymptotic merchantable height for softwoods exceeds that of the hardwood group, again indicating a comparatively steeper slope, but also reflecting the fact that tree form is becoming a limiting factor for the hardwood group. This trend is further defined in the comparison of heights associated with a 9-in. top diameter, where the trend for the hardwood group has notably less slope and a much lower asymptote than the softwood group. These comparisons suggest that for smaller trees, total and merchantable heights for a given dbh are lower for softwoods than those of hardwood species. Given that tree form plays a less significant role for smaller trees, these results imply differences in rates of tree taper. As trees grow larger, the merchantable heights for softwood become higher that those of similarly sized hardwood, which illustrates the effect of tree form on merchantability.

The validation data also were used to evaluate the ability of the models to predict new observations. Root mean squared error (RMSE) values were computed for each species group. The magnitude of errors was similar for all species groups and was comparable to results reported by Ek et al. (1984). Table 5 provides RMSE values for each species group by 2-in. diameter class. RMSE estimates for the higher diameter classes are based on few observations, and there is a trend toward increasing RMSE values as diameter class increases. This was expected, as the heights of large-diameter trees tend to be more variable than that of smaller trees. Although the results are not shown, examination of RMSE values for each of the height locations revealed similar consistency, i.e., there was no apparent difference in predictive ability across top diameter limits.

Discussion

A common underlying assumption for tree-height models is that dbh is positively correlated with tree height. Essentially, dbh describes the height-diameter curve. Estimates for individual trees deviate as a function of the other tree attributes included in the model.

The model form used is capable of assuming a variety of shapes, each of which approaches a maximum asymptotically. In our specification, this upper limit is defined by the top-diameter limit (D) and crown class (CC). As evidenced by the sign of the estimated parameter, specification of larger D results in lower predicted height values. For crown class, movement away from

Table 4. Estimated fixed-effects parameters (SE in parentheses) and variance components for 18 species groups fitted to model 2.

Group number	β_0	β_1	β_2	β_3	β_4	β_5	β_6	β_7	σ^2	σ_1^2	σ_2^2
1	−4.0092 (0.1585)	80.4990 (2.5373)	89.3969 (1.5573)	65.0335 (3.3668)	0.0969 (0.0053)	0.0124 (0.0007)	0.3150 (0.0209)	1.8500 (0.1385)	6.6272	1.3211	0.0006
2	−5.1720 (0.1856)	86.0820 (2.4237)	92.5925 (1.5894)	67.8563 (9.4091)	0.1161 (0.0073)	0.0116 (0.0010)	0.2785 (0.0208)	2.2087 (0.1943)	12.6592	2.1852	0.0010
3	−4.9167 (0.0634)	97.4497 (0.9305)	102.9998 (0.6262)	89.3026 (1.0128)	0.0762 (0.0016)	0.0126 (0.0003)	0.2908 (0.0058)	2.4458 (0.0650)	11.3670	1.7857	0.0003
4	−4.2800 (0.1111)	86.6051 (1.2062)	94.3969 (0.8120)	78.3769 (1.3751)	0.0881 (0.0025)	0.0105 (0.0003)	0.3172 (0.0096)	2.3064 (0.0817)	7.6704	2.3709	0.0004
5	−4.1837 (0.0871)	70.5456 (0.9497)	75.2077 (0.7033)	65.3523 (1.5069)	0.1372 (0.0052)	0.0119 (0.0006)	0.3175 (0.0119)	2.7234 (0.1409)	11.5519	0.7954	0.0009
6	−3.7487 (0.0350)	76.0761 (0.4392)	82.3788 (0.3103)	68.9345 (0.4860)	0.1076 (0.0015)	0.0119 (0.0002)	0.3012 (0.0045)	2.5734 (0.0464)	9.8571	1.0774	0.0006
7	−4.3376 (0.0512)	76.3004 (0.4414)	81.1055 (0.3709)	69.5792 (0.4475)	0.1159 (0.0017)	0.0089 (0.0002)	0.3524 (0.0051)	2.2889 (0.0487)	9.3069	1.5431	0.0007
8	−3.9480 (0.0472)	74.3455 (0.6317)	80.9280 (0.4593)	69.3210 (0.6385)	0.0947 (0.0018)	0.0085 (0.0003)	0.3092 (0.0070)	2.0523 (0.0499)	7.4662	1.2285	0.0006
9	−3.9131 (0.0340)	65.9403 (0.4001)	68.0198 (0.3224)	63.9439 (0.4587)	0.1014 (0.0016)	0.0069 (0.0002)	0.3495 (0.0055)	2.4178 (0.0509)	6.8761	0.8156	0.0005
10	−4.3957 (0.0476)	75.4191 (0.8419)	81.4198 (0.6689)	71.8111 (1.0414)	0.0904 (0.0027)	0.0025 (0.0003)	0.0377 (0.0106)	0.1791 (0.0084)	22.3565	0.5609	0.0006
11	−4.4621 (0.1156)	81.1349 (1.7332)	88.7389 (1.4926)	76.6944 (2.4899)	0.1158 (0.0064)	0.0034 (0.0007)	0.2252 (0.0296)	0.2315 (0.0257)	22.7739	0.8479	0.0008
12	−4.2401 (0.0620)	84.2529 (1.2424)	91.5048 (0.9913)	78.7788 (2.2708)	0.1023 (0.0037)	0.0054 (0.0004)	0.0638 (0.0136)	0.1422 (0.0077)	18.2092	0.7537	0.0006
13	−4.8748 (0.0859)	85.1930 (1.6789)	91.5190 (1.4055)	81.2123 (2.1023)	0.0810 (0.0038)	0.0022 (0.0005)	0.0560 (0.0150)	0.1471 (0.0092)	18.0889	0.7026	0.0005
14	−4.2234 (0.0535)	69.5686 (0.8925)	74.5607 (0.7499)	66.4175 (1.1748)	0.0831 (0.0025)	0.0028 (0.0003)	0.0465 (0.0093)	0.2132 (0.0098)	22.0562	0.3929	0.0006
15	−4.2171 (0.0649)	75.1335 (1.0204)	82.3602 (0.9584)	77.6651 (1.5500)	0.1075 (0.0038)	0.0055 (0.0004)	0.1000 (0.0129)	0.1828 (0.0098)	17.1442	0.5007	0.0008
16	−4.4116 (0.2640)	82.9939 (2.9170)	87.7027 (2.9200)	78.4396 (2.9867)	0.0762 (0.0057)	0.0025 (0.0004)	0.1215 (0.0158)	0.1929 (0.0149)	18.0385	0.5573	0.0004
17	−4.2438 (0.0399)	75.3416 (0.6286)	80.8032 (0.5383)	74.9467 (0.8481)	0.1015 (0.0022)	0.0033 (0.0003)	0.1090 (0.0085)	0.2142 (0.0080)	21.1825	0.5707	0.0007
18	−4.1582 (0.0831)	75.5070 (1.2566)	80.7061 (1.2105)	73.3890 (1.3942)	0.0811 (0.0032)	0.0017 (0.0002)	0.0796 (0.0106)	0.1738 (0.0086)	18.6266	0.7519	0.0006

dominant classification produces smaller estimates of tree height. As would be expected in forested sample plots, there were few open-grown trees in the data, so model predictions for open-grown trees should be used with caution.

The shape of the curve depends on crown ratio (CR), tree class (TC), and the ratio of the top-diameter limit to dbh (D/DBH). As expected, increasing crown ratio results in a less steep ascent to the asymptote, which, all else being equal, provides smaller estimates of height (Figure 3). For tree class, movement away from preferred status lowers predicted heights (Figure 4). Increases in the ratio of top-diameter limit to dbh have the same effect, as smaller predicted heights are necessary when the top-diameter limit increases. These trends are consistent with the expected effects of these variables on tree heights.

Unlike many other tree-height modeling efforts, we also incorporated the ability to predict height of standing dead trees. Our motivation for this approach was to provide estimates of tree heights for use in computation of stem biomass (Wharton and Griffith 1998). The primary difficulty in prediction for dead trees is that the values of predictor variables often differ from those of live trees of similar size and shape, e.g., crown variables. Both the crown class and tree class predictors include dead trees. Trees suffer mortality for a variety of reasons (suppression, disease, weather, etc.), so dead trees were considered to be equivalent to intermediate crown class trees. Similarly, mortality may often have a detrimental effect on tree quality, such that assignment of tree class would be similar to that of rough/rotten cull. There is no contribution from the crown ratio term, as dead trees are appropriately assigned a value of 0.

Example Calculation

To illustrate the use of the model, we provide an example based on a poplar tree (species group = 12) with the following attributes: dbh = 15.5 in.; compacted crown ratio = 40 percent; tree class = acceptable (TC = 2); crown class = codominant (CC_1 = 0; CC_2 = 1; CC_3 = 0). As mentioned earlier, the random-effects parameters assume their expected value (0) when making predictions for a new tree. Thus, the form of the model is given by model 1.

1) Prediction of total height:

$$H_{i0} = (-4.2401(0) + 84.2529(0) + 91.5048(1) + 78.7788(0)) \cdot (1 - \exp(-0.1023(15.5))) \cdot \exp(0.0054(40) + 0.0638(2) + ((0/15.5) + 0.01)^{0.1422})$$

$$H_{i0} = (91.5048) \times (0.7943)\exp(0.3436 + (0.01)^{0.1422})$$

$$H_{i0} = 75.0 \text{ ft}$$

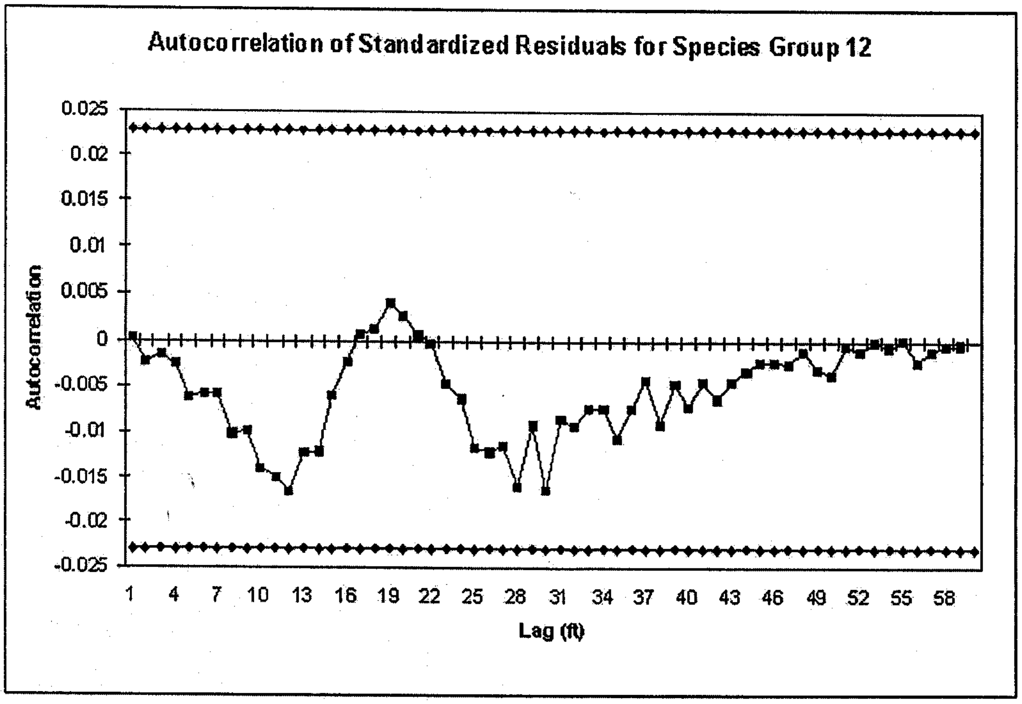


Figure 1. Autocorrelation of standardized residuals from model 2 fitted to data from species group 12.

2) Prediction of bole height (4-in. top diameter):

$$H_{i4} = (-4.2401(4) + 84.2529(0) + 91.5048(1) + 78.7788(0)) \cdot (1 - \exp(-0.1023(15.5))) \cdot \exp(0.0054(40) + 0.0638(2) + ((4/15.5) + 0.01)^{0.1422})$$

$$H_{i4} = (74.5444) \times (0.7943)\exp(0.3436 + (0.2681)^{0.1422})$$

$$H_{i4} = 56.9 \text{ ft}$$

3) Prediction of sawlog height (9-in. top diameter):

$$H_{i9} = (-4.2401(9) + 84.2529(0) + 91.5048(1) + 78.7788(0)) \cdot (1 - \exp(-0.1023(15.5))) \cdot \exp(0.0054(40) + 0.0638(2) + ((9/15.5) + 0.01)^{0.1422})$$

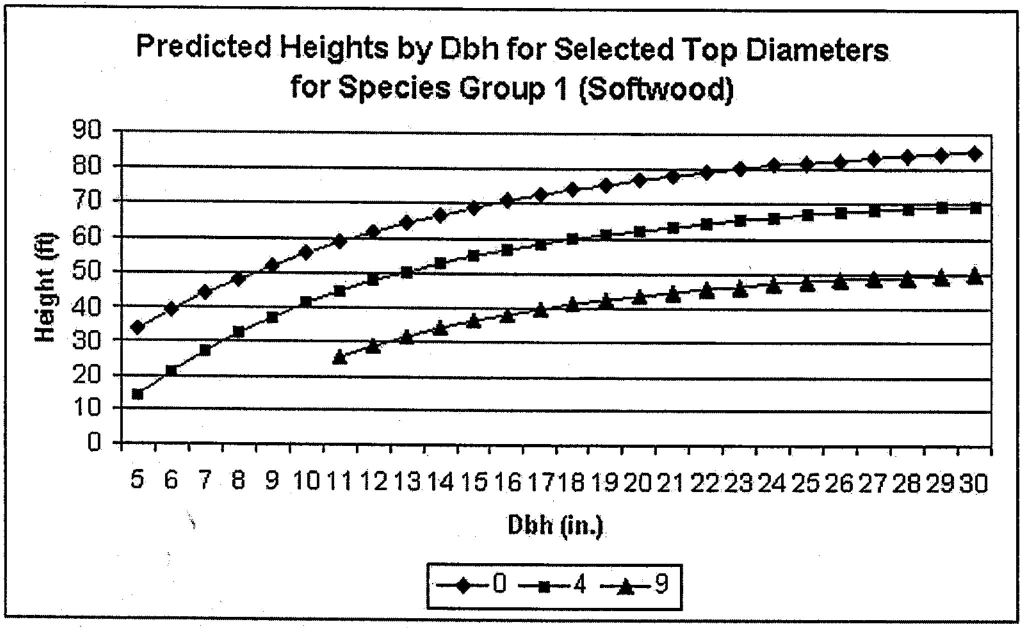
$$H_{i9} = (53.3439) \times (0.7943)\exp(0.3436 + (0.5906)^{0.1422})$$

$$H_{i9} = 39.8 \text{ ft}$$

Comparison with Taper-Derived Heights

For merchantable heights, the models inherently account for effects of tree characteristics due to the prescribed protocols for data collection. Derivation of these height measures with a taper model will not take tree form into account, and merchantable heights likely will be overestimated, particularly for hardwood species. To illustrate this phenomenon, we fitted the segmented polynomial taper model developed by Max and Burkhart (1976) to tree-taper data collected on 30 paper birch trees in the Northeast. A total of 379 height-diameter pairs were taken on standing trees using a Barr & Stroud[2] dendrometer. This sample size is somewhat small; however, it serves to demonstrate the potential bias trend.

The fitted taper model was used to predict height to a 4-in. top diameter for all paper birch trees in the NE-FIA data. Comparison between the taper-derived and observed heights associated with a 4-in. top-diameter limit shows that there is fairly good agreement (within expected range of measurement error) for trees in the lower dbh classes (Figure 5). This is expected, as form issues are not as great for smaller trees. However, as trees become larger, more forks, crooks, etc. occur as the height along the bole increases. This results in merchantability limits below the 4-in. top-diameter limit. This difference seems to increase as tree size increases, and predictions of merchantable height from the taper model tend to overestimate the actual merchantable portion of the stem.



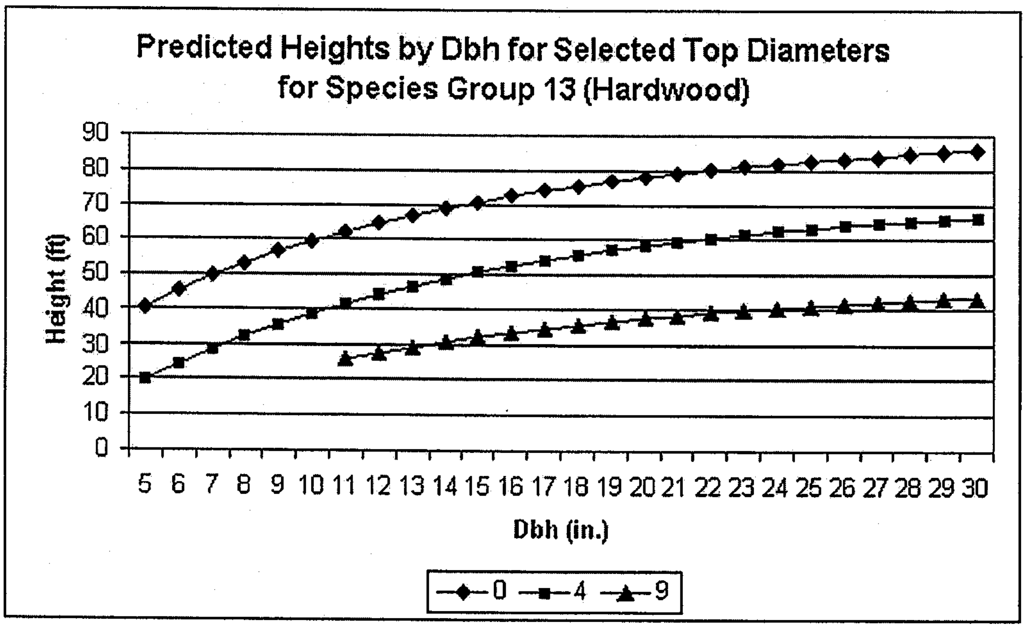


Figure 2. Predicted heights by dbh for top diameters 0, 4, and 9 for species group 1 (miscellaneous softwood) and species group 13 (miscellaneous hardwood).

Conclusion

The model presented here can be used to estimate heights to a specified upper-stem diameter while accounting for individual tree attributes that affect the actual amount of merchantable bole. The predictor variables in the model generally are easier to measure and are also more repeatable than height measurements (Frieswyk 2001). Results can be used to predict desired heights for most tree species in Maine. Caution should be used when applying the model outside the range of top-diameter limits used in this study (>7.0 in. for softwoods and >9.0 in. for hardwoods). The general principles applied in developing this model should be applicable to other geographical areas.

[1] Compacted crown ratio is that portion of the tree supporting live foliage and is expressed as a percentage. It is estimated by visually transferring lower live branches to fill in any gaps in the upper portion of the tree (USDA 2004).

[2] The use of trade, firm, or corporation names in this paper is for the information and convenience of the reader. Such use does not constitute an official endorsement or approval by the USDA or Forest Service of any product or service to the exclusion of others that may be suitable.

Table 5. Number of observations and root mean squared error of height from model 2 by 2-in. diameter class from validation data for 18 species groups.

Group number	*n*	Diameter class (in.) 6	8	10	12	14	16	18	20	22[a]	Group mean
1	304	5.91	8.42	11.45	9.62	10.87	8.28	9.83	1.38	9.86	8.49
2	333	7.86	9.57	10.35	9.79	10.31	7.20	12.46		1.16	9.16
3	2611	6.51	8.05	7.91	8.50	10.17	10.49	10.84	8.33	11.60	8.49
4	1068	6.08	6.81	8.19	9.97	8.64	6.30	9.48	10.39	7.06	7.44
5	1381	6.37	7.03	8.04	6.84	10.38	14.50				6.86
6	6200	6.65	7.12	7.39	7.31	7.20	8.45	9.46	10.32	19.72	7.14
7	6771	6.25	7.13	7.37	8.35	9.15	9.42	9.45			6.76
8	2951	6.00	7.38	6.96	7.00	7.62	7.76	9.87	8.74	8.41	7.03
9	5170	4.86	5.45	5.55	5.96	6.05	8.16	8.52	6.58	11.65	5.63
10	2497	6.91	7.80	7.28	7.75	7.86	9.15	9.51	10.54	10.43	7.76
11	859	7.89	8.99	9.43	7.17	10.35	10.36	9.61	19.56	14.83	8.56
12	1841	6.93	8.46	7.46	9.58	8.07	12.49	10.59	12.73	13.78	8.30
13	1166	7.58	7.84	7.87	9.31	9.01	9.84	9.88	14.08	11.53	8.32
14	2164	6.91	7.98	8.12	7.73	8.23	8.14	9.22	9.72	11.67	7.81
15	2691	7.39	8.17	8.88	9.25	9.94	8.93	10.11		31.57	8.11
16	785	6.80	8.32	11.34	10.51	8.18	22.11	21.86	14.66	14.08	7.79
17	5739	7.45	8.21	8.42	8.18	8.79	9.30	10.62	12.66	18.32	8.08
18	2003	7.73	8.25	8.01	8.35	9.22	12.02	9.84	12.77	17.13	8.16
Class mean		6.71	7.51	7.55	7.88	8.25	9.30	9.98	10.12	12.28	

[a] Includes all trees 21.0 in. and larger in diameter at breast height.

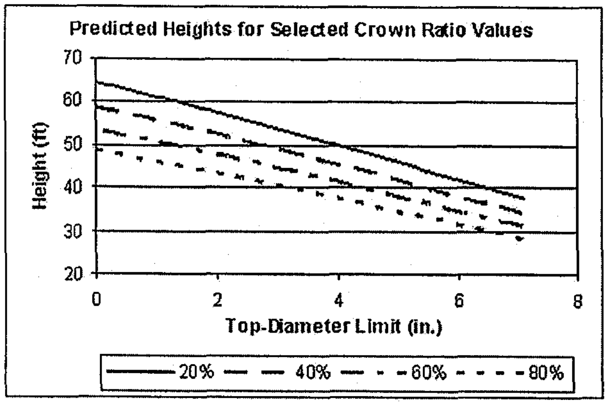


Figure 3. Predicted heights for crown ratios of 20, 40, 60, and 80 percent for live pitch pine tree (species group 1) with 12.0-in. dbh, tree class 2, and codominant crown class.

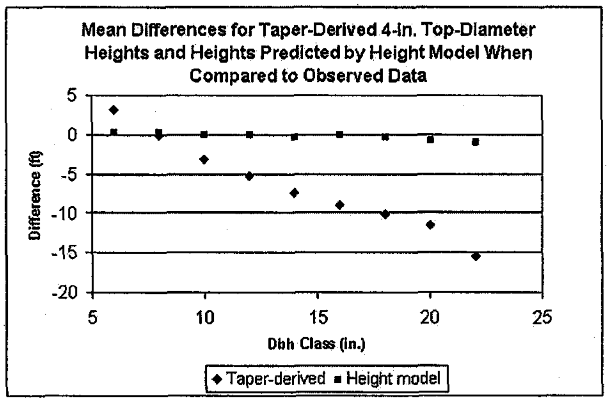


Figure 5. Mean differences for height to 4-in. top diameter estimated with taper equation and height to 4-in. top diameter estimated from model 2 compared with observed height data for paper birch trees in Maine.

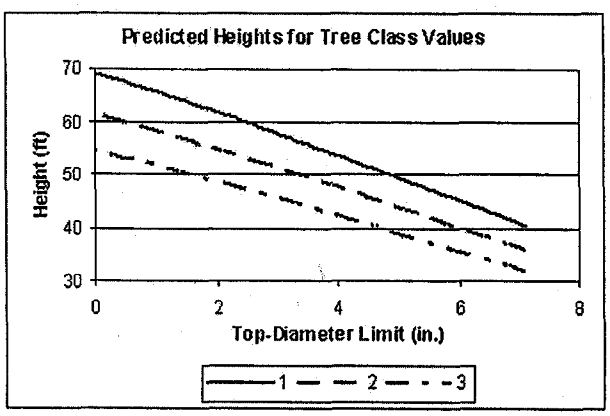


Figure 4. Predicted heights for tree classes 1, 2, and 3 for live pitch pine tree (species group 1) with 12.0-in. dbh, 30 percent crown ratio, and codominant crown class.

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