Available online at www.sciencedirect.com
SCIENCE DIRECT®
Ecological Modelling 180 (2004) 119–133
ECOLOGICAL MODELLING
www.elsevier.com/locate/ecolmodel

A hierarchical fire frequency model to simulate temporal patterns of fire regimes in LANDIS

Jian Yang[a,*], Hong S. He[a], Eric J. Gustafson[b]

[a] School of Natural Resources, University of Missouri-Columbia, 203 Anheuser-Busch Natural Resources Building, Columbia, MO 65211, USA
[b] USDA Forest Service, North Central Research Station, 5985 Highway K, Rhinelander, WI 54501, USA

Received 27 May 2003; received in revised form 15 March 2004; accepted 15 March 2004

Abstract

Fire disturbance has important ecological effects in many forest landscapes. Existing statistically based approaches can be used to examine the effects of a fire regime on forest landscape dynamics. Most examples of statistically based fire models divide a fire occurrence into two stages—fire ignition and fire initiation. However, the exponential and Weibull fire-interval distributions, which model a fire occurrence as a single event, are often inappropriately applied to these two-stage models. We propose a hierarchical fire frequency model in which the joint distribution of fire frequency is factorized into a series of conditional distributions. The model is consistent with the framework of statistically based approaches because it accounts for the separation of fire ignition from fire occurrence. The exponential and Weibull models are actually special cases of our hierarchical model. In addition, more complicated non-stationary temporal patterns of fire occurrence also can be simulated with the same approach. We implemented this approach as an improved fire module in LANDIS and conducted experiments within forest landscapes of northern Wisconsin and southern Missouri. The results of our experiments demonstrate this new fire module can simulate a wide range of fire regimes across heterogeneous landscapes with a few parameters and a moderate amount of input data. The model possesses great flexibility for simulating temporal variations in fire frequency for various forest ecosystems and can serve as a theoretical framework for future statistical modeling of fire regimes.

Keywords: Fire frequency model; Fire regime; Hierarchical modeling; LANDIS

1. Introduction

Disturbances such as fire are often key factors driving the dynamics of forested landscapes in many ecosystems such as boreal forests (Heinselman, 1970), coastal western chaparral shrublands (Keeley et al., 1999; Moritz, 2003), and the northern and central hardwood forests (Bormann and Likens, 1979; Guyette and Larsen, 2000). Small and large fires of varying intensity strongly affect species composition and age distribution (Hough and Forbes, 1943). Fire creates a mosaic of burned and unburned forest patches, leaving complex heterogeneous patterns across the

* Corresponding author. Tel.: +1 573 884 9410.
E-mail address: jym6b@missouri.edu (J. Yang).

0304-3800/$ – see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.ecolmodel.2004.03.017

landscape. The resulting landscape heterogeneity can further influence successional processes, which in turn may affect the spatial spread of subsequent fires (Turner and Romme, 1994). In this light, modeling the temporal and spatial pattern of fire disturbance, as well as the interaction between fire disturbance and landscape heterogeneity, is very important for understanding forest landscape dynamics.

A large variety of fire models have been developed to examine the effects of fire regimes (e.g., fire frequency, severity, and extent of disturbances) on the recovery of disturbed landscapes. Because different models have varying purposes, applicable spatial and temporal extents, and levels of ecological detail, they use different approaches to simulate fire occurrence, behavior and effects. Detailed reviews on the approaches can be found in Albright and Meisner (1999), and Gardner et al. (1999). Here we discuss only one family of these approaches, called statistically based approaches, that are often applied to forest landscape simulations over large spatial and temporal domains. Statistically based approaches use the distribution of fire frequency and fire size, and estimates of the renewal rate of disturbed forests from fire history studies (e.g., Heinselman, 1973) to simulate a given fire regime. The approaches have evolved from the theory of Weibull and exponential fire history models (Van Wagner, 1978; Johnson and Van Wagner, 1985) and are used in many models such as DISPATCH (Baker et al., 1991), the model of Antonovski et al. (1992), REFIRES (Davis and Burrows, 1994), FLAP-X (Boychuk and Perera, 1997), ON-FIRE (Li et al., 1997), LANDIS (He and Mladenoff, 1999), and LADS (Wimberly et al., 2000). All these spatial models simulate fire ignition from certain probability distributions, use fire probability to determine whether an ignition can become an active fire, and randomly generate a pre-defined fire size to simulate the extent of a given fire occurrence. The distributions of fire frequency and fire size are used to estimate fire probability and the pre-defined fire size for the simulation of fire occurrence and spread, respectively. The main differences among these models are that (1) the distributions used to model fire frequency and fire size are different; and (2) the way that fire probability is calculated varies. For example, the fire frequency distribution in DISPATCH is uniform while the distribution in LADS is Poisson, and the fire size distribution in LANDIS is lognormal while the distribution in LADS is exponential.

The term *fire occurrence* here refers to a detected active fire that happens when the fire begins to spread through the forest fuel complex as a surface fire or a crown fire and emits significant amounts of smoke and energy (Anderson et al., 2000). Some modelers use the term *fire ignition* to refer to fire occurrence, and use other terms such as potential fire ignition (Davis and Burrows, 1994) or fire source (Antonovski et al., 1992) to refer to fire ignition as used here. No matter what terms are employed in the models, the essence of implementing a statistically based approach is to divide a fire occurrence into two consecutive events—fire ignition and fire initiation (Li, 2000). Separating fire ignition from fire occurrence helps to separate the abiotic factors influencing fire ignition such as climate, topography, and human activities from the influences on fire spread of biotic factors such as fuel accumulation and vegetation structure. Abiotic factors often stay relatively stable in the statistically based fire regime models, whereas biotic-related processes are typically dynamic.

Although current statistically based fire regime models have been successfully applied in various studies, they typically are not flexible enough to simulate the full range of fire regimes observed in forested ecosystems. This is mainly because the modeling of fire ignition and calculation of fire probability in these models cannot fully account for the fire frequency distribution of various fire regimes. Fire frequency distributions in these models are deduced from the exponential or Weibull model in fire history studies, in which a fire occurrence is treated as one single event rather than two events in fire regime models. It is, therefore, conceptually inaccurate to apply the exponential or Weibull model directly into fire regime modeling. Moreover, in statistically based fire regime models, including the current version of LANDIS (version 3.7), there is a common mistaken practice of using the fire hazard function provided by the exponential or Weibull model as the fire probability function. Fire hazard is defined as the instantaneous rate of burning (Johnson and Van Wagner, 1985). In discrete time, fire hazard, denoted as $h(t)$, is the probability of fire in year t given that a fire has not yet occurred (Clark, 1989). It represents a combination of the rate of ignition and the probability of the fire spreading given the presence of ignition sources (McCarthy et al., 2001). On the other hand, fire probability in fire regime models is the probability of a fire occurrence given the presence of an ignition. It is determined primarily by the process of fuel build-up, which is often assumed to be a function of time since last fire. Thus, fire probability is different from fire hazard. However, many fire regime models assume that fire hazard equals fire probability. This often causes problems in model parameterization because the discrepancy between fire hazard and fire probability is not properly accounted for in the calculation of fire probability in these models.

In this study, we use the theory of hierarchical modeling and mixture distributions to model fire frequency. Hierarchical modeling in statistics refers to modeling a complicated process by a sequence of relatively simple models placed in a hierarchy (Casella and Berger, 2001). It is based on the fact that the joint distribution of a collection of random variables can be decomposed into a series of conditional models. That is, if *A*, *B*, and *C* are random variables, we can write a factorization such as $[A, B, C] = [A|B, C]\ [B|C]\ [C]$. The notation $[A]$ denotes the probability distribution of *A*; $[A|B]$ represents the conditional distribution of *A* given the random variable *B*. Random variable *A* has a mixture distribution, because the distribution of *A* depends on a quantity *B* that also has a distribution. Because statistically based fire regime models simulate fire occurrence as two consecutive stages, it is natural to use the theory of hierarchical modeling to model fire frequency distribution as a mixture distribution.

The objectives of this study are: (1) to design a hierarchical fire frequency model that accounts for the separation of fire ignition from fire occurrence; (2) to implement the hierarchical model as an improved fire module in LANDIS; and (3) to explore the potential of the improved module to simulate a wide range of temporal patterns of various fire regimes on heterogeneous landscapes.

2. Fire frequency models

2.1. Terms describing temporal patterns of fire regimes

The combination of certain aspects of fire disturbance, especially fire frequency, size, severity, and seasonality, may be used to characterize a fire regime. Here we only clarify the terms that will be used in fire frequency models, that include fire frequency, fire interval, fire cycle, and mean fire size. *Fire frequency* is the number of fires per unit time in a specific area (Agee, 1993). The size of the specific area will affect fire frequency: larger areas will have a higher fire frequency (Johnson and Van Wagner, 1985). The reciprocal of fire frequency is *fire interval*, which is the elapsed time between two successive fires in a specific place (McPherson et al., 1990). Fire interval often is modeled using a Weibull distribution or an exponential distribution, a special case of the Weibull distribution where the fire hazard is held constant (Johnson and Van Wagner, 1985). *Fire cycle* is the number of years necessary for an area equal to the entire area of interest to burn (Johnson and Van Wagner, 1985; Turner and Romme, 1994). This definition does not imply that the entire area would burn during a cycle; some sites may burn several times, while others do not burn at all. Fire cycle also is referred to as fire rotation (Agee, 1993). The distribution of fire size is usually difficult to estimate. However, mean fire size also is a common descriptor in the study of fire regimes, and the relationship among the size of study area (AREA), mean fire size (MFS), mean fire frequency (MFF), and fire cycle (FC) is depicted in the following equation (Boychuk et al., 1997).

$$\mathrm{AREA} = \mathrm{MFS} \times \mathrm{MFF} \times \mathrm{FC} \tag{1}$$

2.2. Exponential model

If fire hazard is constant, then fire interval has an exponential distribution, and fire frequency is distributed as a Poisson process (Van Wagner, 1978). Following statistical conventions, random variables are denoted by uppercase letters and their observed numerical values are denoted by lowercase letters. The probability density function (PDF) of a random variable, such as *X*, is denoted as $f_X(x)$, and its cumulative density function (CDF) is denoted as $F_X(x)$. The relational symbol ($\sim$) means "is distributed as". We use *U* to denote the number of fire occurrences per unit time in the study area, and *T* to denote the time since last fire. *U* follows Poisson distribution with parameter α. *T* then has an exponential distribution with parameter β, which is the inverse of α (Appendix A). The probability density function of U and T are Eqs. (2) and (3), respectively.

$$f_U(u) = \frac{e^{-\alpha}\alpha^u}{u!} \quad (2)$$

$$f_T(t) = \frac{1}{\beta}e^{-t/\beta} \quad (3)$$

MFF is the expected value of U that equals α, and mean fire interval (MFI) is the expected value of T that equals β. Fire hazard is independent of time since last fire and it equals $1/\beta$, or α.

2.3. Weibull model

Fire interval is widely modeled as a Weibull distribution because it permits fire hazard to increase or decrease with time since last fire (Johnson and Van Wagner, 1985; Clark, 1989; Johnson and Gutsell, 1994; McCarthy et al., 2001). The probability density function of time since last fire with parameters β and γ is

$$f_T(t) = \frac{\gamma}{\beta}t^{\gamma-1}\,e^{(-1/\beta)t^{\gamma}} \quad (4)$$

The fire hazard function for the Weibull distribution is

$$h(t) = (\gamma/\beta)t^{\gamma-1} \quad (5)$$

When γ equals 1, fire hazard is then a constant $1/\beta$, and Eq. (4) becomes the same as Eq. (3). Hence the exponential model is a special form of Weibull model.

There is no explicit distributional form for fire frequency when fire interval is modeled as a Weibull distribution. Instead, a fairly complicated renewal function, denoted as $R(t)$, is used to give the expected number of fire occurrences during time (0, t) (Clark, 1989).

$$R(t) = F(t) + \int_0^t R(t-x)\,\mathrm{d}F(x) \quad (6)$$

2.4. Hierarchical fire frequency model

Unlike the previous two fire-interval models, our hierarchical fire frequency model divides a fire occurrence into two consecutive events—fire ignition and fire initiation. A fire occurrence begins with an ignition from an external heat source, that heats the forest fuel complex up to its ignition temperature. Fire ignition agents are either natural (lightning) or anthropogenic (e.g., arson or accidental). A fire initiation event starts with the ignition until a certain area whose size is equal to the grain of the model is burned (Li, 2000). Whether a fire ignition can result in fire initiation is dependent on the fuel loading, fuel arrangement, and fuel moisture content.

Let X denote the number of fire ignitions per unit time in a specific area. X is a discrete random variable and follows a Poisson distribution with the parameter, intensity λ, that is the expected number of ignitions per unit time (Cunningham and Martell, 1973; Van Wagner, 1978; Anderson et al., 2000; Pennanen and Kuuluvainen, 2002). The fire initiation process can be modeled as a Bernoulli trial. Given x ignitions during the unit time, we assume there exist x random variables Y_i ($i = 1, 2, \ldots, x$) taking a value 1 if a fire ignition results in fire initiation and 0 otherwise. Each Y_i has a Bernoulli distribution with the parameter fire probability (P_i). The sum of these Bernoulli trials (i.e., fire occurrence per unit time) is fire frequency (U). The conditional distribution of $U|X$ (conditional distribution of fire occurrence given the fire ignitions) primarily is determined by how we define the fire probability function. If we assume fire probability (P) is independent of time since last fire, and constant across the fire regime, then $U|X$ follows a binomial distribution with the parameters X and P, and U follows a Poisson distribution with the parameter λP (Appendix B). In this case, the fire frequency distribution is identical to the one in the exponential model, where fire hazard (α) equals the product of fire ignition intensity (λ) and fire probability (P). Hence our hierarchical fire frequency model is consistent with previous fire frequency models, except that fire hazard is more accurately modeled as the combination of ignition rate and fire probability.

Fire probability also can be a function of time since last fire. Here the form of the fire probability function is determined primarily by the fuel accumulation within the ecosystem that can be estimated from data on the rate of fuel accumulation and occurrences of fire. As long as fire probability is not a constant, fire occurrence is a complex heterogeneous Poisson process, whose distribution is often difficult to explicitly formulate in a single equation. However, this fairly complex process can be factorized into much simpler probabilistic distributions as shown in Eqs. (7)–(10).

$$[U] = [U|X][X|\lambda][\lambda] \quad (7)$$

$$[X|\lambda] \sim \text{Poisson}(\lambda) \quad (8)$$

$$U|X = \sum_{1}^{X} Y_i \quad (9)$$

$$[Y_i|P_i] \sim \text{Bernoulli}(P_i) \quad (10)$$

3. Fire module in LANDIS

3.1. LANDIS overview

LANDIS is a spatially explicit and stochastic raster-based model that simulates forest landscape change over long time domains (10^1–10^3 years) and large heterogeneous landscapes (10^3–10^7 ha). The model currently operates on 10-year time step. It is designed to model ecological dynamics and interactions of temporal processes such as succession, and spatial processes such as seed dispersal, disturbances, and forest management (Mladenoff et al., 1996; Mladenoff and He, 1999, Gustafson et al., 2000). In LANDIS, a large landscape is stratified into several small relatively homogeneous fire regime units such as ecoregions or land types where the meteorological, physical, and biological properties as well as ecological factors are uniform. LANDIS simulates fire regime units based on their fire cycles and statistics of fire sizes that are specified by the users. Further details about the simulation of fire regime and its interactions with succession can be found in He and Mladenoff (1999).

3.2. Fire module design

We incorporated the hierarchical fire frequency model into LANDIS by dividing the fire process into three stages: fire ignition, fire initiation, and fire spread. For a given time step (i.e., 10 years), LANDIS first generates the number of ignitions (X) in the given fire regime unit from the Poisson distribution with the parameter λ (i.e., average fire ignitions per decade). For each ignition, LANDIS performs a Bernoulli trial, whose result is denoted by Y_i, with the parameter fire probability P_i, whose value is determined by the time since last fire of the ignited cell. If the ignition becomes an initiation, LANDIS will select a pre-defined fire size, denoted by Z, from a lognormal distribution with parameters μ and σ^2 to simulate fire spread (Fig. 1).

LANDIS uses a percolation algorithm similar to the algorithms of Gardner et al. (1987), Clarke et al. (1994), Hargrove et al. (2000), and Wimberly et al. (2000) to simulate fire spread. Fires simulated by the percolation algorithms spread from a burning cell to forested cells in the cardinal directions (up, down, left and right). These cells are entered into a priority queue in a random order. The first cell in the queue has a higher priority of fire spread. The fire will continue to spread until it reaches its predetermined size. LANDIS does not allow a forested site to be burned more than once within one

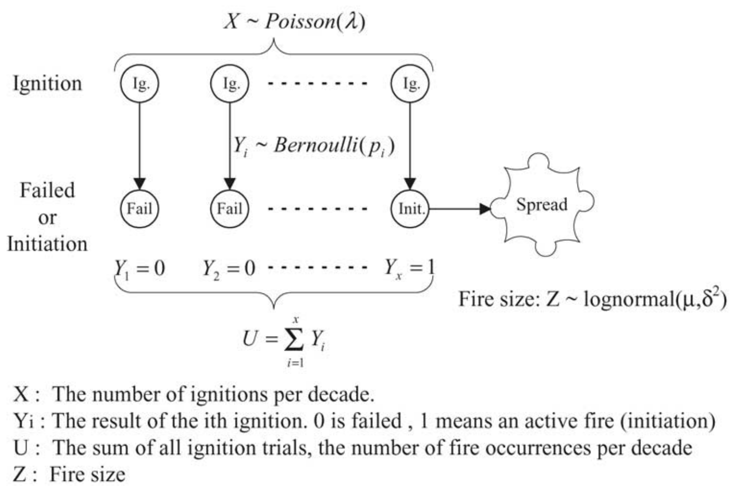

Fig. 1. The overview of the fire module design.

time step, and non-active land types or ecoregions (e.g., roads, lakes) may serve as fuel breaks in the landscape. Therefore, it is possible for a fire to be extinguished prior to burning its predetermined size if the fire reaches fuel breaks or newly burned sites. In a real landscape, fires may spread across the boundaries of fire regime units where the fire size distribution changes. When a fire spreads into a different fire regime unit, the module will simulate a new ignition. If the new ignition results in an active fire, a new predetermined fire size will be selected based on the fire size distribution for the new fire regime unit.

For each fire regime unit, LANDIS needs to know its size of area, fire cycle, mean fire size, and the standard deviation of fire size (DFS). Mean fire frequency can be calculated using Eq. (1). Fire size follows a lognormal distribution that is negatively skewed, consisting of many small fires and some rare large fires (He and Mladenoff, 1999; Wimberly, 2002). The parameters μ and σ^2 can be derived from the MFS and DFS:

$$\mu = 2\log \mathrm{MFS} - \tfrac{1}{2}\log(\mathrm{DFS}^2 + \mathrm{MFS}^2) \quad (11)$$

$$\sigma^2 = \log(\mathrm{DFS}^2 + \mathrm{MFS}^2) - 2\log \mathrm{MFS} \quad (12)$$

The fire probability function is essential to the distribution of fire frequency and hence very important in determining the realism of fires simulated by LANDIS. Different forest ecosystems may have different fire probability functions because of fuel accumulation. Our model uses Olson's approach (Olson, 1963; McCarthy et al., 2001), in that fuel accumulation is assumed as a constant rate of litter input and constant decomposition of a proportion of the litter. Assuming fire probability is proportional to fuel load, the change in fire probability is given in the following equation, where t is the time since last fire, and FC is the fire cycle.

$$P(t) = 1 - \mathrm{e}^{-t/\mathrm{FC}} \quad (13)$$

4. Experimental design and analysis

4.1. Case study landscapes

To demonstrate the capability of the LANDIS fire module to simulate multiple fire regimes on heterogeneous landscapes at various spatial resolutions, we applied the module to two landscapes with distinct fire regimes. The first landscape is characterized by 2 forested ecoregions and 23 species of trees, and is located in northern Wisconsin, USA (Fig. 2). The area comprises more than 700,000 ha, of which the pine barren ecoregion is about 90,000 ha and the lakeshore ecoregion is about 150,000 ha. Non-forested ecoregions (e.g., agriculture lands, lakes), shown as background in the map, are treated as fuel breaks in the simulation. Ecoregion boundaries are derived from an existing quantitative ecosystem classification (Host et al., 1996). This area is a largely forested, glacial region, with little topographic relief. Dominant tree species include sugar maple (*Acer saccharum*), northern red oak (*Quercus rubra*), eastern hemlock (*Tsuga canadensis*), yellow birch (*Betula alleghaniensis*), paper birch *Betula papyrifera* (*B. papyrifera*), quaking aspen (*Populus tremuloides*), white pine (*Pinus strobus*), red pine *Pinus resinosa* (*P. resinosa*), and jack pine *Pinus banksiana* (*P. banksiana*) (Curtis, 1959).

The second landscape for our case study has eight land types and four dominant species, and is located in southern Missouri, USA. The study area is in the Ozark Highlands Section (Kabrick et al., 2000), approximately 130,000 ha. The area is largely forested, with white oak *Quercus alba* (*Q. alba*), post oak *Quercus stellata* (*Q. stellata*), black oak *Quercus velutina* (*Q. velutina*) and scarlet oak *Quercus coccinea* (*Q. coccinea*) as the dominant species. Forest age structure is relatively simple due to historical harvesting practices. Topographic variation is high, with elevations ranging from 140 to 410 m, and many slopes are greater than 30°. There are eight land types: private land, southwest slopes, northeast slopes, ridges or flat uplands, upland drainages, mesic coves or bottoms, sites with limestone, savannas, and glades (Fig. 3). Private land is the collection of the sites where there is no national forest; we include it in our study to allow fires to spread across the entire forested landscape.

Historical fire regime statistics (FC, MFS, DFS) for the fire regime units are interpreted from empirical studies in the Wisconsin and the Missouri regions (Heinselman, 1973, 1981; Cleland et al., 1997; He and Mladenoff, 1999; Gustafson et al., 2000; Shifley et al., 2000; Guyette et al., 2002) (Table 1). Mean fire frequency (MFF) is calculated from Eq. (2). Fires in the lakeshore ecoregion are very infrequent and fire sizes

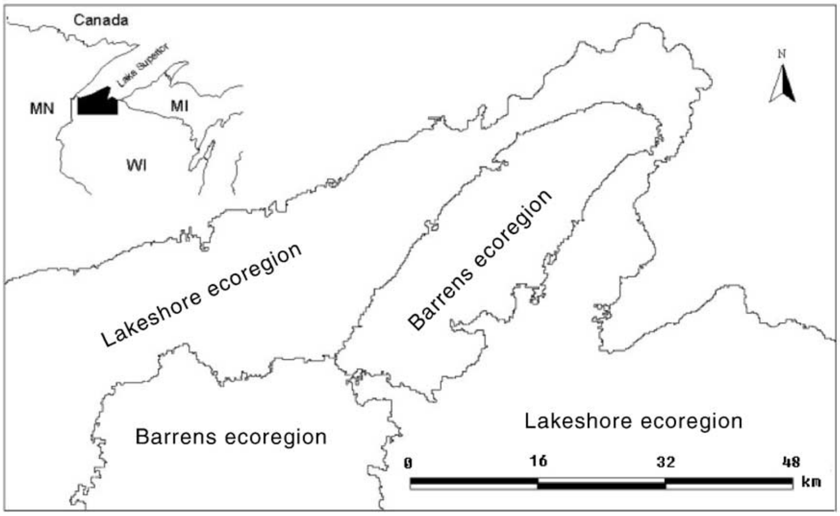

Fig. 2. Study region, location, and land types within the study area in northern Wisconsin.

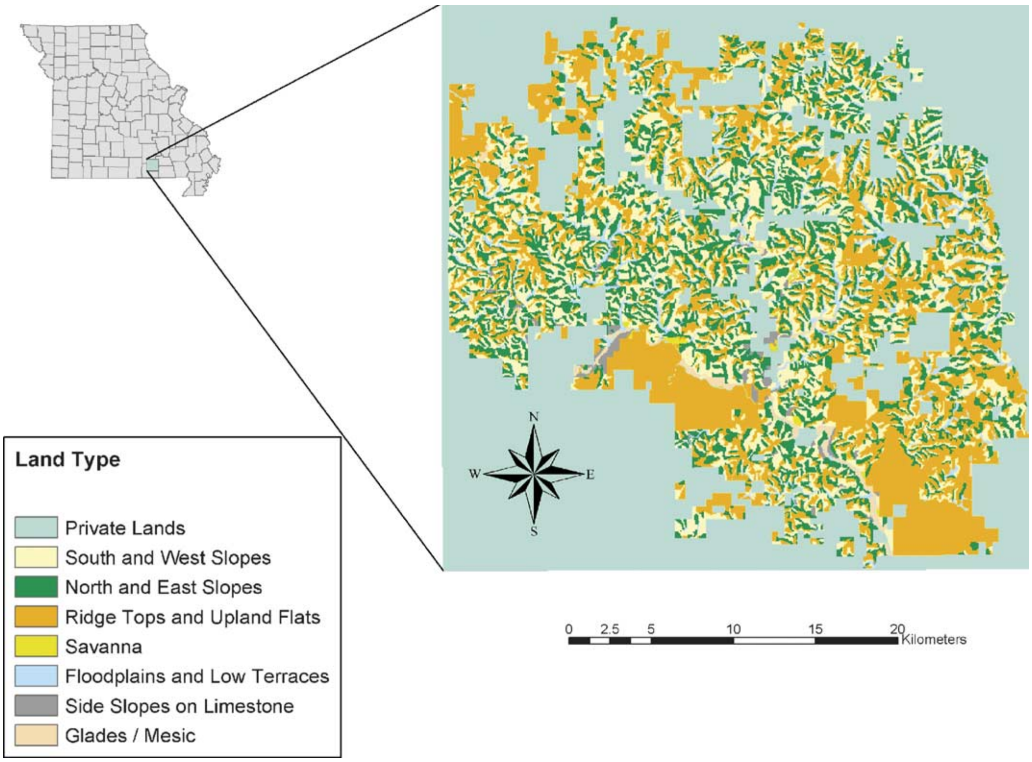

Fig. 3. Study region, location, and land types within the study area in southern Missouri.

Table 1
Characteristics of fire regimes of test landscapes for LANDIS simulations

Fire regime unit	Area (ha)	FC (years)	MFS (ha)	DFS (ha)	MFF (number per year)
Barren, WI	87752	100	400	300	2.19
Lakeshore, WI	151200	800	2000	2000	0.09
Private land, MO	57585	300	5.4	1.8	35.55
Southwest slope, MO	21054	415	2.7	1.8	18.79
Northeast slope, MO	18177	415	2.3	1.8	19.04
Flat, MO	26141	415	2.3	1.8	27.39
Savanna, MO	255	10	1.5	0.9	17
Updrain, MO	3484	415	2.3	0.9	3.65
Lime, MO	842	415	1.8	0.9	1.13
Mesic, MO	1189	415	1.8	0.9	1.59

Table 2
Calibrated parameters of fire regimes for LANDIS simulations

Fire regime unit	Initial time-since-last-fire (years)	Ignition density (number per decade)	Input MFS (ha)	Input DFS (ha)
Barren, WI	50	48	500	200
Lakeshore, WI	200	3.15	3000	2000
Private land, MO	50	1520	5.4	1.8
Southwest slope, MO	50	1020	4.5	1.8
Northeast slope, MO	50	920	3.33	1.8
Flat, MO	50	1300	2.7	1.8
Savanna, MO	10	350	25	7
Updrain, MO	50	210	3	0.9
Lime, MO	50	46	2.5	0.9
Mesic, MO	50	61	2.5	0.9

tend to be very large, whereas fires in the barrens ecoregion are relatively frequent and fires tend to be smaller. Fires in the land types of the Ozark region are very frequent and fire sizes are very small.

4.2. *Parameterization and simulation of fire disturbance*

According to the dendrochronological study and other historic records of fires of the two tested landscapes (He and Mladenoff, 1999; Guyette and Larsen, 2000), initial time-since-last-fire for the barrens and lakeshore ecoregions in northern Wisconsin landscape was set to 50 and 200 years, respectively; initial time-since-last-fire for the land types in the Missouri landscape was set to 50 years, except that it was set to 10 years for savanna. The ignition rate for each fire regime unit was held constant through simulation time with the assumption that there were no changes of climate and human dimensions. The input of mean fire size was larger than the expected mean fire size due to landscape fragmentation and configuration. Simulation runs of northern Wisconsin were carried out on a 328 × 535 grid of 200 m × 200 m cells for 400 years. Simulation runs of Missouri Ozark highlands were carried out on an 1185 × 1207 grid of 30 m × 30 m cells, for 300 years using the calibrated parameters (Table 2). The reasons for choosing different resolutions of analysis of the two study areas were that: (1) the ecoregions in northern Wisconsin are fairly contiguous, which reduces the need for analyzing fire spread at a finer resolution; and (2) an important purpose of the excise was to demonstrate that the new fire module in LANDIS can simulate forest landscape dynamics at various spatial resolutions.

5. Results

5.1. *Frequency and extent of fire disturbance*

Simulation results show distinct differences between the Wisconsin and the Missouri fire regime units

Table 3
Simulated results of fire regimes from LANDIS simulations

Fire regime unit	MFF (number per year)	MFS (ha)	FC (years)	Error of MFF (%)	Error of MFS (%)	Error of FC (%)
Barren, WI	2.24	390	100	5.0	−2.5	0.0
Lakeshore, WI	0.095	1997	797	5.6	−0.2	−0.4
Private land, MO	38.34	5.17	290	7.8	−4.3	−3.3
Southwest slope, MO	19.39	2.54	428	3.2	−5.9	2.9
Northeast slope, MO	19.22	2.16	438	0.9	−6.1	5.1
Flat, MO	26.59	2.14	459	−3.0	−7.0	9.6
Savanna, MO	18.23	1.36	425	7.2	−9.3	2.4
Updrain, MO	3.67	2.24	424	0.5	−2.6	2.1
Lime, MO	1.15	1.78	411	1.8	−1.1	−0.9
Mesic, MO	1.47	1.96	412	−7.5	8.9	−0.6

in terms of mean fire frequency and mean fire size, as expected (Table 3). The module also captures subtle differences of fire pattern at the finer land type scale. For instance, in Missouri, simulated mean fire size on southwest slopes is slightly larger than on northeast slopes, whereas simulated mean fire frequency is less than on northeast slopes. After calibration, the percentage absolute errors of simulated MFS and MFF on the fire regime units are very small (less than 10%), and simulated fire cycles on the fire regime units are close to the expected fire cycles (Table 3). Meanwhile, the temporal dynamics of fire frequency exhibit high variability, reflecting the non-stationary behavior of fire occurrences (Figs. 4 and 5). Although fire ignition is simulated as a stationary process in our model (i.e., ignition rate is held constant during the simulation time), the simulated fire occurrences on the tested landscapes are still non-stationary because fire probability in our model increases exponentially with time-since-last-fire rather than being held constant.

5.2. Fire-spread behavior over heterogeneous landscape

Simulated mean fire size for the lakeshore ecoregion seems to have a negative temporal autocorrelation (Fig. 6), which suggests that if few fires occur for a relatively long period, then there will be more suitable,

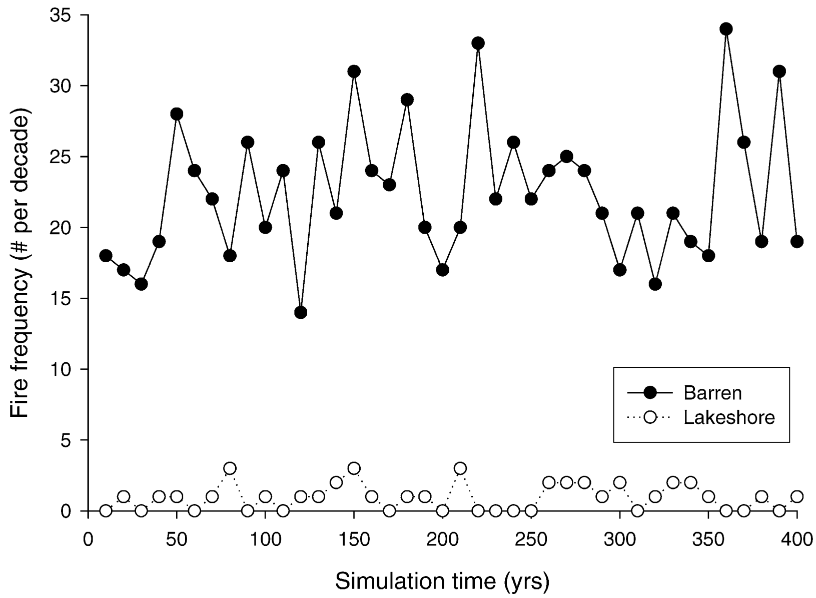

Fig. 4. Simulated fire frequency for barrens land type in northern Wisconsin and lakeshore land type in northern Wisconsin.

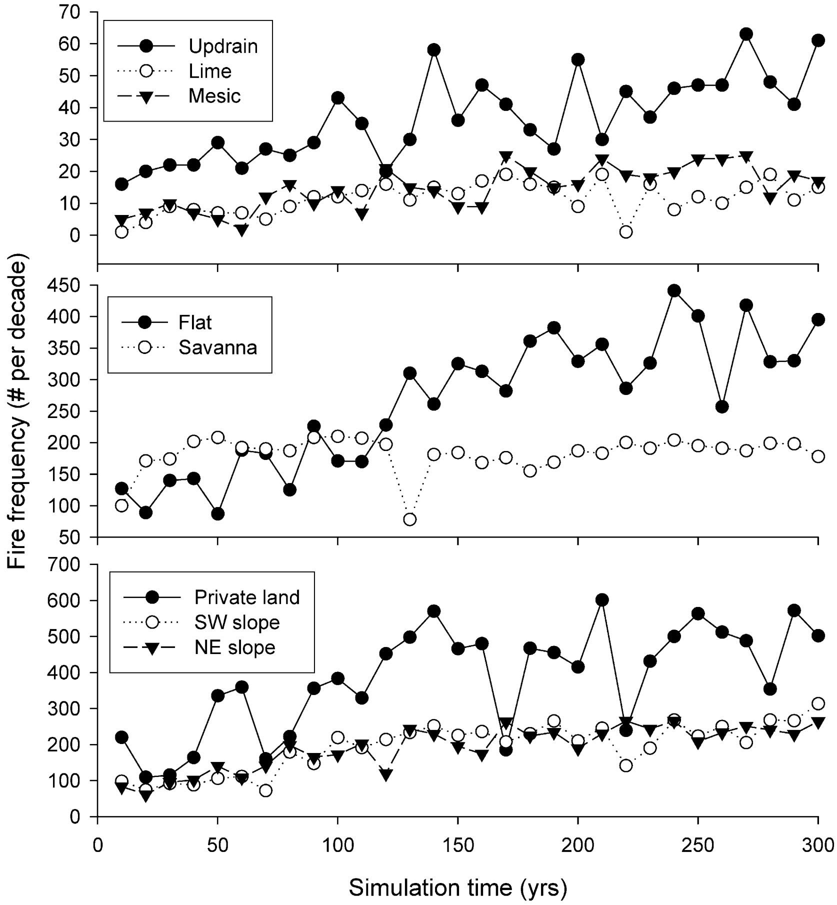

Fig. 5. Simulated fire frequencies for the eight land types in southern Missouri.

contiguous fuel left in the lakeshore, resulting in large, severe fires during the subsequent period. A similar pattern also is found in the simulation results for the relatively contiguous land types (e.g., Flat) in southern Missouri (Fig. 7). This demonstrates that our module can simulate the correlations between fire frequency and fire size to some extent by having fires spread across the patches with different time since last fire, thus different fire probabilities within a fire regime unit.

Another scale of landscape heterogeneity also affects fire spread—the spatial configuration of patches of different fire regime units. In our simulations of northern Wisconsin fires, fires initiated in the barrens ecoregion often stop in the lakeshore ecoregion or at the boundary of non-forest ecoregions, which serve as fuel breaks. Fire occurrences in the lakeshore ecoregion are usually caused by fire spreading from the barrens ecoregion. Similar fire-spreading patterns occur in the southern Missouri highlands, where land types are highly dispersed. For example, only 35% of fires reach their predetermined size completely within the southwest slope land type in the simulation; the other 65% of the fires spread into other land types, especially the northeast slope land type.

6. Discussion

6.1. Implications of the hierarchical fire frequency model

The hierarchical fire frequency model presented here depicts fire frequency as a mixture distribution

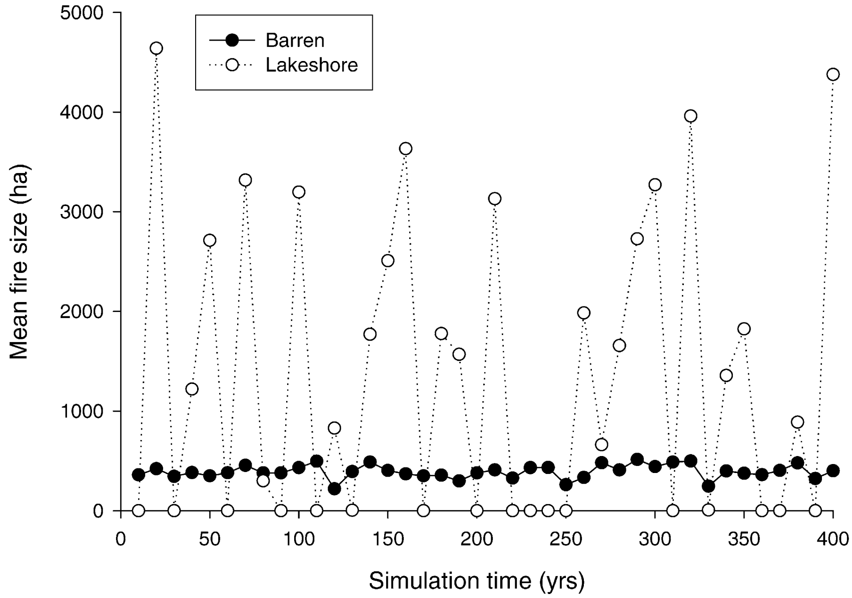

Fig. 6. Simulated dynamics of mean fire size (MFS) for barren and lakeshore land types in northern Wisconsin.

where parameters also follow relatively simple distributions. Although our model does not have an explicit fire frequency distribution or fire-interval distribution as do the exponential and Weibull models, it is more flexible in that it can represent a wider range of fire regimes than these other models. The exponential and Weibull models are stationary in the sense that the parameters are fixed and the sampling occurs from a single fixed probability distribution assumed static (Polakow and Dunne, 1999). On the other hand, our hierarchical model incorporates variability about the parameter estimates; hence it can be used to replicate more complicated, non-stationary temporal patterns of fire occurrence. Moreover, the hierarchical model conceptually is consistent with statistically based fire regime models, because it separates fire ignition from fire occurrence as most of these models do. Therefore, ours is a better model for application in statistically based modeling of fire regimes than widely used exponential or Weibull fire-interval models.

6.2. Improvements in LANDIS fire simulation using the hierarchical fire frequency model

The fire module implemented in LANDIS (version 4.0) is an application of our hierarchical fire frequency model. The results of our experiments demonstrate this new fire module can simulate multiple fire regimes across heterogeneous landscapes with a few parameters and a moderate amount of input data. The simulated fire cycle, fire frequency distribution, and fire size distribution are consistent with historical data on fire occurrences. Compared to previous fire simulations using LANDIS 3.x, which applies the exponential model to simulate fire occurrences, four major advances have been achieved: (1) In earlier versions of LANDIS, the fire algorithm fails to simulate fire regimes characterized by many small fires, and some applications of LANDIS have had to artificially modify the fire algorithm to circumvent such limitations (e.g., Sturtevant et al., 2004). As our results demonstrate, the new fire module can simulate a much wider range of fire regimes. (2) Earlier versions of LANDIS assume fire hazard is constant across the fire regime throughout the entire simulation period. Results attained from these versions simulated stationary temporal patterns of fire disturbance (He and Mladenoff, 1999). However, even fire ecologists who utilize the exponential model assess fire frequency in terms of temporally distinct epochs and recognize the variability in the parameters for fire frequency distribution (Johnson and Gutsell, 1994; Reed et al., 1998). The new LANDIS fire module simulates

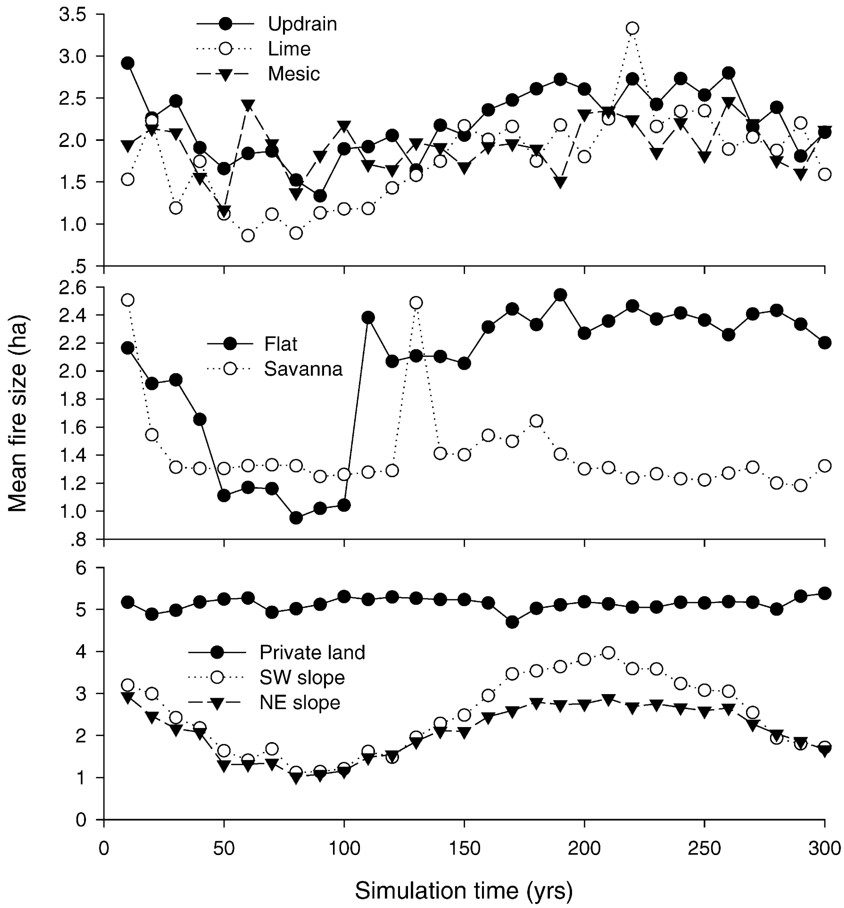

Fig. 7. Simulated dynamics of mean fire size for the eight land types in southern Missouri.

fire probability as increasing with fire interval, and hence can simulate non-stationarity of temporal patterns of fire disturbance. (3) Our new fire module is able to simulate subtle differences among multiple fire regime units within a large landscape, which are often hard to simulate in earlier versions of LANDIS due to the difficulty in parameterization. For instance, simulated mean fire size on southwest slopes is slightly larger than on northeast slopes; this is consistent with the fact that prevailing winds in the Ozark highlands are southerly and strongest in the spring season (Kabrick et al., 2000). (4) The new fire module captures more realistic fire-spread patterns than do previous approaches that use only one distribution of fire size for the entire landscape. From the simulation results, we observe that if few fires occurred in some decades, then the subsequent fires tend to be larger and more intense (unpublished data), and that fires often extinguish near the boundaries of less flammable fire regime units. This is consistent with empirical observations by Bergeron (1991) on the influence of island and lakeshore landscapes on boreal fire regimes.

6.3. Further research needs

The new LANDIS fire module assumes ignition density is uniform within a fire regime unit without explicitly considering the effects of human population, site, topography, and vegetation. A generalized linear mixed model (GLMM) can be used to describe temporal and spatial distributions of ignition density (Diaz-Avalos et al., 2001). The module also assumes fire probability increases exponentially with the time-since-last-fire. This assumption may not be valid for forest ecosystems different from those selected (McCarthy et al., 2001). A model of fire probability with respect to fuel loading and weather conditions can be incorporated into the module. Such future work will allow us to model fire regimes more dynamically with fewer predetermined fire regime statistics required as inputs to the module; simulated fire cycle and fire size will be emergent properties of the simulations rather than predetermined inputs.

Acknowledgements

This research was funded in part through a cooperative agreement with the USDA Forest Service North Central Research Station. We appreciate suggestions to this work from Richard P. Guyette, Rob Doudrick, David R. Larsen, Bernard J. Lewis, Bo Z. Shang, Stephen R. Shifley, and Farroll T. Wright. We particularly wish to thank Ajith H. Perera, Brian R. Sturtevant, and an anonymous reviewer for their comments on previous versions of the manuscript.

Appendix A. Relation between poisson process and exponential process

We use U to denote the number of fire occurrence per year, and T to denote the time since last fire. If U follows Poisson distribution with parameter α, then T has an exponential distribution with $\beta = 1/\alpha$.

Proof. The cumulative distribution function for T is given by

$$F_T(t) = \Pr[T \le t] = 1 - Pr[T > t]$$

$\Pr[T > t]$ is the probability of having the first fire occurring after time t, which means no fire occurrences in the time interval $[0, t]$. Let W denote the number of fire occurrences in this time interval. W is a Poisson process with parameter αt. Thus

$$\Pr[T > t] = \Pr[W = 0] = \frac{e^{-\alpha t}(\alpha t)^0}{0!} = e^{-\alpha t}$$

By substitution we obtain

$$F_T(t) = 1 - e^{-\alpha t}$$

Because the PDF is the derivative of CDF for a continuous random variable

$$f_T(t) = F_T(t) = \alpha\, e^{-\alpha t} = \frac{1}{\beta} e^{-t/\beta}$$

This is the PDF for an exponential random variable with $\beta = 1/\alpha$. □

Appendix B. A special case of hierarchical fire frequency model

When both ignition rate and fire probability are constant, fire ignition is a Poisson process with λ, conditional distribution of fire occurrence given fire ignition is binomial with X and P, i.e.,

$$[X|\lambda] \sim \text{Poisson}(\lambda)$$

$$U|X \sim \text{Binomial}(X, P)$$

Then the mixture distribution of fire occurrence becomes a Poisson distribution with λP.

Proof. The PDF of discrete random variable U (fire frequency) is

$$f_U(u) = \Pr[U = u] = \sum_{x=0}^{\infty} \Pr[U = u,\ X = x]$$

(definition of marginal distribution)

$$= \sum_{x=0}^{\infty} \Pr[U = u|X = x] \Pr[X = x]$$

(definition of conditional distribution)

$$= \sum_{x=u}^{\infty} \left[\binom{x}{u} p^u (1-p)^{x-u} \right] \left[\frac{e^{-\lambda}\lambda^x}{x!} \right],$$

(conditional probability is 0 if $x < u$)

where we substitute PDFs of binomial and Poisson distribution into the expression. If we now simplify this expression, we get

$$f_U(u) = Pr[U = u] = \frac{(\lambda p)^u e^{-\lambda}}{u!} \sum_{x=u}^{\infty} \frac{((1-p)\lambda)^{x-u}}{(x-u)!}$$

$$= \frac{(\lambda p)^u e^{-\lambda}}{u!} \sum_{t=0}^{\infty} \frac{((1-p)\lambda)^t}{t!} \quad \text{(change of variable)}$$

$$= \frac{(\lambda p)^u e^{-\lambda}}{u!} e^{(1-p)\lambda}$$

(Maclaurin series for exponential function)

$$= \frac{(\lambda p)^u}{u!} e^{-\lambda p} \quad \text{(a kernel for a Poisson distribution),}$$

therefore

$$[U] \sim \text{Poisson}(\lambda p)$$

□

References

Agee, J.K., 1993. Fire Ecology of Pacific Northwest Forests. Island Press, Washington DC.

Albright, D., Meisner, B.N., 1999. Classification of fire simulation systems. Fire Manage. Notes 59 (2), 5–12.

Anderson, K., Martell, D.L., Flannigan, M.D., Wang, D., 2000. Modeling of fire occurrence in the boreal forest region of Canada. In: Kasischke, E.S., Stocks, B.J. (Eds.), Fire, Climate Change and Carbon Cycling in the Boreal Forest. Springer-Verlag, New York, pp. 357–367.

Antonovski, M.Y., Ter-Mikaelian, M.T., Furyaev, V.V., 1992. A spatial model of long-tem forest fire dynamics and its applications to forests in western Siberia. In: Shugart, H.H., Leemans, R., Bonan, G.B. (Eds.), A Systems Analysis of the Global Boreal Forest. Cambridge University Press, pp. 373–403.

Baker, W.L., Egbert, S.L., Frazier, G.F., 1991. A spatial model for studying the effects of climatic change on the structure of landscapes subject to large disturbances. Ecol. Model. 56, 109–125.

Bergeron, Y., 1991. The influence of island and mainland lakeshore landscapes on boreal forest fire regimes. Ecology 72 (6), 1980–1992.

Bormann, F.H., Likens, G.E., 1979. Pattern and Process in a Forested Ecosystem. Springer-Verlag, New York, 253 pp.

Boychuk, D., Perera, A.H., 1997. Modeling temporal variability of boreal landscape age-classes under different fire disturbance regimes and spatial scales. Can. J. Forest Res. 27 (7), 1083–1094.

Boychuk, D., Perera, A.H., Ter-Mikaelian, M.T., Martell, D.L., Li, C., 1997. Modelling the effect of spatial scale and correlated fire disturbances on forest age distribution. Ecol. Model. 95, 145–164.

Casella, G., Berger, R.L., 2001. Statistical Inference, 2nd ed. Duxbury Press.

Clark, J.S., 1989. Ecological disturbance as a renewal process: theory and application to fire history. Oikos 56, 17–30.

Clarke, D.C., Brass, J.A., Riggan, P.J., 1994. A cellular automaton model of wildfire propagation and extinction. Photogramm. Eng. Remote Sens. 60, 1355–1367.

Cleland, D., Avers, P., McNab, W., Jensen, M., Bailey, R., King, T., Russell, W., 1997. National hierarchical framework of ecological units. In: Boyce, M.S., Haney, A. (Eds.), Ecosystem Management Applications for Sustainable Forest and Wildlife Resources. Yale University Press, New Haven, CT, pp. 181–200.

Cunningham, A.A., Martell, D.L., 1973. A stochastic model for the occurrence of mancaused forest fires. Can. J. Forest Res. 3, 282–287.

Curtis, J.T., 1959. The Vegetation of Wisconsin. The University of Wisconsin Press, Madison.

Davis, F.W., Burrows, D.A., 1994. Spatial simulation of fire regime in mediterranean-climate landscapes. In: Talens, M.C., Oechel, W.C., Moreno, J.M. (Eds.), The Role of Fire in Mediterranean-Type Ecosystems. Springer-Verlag, New York, pp. 117–139.

Diaz-Avalos, C., Peterson, D.L., Alvarado, E., Ferguson, S.A., Besag, J.E., 2001. Space-time modelling of lightning-caused ignitions in the Blue Mountains. Oregon. Can. J. Forest Res. 31, 1579–1593.

Gardner, R.H., Milne, B.T., Turner, M.G., O'Neill, R.V., 1987. Neutral models for the analysis of broad-scale landscape pattern. Landsc. Ecol. 1, 19–28.

Gardner, R.H., Romme, W.H., Turner, M.G., 1999. Predicting forest fire effects at landscape scales. In: Mladenoff, D.J., Baker, W.L. (Eds.), Spatial Modeling of Forest Landscapes: Approaches and Applications. Cambridge University Press, Cambridge, UK, pp. 163–185.

Gustafson, E.J., Shifley, S.R., Mladenoff, D.J., Nimerfro, K.K., He, H.S., 2000. Spatial simulation of forest succession and timber harvesting using LANDIS. Can. J. Forest Res. 30, 32–43.

Guyette, R.P., Larsen, D., 2000. A history of anthropogenic and natural disturbances in the area of the Missouri Ozark Forest Ecosystem Project. In: Shifley, S.R., Brookshire, B.L. (Eds.), Missouri Ozark Forest Ecosystem Project: Site History, Soils, Landforms, Woody and Herbaceous Vegetation, Down Wood and Inventory Methods for the Landscape Experiment. Gen. Tech. Rep. NC-208. USDA, Forest Service, North Central Research Station, St. Paul, MN, pp. 19–40.

Guyette, R.P., Muzika, R.M., Dey, D.C., 2002. Dynamics of an anthropogenic fire regime. Ecosystems 5, 472–486.

Hargrove, W.W., Gardner, R.H., Turner, M.G., Romme, W.H., Despain, D.G., 2000. Simulating fire patterns in heterogeneous landscapes. Ecol. Model. 135, 243–263.

He, H.S., Mladenoff, D.J., 1999. Spatially explicit and stochastic simulation of forest landscape fire disturbance and succession. Ecology 80 (1), 81–99.

Heinselman, M.L., 1970. The natural role of fire in northern conifer forests. Naturalist 21 (4), 14–23.

Heinselman, M.L., 1973. Fire in the virgin forests of the Boundary Waters Canoe Area. Minn. Q. Res. 3, 329–382.

Heinselman, M.L., 1981. Fire intensity and frequency as factors in the distribution and structure of northern ecosystems. In: Fire Regimes and Ecosystem Properties. U.S. For. Serv. Gen. Tech. Re WO-26, pp. 7–57.

Host, G.E., Polzer, P.L., Mladenoff, D.J., White, M.A., Crow, T.R., 1996. A quantitative approach to developing regional ecosystem classifications. Ecol. Appl. 6 (2), 608–618.

Hough, A.F., Forbes, R.D., 1943. The ecology and silvics of forests in the high plateaus of Pennsylvania. Ecol. Monogr. 13 (3), 299–320.

Johnson, E.A., Gutsell, S.L., 1994. Fire frequency models, methods and interpretations. Adv. Ecol. Res. 25, 239–287.

Johnson, E.A., Van Wagner, C.E., 1985. The theory and use of two fire history models. Can. J. Forest Res. 15, 214–220.

Kabrick, J., Meinert, D., Nigh, T., Gorlinsky, B.J., 2000. Physical environment of the Missouri Ozark forest ecosystem project sites. In: Shifley, S.R., Brookshire, B.L. (Eds.), Missouri Ozark Forest Ecosystem Project: Site History, Soils, Landforms, Woody and Herbaceous Vegetation, Down Wood and Inventory Methods for the Landscape Experiment. USDA, Forest Service, North Central Research Station, St. Paul, MN, pp. 41–70.

Keeley, J.E., Fotheringham, C.J., Morais, M., 1999. Reexamining fire suppression impacts on brushland fire regimes. Science 284, 1829–1832.

Li, C., 2000. Reconstruction of natural fire regimes through ecological modelling. Ecol. Model. 134, 129–144.

Li, C., Ter-Mikaelian, M.Y., Perera, A.H., 1997. Temporal fire disturbance patterns on a forest landscape. Ecol. Model. 99, 137–150.

McCarthy, M.A., Gill, A.M., Bradstock, R.A., 2001. Theoretical fire interval distributions. Int. J. Wildl. Fire 10, 73–77.

McPherson, G.R., Wade, D.D., Phillips, C.B., 1990. Glossary of Wildland Fire Management Terms Used in the United States. Society of American Foresters, Washington, DC.

Mladenoff, D.J., He, H.S., 1999. Design and behavior of LANDIS, an object-oriented model of forest landscape disturbance and succession. In: Mladenoff, D.J., Baker, W.L. (Eds.), Spatial Modeling of Forest Landscapes: Approaches and Applications. Cambridge University Press, Cambridge, UK, pp. 125–162.

Mladenoff, D.J., Host, G.E., Boeder, J., Crow, T.R., 1996. LANDIS: a spatial model of forest landscape disturbance, succession, and management. In: Goodchild, M.F., Steyaert, L.T., Parks, B.O., Johnston, C., Maidment, D., Crane, M., Glendining, S. (Eds.), GIS and Environmental Modeling: Progress and Research Issues. GIS World Books, Fort Collins, CO, pp. 175–180.

Moritz, M.A., 2003. Spatiotemporal analysis of controls on shrubland fire regimes: age dependency and fire hazard. Ecology 84, 351–361.

Olson, J.S., 1963. Energy storage and the balance of producers and decomposers in ecological systems. Ecology 44, 322–331.

Pennanen, J., Kuuluvainen, T., 2002. A spatial simulation approach to natural forest landscape dynamics in boreal Fennoscandia. Forest Ecol. Manage. 164, 157–175.

Polakow, D.A., Dunne, T.T., 1999. Modelling fire return interval T: stochasticity and censoring in the two-parameter Weibull model. Ecol. Model. 121, 78–102.

Reed, W.J., Larsen, C.P.S., Johnson, E.A., MacDonald, G.M., 1998. Estimation of temporal variations in historical fire frequency from time-since-fire map data. Forest Sci. 44, 465–475.

Shifley, S.R., Thompson III, F.R., Larsen, D.R., Dijak, W.D., 2000. Modeling forest landscape change in the Missouri Ozarks under alternative management practices. Comp. Electron. Agric. 27, 7–24.

Sturtevant, B.R., Zollner, P.A., Gustafson, E.J., Cleland, D.T., 2004. Human influence on fuel connectivity and the risk of catastrophic fire in mixed forests of northern Wisconsin. Landsc. Ecol. 19 (3), 235–254.

Turner, M.G., Romme, W.H., 1994. Landscape dynamics in crown fire ecosystems. Landsc. Ecol. 9 (1), 59–77.

Van Wagner, C.E., 1978. Age-class distribution and forest fire cycle. Can. J. Forest Res. 8, 220–227.

Wimberly, M.C., 2002. Spatial simulation of historical landscape patterns in coastal forests of the Pacific Northwest. Can. J. Forest Res. 32, 1316–1328.

Wimberly, M.C., Spies, T.A., Long, C.J., Whitlock, C., 2000. Simulating historical variability in the amount of old forests in the Oregon Coast Range. Conserv. Biol. 14, 167–180.