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One- and two-objective approaches to an area-constrained habitat reserve site selection problem

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Abstract

We compare several ways to model a habitat reserve site selection problem in which an upper bound on the total area of the selected sites is included. The models are cast as optimization coverage models drawn from the location science literature. Classic covering problems typically include a constraint on the number of sites that can be selected. If potential reserve sites vary in terms of area, acquisition cost or land value, then sites need to be differentiated by these characteristics in the selection process. To address this within the optimization model, the constraint on the number of selected sites can either be replaced by one limiting the total area of the selected sites or area minimization can be incorporated as a second objective. We show that for our dataset and choice of optimization solver average solution time improves considerably when an area-constrained reserve site selection problem is modeled as a two objective rather than a single objective problem with a constraint limiting the total area of the selected sites. Computational experience is reported using a large dataset from Australia.
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1. Introduction

Escalating rates of land conversion from open space to more developed and extractive uses are threatening biodiversity in many parts of the world today. A common strategy to reduce the loss of biodiversity on the landscape is to establish a system of biological reserves to preserve key habitat, species and ecological features (Pimm and Lawton, 1998). Because inclusion of sites in a protected reserve network often limits or precludes other uses of that land, trade-offs must be carefully considered and weighed. This is particularly important as conservation planners are often constrained by budgets and restrictions on the total area that may be devoted to reserves. Optimization models for reserve site selection have emerged as tools that can assist decision makers in making difficult land use protection decisions.

Over the past 20 years many optimization models and solution methodologies have been developed to facilitate the efficient and effective selection of sites for inclusion in a protected reserve network. One of the most common expressions of the reserve site selection problem has been to maximize species or habitat type representation for a given budget, where a species is represented or covered if it is present in one or more of the protected sites. This maximal species' covering problem, formulated as a linear integer-programming model, is derived from the classic Maximal Covering Location Problem specified in the location science literature (Church and ReVelle, 1974).

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Many researchers have solved examples of the reserve site selection problem cast as a maximal covering type formulation (including, Underhill, 1994; Camm et al., 1996; Csuti et al., 1997; Önal, 2004). Additionally, Church et al. (1996) and Arthur et al. (1997) also solved the equivalent minimal uncovering model for a reserve site selection problem, noting computational advantages of this model structure over the maximal covering structure.

Many of the reserve site selection applications in the literature have assumed that the eligible sites did not differ in size, quality, cost or land value. Thus, a constraint would be included in the model limiting the number of sites that could be selected. A more realistic approach to this site selection problem would be to differentiate eligible parcels by some of these other features. To do so in the context of the maximal covering formulation requires that the constraint limiting the number of sites selected be replaced by a constraint limiting the total area or cost of the selected sites. Some examples of this problem specification can be found in the conservation literature (for example, Snyder et al., 1999; Polasky et al., 2001). Snyder et al. (1999) solved a reserve site selection model that included a constraint on the maximum total area of selected reserve sites. Their model was applied to a small problem on the Superior National Forest in Minnesota with 33 potential reserves and 63 vegetation communities. Polasky et al. (2001) solved a reserve site selection model that included a constraint on total land acquisition costs. Their model was applied to a medium-sized problem of the Oregon Gap Analysis data with 289 sites and 415 species. Neither reported any computational difficulties solving their applications using the budget or area-constrained maximal covering formulation. However, as ReVelle et al. (2002) point out, a covering formulation with a conventional budget constraint in which the cost or area coefficients are not 0 or 1 is not likely to be integer-friendly or readily able to be solved to optimality quickly (ReVelle, 1993). That is, significant branching and bounding is likely required to find an integer-optimal solution, particularly for large problems. Further, research has begun to suggest that the data structure of reserve site selection problems plays a significant role in ease of solution (Pressey et al., 1999; ReVelle et al., 2002). Thus, while Snyder et al. (1999) and Polasky et al. (2001) reported no difficulties in solving their instances of budget-constrained maximal covering formulations, other reserve selection datasets are likely to exist in which computational difficulties will occur (Church et al., 1996; Rodrigues and Gaston, 2002; Önal, 2004).

To address this, an alternative way to include the requirements of a budget or area constraint is to cast it as a second objective of cost or area minimization. A two-objective reserve site selection problem can then be formulated in which species' or habitat coverage is traded off against total area or cost of selected sites, with solution by the multiobjective weighting method (Cohon, 1978). With this approach, the non-integer-friendly budget constraint is taken out of the constraint set, often leading to a much easier integer-programming problem to solve (ReVelle, 1993). In one of the few multiobjective reserve selection applications in the literature, Rothley (1999) developed and solved a three-objective maximal covering reserve site selection problem to maximize connectedness and total area of the reserve system, and to maximize the number of rare plant species represented. A constraint was included to limit the number of sites selected, along with other logic constraints. The model was applied to a small problem with 20 potential reserve sites and solved using the multiobjective constraint method. Church et al. (1996) and ReVelle et al. (2002) suggested that cost or size differentials of parcels could be handled as a second objective in a reserve site selection problem. However, neither formulated, solved nor reported any computational experience with this two-objective reserve site selection problem specification. It is here that our work makes a contribution.

In this paper, we formulate, solve and compare solution effort for several equivalent one- and two-objective specifications of an area-constrained, reserve site selection problem. Within this context, we also compare the solution performance of the maximal covering formulation with the equivalent minimal uncovering formulation in both one- and two-objective models. The computational impact of designating the coverage decision variables as either binary or non-negative is also explored. Computational experience is reported using a large, real dataset from Australia. The purpose of our research is to demonstrate that gains in computational efficiency may be possible if a modeler is willing to explore alternative, but equivalent formulations for a particular decision problem, dataset and choice of optimization solver.

2. Methods

2.1. Data

We used a dataset from the Western Division of New South Wales, Australia (Pressey and Logan, 1995; Pressey et al., 1999). The data consisted of a set of 1886 pastoral land holdings, their area, and the occurrence of each of 248 land systems (recurring patterns of landforms, soils and vegetation) within the set of sites (Mabbutt, 1968; Walker, 1991). The total area of the set of sites was 325,058 km^2, with a range from 0.25 to 507.25 km^2 and a mean area of 262.875 km^2. Refer to Pressey and Logan (1995) for a map.

2.2. Models

To address the problem of selecting reserve sites under area limitations, we formulated several 0–1 integer-optimization models. The models are based upon the maximal covering location problem (Church and ReVelle, 1974). The following notation was used:

2.2.1. Single objective, area-constrained, maximal covering reserve site selection model

i, I are the index and set of land systems,
j, J are the index and set of sites,
L is specified upper bound on total area of the selected reserve sites,
A_j is the area of site j,
N_i is the set of sites, j, that contain land system i,
X_j = {a 0–1 decision variable equal to 1 if site j is selected for protection, and 0 otherwise}
Y_i = {a 0–1 decision variable equal to 1 if land system i is represented by the selected set of sites, and 0 otherwise}

The model was formulated as follows:

$$\text{Model 1:} \quad \text{maximize} \quad \sum_{i \in I} Y_i \tag{1}$$

$$\text{subject to:} \quad \sum_{j \in J} A_j X_j \leqslant L, \tag{2}$$

$$Y_i \leqslant \sum_{j \in N_i} X_j \quad \forall i \in I, \tag{3}$$

$$X_j, Y_i \in \{0, 1\}. \tag{4}$$

The objective (1) maximizes the number of unique land systems represented or covered by the set of selected sites. The first constraint (2) ensures that the total area of the selected set of sites does not exceed L, the specified upper bound on total area of selected sites. The second set of constraints (3) defines the conditions of land system coverage. That is, this constraint stipulates that a land system is covered if at least one of the eligible sites containing it is selected for protection. Constraint (4) defines the integer restrictions for the decision variables. Note, however, that research with the maximal covering formulation has shown that the coverage variables, y_i, do not have to be declared as binary variables for them to solve with these values. The structure of the formulation will naturally force these variables to the values of 0 or 1 if they are simply defined as non-negative variables with an upper bound of 1 (Church et al., 1996). The significance of this is that reducing the number of binary variables is typically thought to create an easier optimization problem to solve. So, for our analysis, the binary restriction on these variables was replaced with the requirement that the variables be non-negative with an upper bound of 1.

2.2.2. Single objective, area-constrained, minimum uncovering reserve site selection model

We next specified the equivalent, 'uncovering' version of the one-objective reserve site selection model. The model is based upon work by Church et al. (1996) and Arthur et al. (1997) in which a reserve site selection model with a limit on the number of sites selected was developed.

New notation. U_i = {a 0–1 decision variable equal to 1 if land system i is *not* covered by the selected set of sites, and 0 otherwise}

Our area-constrained, uncovering model is formulated as follows:

$$\text{Model 2:} \quad \text{minimize} \quad \sum_{i \in I} U_i \tag{5}$$

$$\text{subject to:} \quad \sum_{j \in J} A_j X_j \leqslant L, \tag{6}$$

$$U_i + \sum_{j \in N_i} X_j \geqslant 1 \quad \forall i \in I, \tag{7}$$

$$X_j, U_i \in \{0, 1\}. \tag{8}$$

The objective (5) minimizes the number of unique land systems that are *not* represented or covered by the set of selected sites. Constraint (6), as in the previous formulation, constrains the total area of the selected set of sites. Constraint set (7) defines the conditions under which a land system is not covered. That is, this constraint stipulates that a land system remains uncovered if none of the eligible sites containing it are selected for protection. Constraint (8) defines the integer restrictions for the site selection decision variable. As with the previous formulation, the 'uncoverage' variable, U_i, does not actually have to be declared binary in this specification of the problem, nor does it have to be upper-bounded at 1.0. The variable can simply be declared a non-negative variable. The lack of a required upper bound on this variable is the feature of the Minimum Uncovering formulation that makes it more computationally efficient than the equivalent Maximum Covering formulation (Church et al., 1996). As with the previous formulation, we relax the binary requirements for the coverage variable in this model.

2.2.3. Two-objective maximal covering reserve site selection model

We next formulate a two-objective version of the area-constrained maximal coverage model.

New notation.
B is the total area of selected sites,
S is the total number of covered land systems,
w_1 and w_2 are non-negative objective function weights whose sum equal 1 ($w_2 = (1 - w_1)$).
The model is formulated as follows:

$$\text{Model 3:} \quad \text{maximize} \quad \left(w_{1*}\left(\sum_{i\in I} Y_i\right)\right) - \left(w_{2*}\left(\sum_{j\in J} A_j X_j\right)\right) \qquad (9)$$

$$\text{subject to:} \quad \sum_{j\in J} A_j X_j = B, \qquad (10)$$

$$\sum_{i\in I} Y_i = S, \qquad (11)$$

$$Y_i \leqslant \sum_{j\in N_i} X_j \quad \forall i \in I, \qquad (12)$$

$$X_j, Y_i \in \{0,1\}, \quad B \geqslant 0, \; S \geqslant 0. \qquad (13)$$

The objective (9) is a statement of our two objectives to maximize the number of land systems covered while minimizing the total area of the selected sites. Maximizing the negative of the budget is equivalent to minimizing this value. The weights, w_1 and w_2, are non-negative values whose sum is equal to 1. The first constraint (10) is a definitional constraint that calculates the total area of the selected set of sites, B. Unlike the single objective models, B is now a decision variable rather than a known parameter. Constraint (11) is another definitional constraint that sums the number of covered land systems. Strictly speaking, constraints (10) and (11) do not need to be included in the model. They are included for our analysis so the values of each of the objective functions can be readily calculated. Both values could be determined by manually summing the Y_i and A_jX_j from the model output. Constraint set (12) defines the conditions of coverage. Constraint (13) defines the integer restrictions for the coverage and site selection decision variables, and the non-negativity requirement for the area variable, B, and the variable defining the sum of the covered land systems, S. As with the single objective model, the binary restrictions on the coverage variable can be replaced by non-negativity requirements and an upper bound of 1.

2.2.4. Two-objective minimum uncovering reserve site selection model

Finally, we specify a minimum uncovering version of the two-objective model. The model is formulated as follows using previously defined notation:

$$\text{Model 4:} \quad \text{minimize} \quad \left(w_{1*}\left(\sum_{i\in I} U_i\right)\right) + \left(w_{2*}\left(\sum_{j\in J} A_j X_j\right)\right) \qquad (14)$$

$$\text{subject to:} \quad \sum_{j\in J} A_j X_j = B, \qquad (15)$$

$$\sum_{i\in I} U_i = S, \qquad (16)$$

$$U_i + \sum_{j\in N_i} X_j \geqslant 1, \quad \forall i \in I \qquad (17)$$

$$X_j, U_i \in \{0,1\}, \quad B \geqslant 0, \; S \geqslant 0. \qquad (18)$$

The objective (14) is a statement of our two objectives to minimize the number of uncovered land systems covered while also minimizing the total area of the selected sites. Constraint (15) and (16) are the definitional constraints for the two-objective function values. Constraint set (17) defines the coverage conditions. Constraint (18) defines the variable types. Again, the binary restrictions on the coverage variables are relaxed.

2.3. Solution method

For the two-objective problems the multiobjective weighting was used as the method of solution (Cohon, 1978). The weights, w_1 and w_2, were systematically varied and the problem re-solved for each set of weights to produce a trade-off curve between the number of land systems covered and the total area of the selected sites. As w_1 was increased relative to w_2, more weight was given to the first objective, causing both coverage to go up as well as the total area of the selected sites. The opposite trend occurs as w_2 increased relative to the value of w_1. The multiobjective problems were run first, producing a value for the total selected area variable, L, for each set of weights. These values were then used to solve the single objective problems as the specified upper bound on total selected area. That is, the single objective problem was solved successively with the same values of L previously generated through the two-objective problems, only now used as a known parameter. With this approach, the same 25 solutions were generated by both the single and two-objective models. Therefore, direct comparisons can be made of the computational efficiency of each model in generating those 25 solutions.

On a further computational note for the one-objective problems, the optimality criteria was set to a value of 0.9999 in all of the runs in the branch-and-bound solution algorithm. This means that once the absolute optimality gap was less than 1.0, the program could be prematurely terminated and the resulting solution would be optimal. Önal (2004) made this observation, noting that since the objective function of maximal covering formulations can only take on integer values (e.g., number of land types or species covered), a current solution could not be improved upon once the gap was less than 1. This intervention technique can result in significant timesavings when solving 0–1 integer-programming problems of this nature. Exploiting this characteristic of the maximal covering/minimal uncovering formulation in our research resulted in significant timesavings for the one-objective problems. This protocol, however, could not be applied to the two-objective problem since the objective function was no longer integer in nature.

2.4. Software

All of the problems were solved on an IBM Pentium™ 4 personal computer, using the integrated solution package GAMS/OSL 2.25 (GAMS Development Corp, 1990), which was designed for large and complex linear and mixed integer-programming problems. Input files were created using GAMS (General Algebraic Modeling System), a program designed to generate data files in a standard format that optimization programs can read and process. The models were solved using IBM's OSL (Optimization Subroutine Library), a Fortran-based subroutine library designed to solve optimization problems. The revised primal simplex algorithm, in conjunction with the branch-and-bound algorithm for integer-variable problems, were used to solve the models.

3. Results

3.1. General results

Each of the models was solved 25 times, varying either the set of objective weights for the two-objective problem or the corresponding parameter, L, for the bound on total selected area for the single objective problem. The weights, total area parameter values and levels of coverage are contained in Table 1 and correspond to the 25 runs for each model. The trade-off curve that corresponds to the output of each model solved in this analysis is shown in Fig. 1. Each point on the curve represents one of the 25 solutions, illustrating the number of land systems that are (un)covered for each upper bound on total selected area.

Obviously, as the total area of selected sites increases, coverage increases as well. This is achieved in the two-objective models by incrementally increasing the value of w_1 while simultaneously decreasing the value of w_2. In the one objective models, this is achieved by incrementally increasing the bound on total selected area. All 248 land systems can be covered by selecting sites whose total area is 12075.5 km, 3.7% of the total area of 1886 eligible sites. Thus, complete coverage, as coverage is defined here, can be achieved for a relatively small percentage of the landscape. Additionally, a decision-maker can see what levels of coverage are possible for lower investments in total area. We should note that the solutions displayed in Fig. 1 are by no means exhaustive, but rather a representative sample meant to illustrate the range of the values of the two objectives. Additional

Table 1
Values and relationships between weights, total selected area, and number of (un)covered land systems

Solution number	w_1	Total area (km^2)	Land systems covered	Land systems uncovered
1	0	0	0	248
2	0.3	1	4	244
3	0.5	2.5	6	242
4	0.6	6.25	9	239
5	0.7	15	13	235
6	0.8	40.75	21	227
7	0.9	60.5	24	224
8	0.92	117.5	29	219
9	0.93	219.5	37	211
10	0.94	497	56	192
11	0.945	709.75	69	179
12	0.95	817.5	75	173
13	0.955	1025.5	85	163
14	0.96	1118.5	89	159
15	0.965	1599	108	140
16	0.97	2772.5	147	101
17	0.975	2915.5	151	97
18	0.98	3675.5	167	80
19	0.982	3978	174	74
20	0.985	5723.25	204	44
21	0.99	7012.25	219	29
22	0.992	7665	225	23
23	0.995	10268.5	242	6
24	0.997	11231.25	246	2
25	0.998	12070.75	248	0

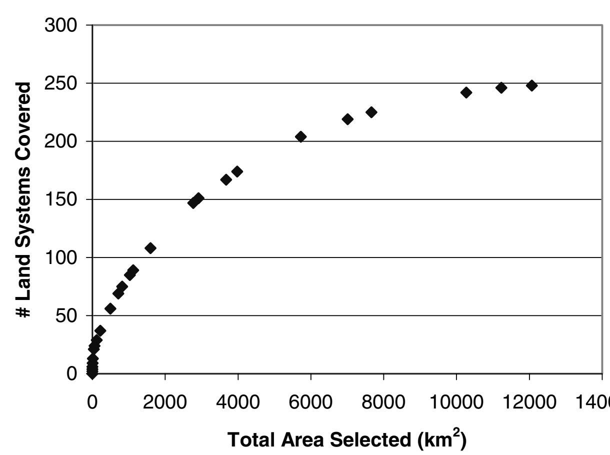

Fig. 1. Trade-off curve between land system coverage and total protected reserve area.

solutions may exist for different values of the parameter on total selected area and different sets of objective weights. Once an approximation of the trade-off curve has been generated, a decision-maker may then decide to focus more closely on a portion of the curve, generating additional solutions in a specific range.

3.2. One- and two-objective maximal covering problems

The results of the comparison between the one- and two-objective maximal covering problems, models 1 and 3, are listed in Table 2. We compared average solution time over the set of solutions and number of branch-and-bound nodes and iterations. We found that average solution time for the two-objective maximal covering formulation was more than three orders of magnitude faster than the one-objective version, 3.17 versus 7962.79 s. Note that for runs when the maximum area limit was less than or equal to 500 km^2, both models solve very rapidly with no required branch-and-bound nodes. With budget levels greater than 500 km^2, computational effort increases for both models, but at a much higher rate for the single objective formulation. The average number of iterations and branch-and-bound nodes are more than two orders of magnitude less for the two-objective version of this problem.

3.3. One- and two-objective minimal uncovering problems

A similar comparison was done of the one- and two-objective minimum uncovering problems, models 2 and 4. Results are listed in Table 3. Average solution time for the two objective, minimal uncovering formulation is again over three orders of magnitude faster than the corresponding one-objective model, 2.53 versus 9297.92 s. As with the previous set of runs, both models solve easily and quickly when the maximum budget level is less than or equal to 500 km^2. Above that, computational effort increases for both models, but again at a much higher rate for the single objective formulation. The average number of iterations and branch-and-bound nodes are over two orders of magnitude less for the two-objective version of this problem.

These results clearly illustrate that for our data set, the two-objective formulations are much more efficient than the equivalent one-objective formulations. Our findings when comparing the maximal covering to minimal uncovering formulation differ somewhat from findings made by Church et al. (1996) and Arthur et al. (1997). They both found that the minimum uncovering reserve site selection formulation when a constraint on the number of sites is included solves more quickly than the maximal covering formulation. We found virtually

Table 2
Comparison of solution characteristics of the one and two-objective, maximal covering formulation with coverage variables declared non-negative

Solution number	One objective			Two objective		
	Time (s)	B&B nodes	Iterations	Time (s)	B&B nodes	Iterations
1	0.003	0	0	0.152	0	0
2	0.090	0	8	0.152	0	253
3	0.109	0	16	0.152	0	256
4	0.09	0	23	0.16	0	258
5	0.09	0	42	0.148	0	263
6	0.09	0	67	0.152	0	275
7	0.098	0	91	0.148	0	281
8	0.113	0	167	0.16	0	289
9	0.152	0	206	0.16	0	298
10	0.23	0	386	0.16	0	317
11	2.281	126	2198	0.188	7	343
12	4.527	324	4670	0.219	10	375
13	26.008	2448	19,431	0.258	7	445
14	6.332	470	6454	0.238	3	510
15	16.078	1657	7304	0.297	4	569
16	25.898	925	30,458	0.559	27	775
17	141.539	8093	159,132	0.551	31	948
18	587.961	40,477	571,719	1.281	82	2026
19	804.48	49,052	861,436	2.309	191	3008
20	631.398	17,646	511,338	2.082	112	3248
21	52884.605	870,391	10,425,376	11.508	1180	10,892
22	20245.08	320,945	6,394,809	20.199	2046	17,958
23	799.281	36,120	648,600	1.93	137	3233
24	86248.451	906,914	8,922,934	11.551	1206	8172
25	36644.777	917,903	17,066,480	24.367	2673	14,486
	Average:					
	7962.79	127751.6	1,825,334	3.16684	308.64	2779.12

Table 3
Comparison of solution characteristics of the one and two-objective, minimal uncovering formulation with the uncoverage variables declared non-negative

Solution number	One objective			Two objective		
	Time (s)	B&B nodes	Iterations	Time (s)	B&B nodes	Iterations
1	0.27	0	0	0.129	0	42
2	0.24	0	13	0.121	0	9
3	0.229	0	15	0.141	0	21
4	0.240	0	21	0.133	0	23
5	0.25	0	34	0.129	0	32
6	0.24	0	39	0.117	0	44
7	0.26	0	51	0.133	0	45
8	0.27	0	64	0.129	0	51
9	0.289	0	108	0.141	0	72
10	0.299	0	106	0.148	0	89
11	2.738	126	1492	0.188	6	112
12	5.109	325	3018	0.238	10	132
13	40.23	2449	13,741	0.199	7	131
14	7.07	399	4480	0.199	3	221
15	21.691	1659	6028	0.227	4	222
16	29	912	24,452	0.48	27	466
17	174.93	7808	146,681	0.457	31	593
18	935.799	27,469	686,545	0.949	70	1053
19	1054.66	48,827	849,488	2.23	208	2356
20	1295.562	34,902	781,372	2.039	148	2061
21	79929.234	953,362	11,134,104	12.762	1383	10161
22	26683.75	278,263	500,2856	9.391	958	8527
23	2478.152	45,110	919,060	1.781	145	2097
24	8187.449	128,077	2,639,414	10.34	1013	7547
25	111,600*	1,389,535*	10,793,021*	20.371	2255	11808
	Average:					
	9297.922	116768.9	1,320,248	2.52688	250.72	1916.6

* *Note.* This run terminated after 31 h without an optimal solution because OSL encountered solution difficulties.

no difference in average solution time when comparing the two-objective maximal covering problem with the two-objective minimal uncovering formulations of our model specifications with area considerations. Further, we found that the average solution time of the area-constrained maximal covering formulation was over 20 min faster than the equivalent minimal uncovering formulation, 7962.79 versus 9297.92 s.

3.4. Binary versus non-negative coverage decision variables

As pointed out by Church et al. (1996), the coverage variables, y_i, and the uncoverage variables, u_i, do not have to be declared binary in order for them to solve as such in the conventional maximal covering location problem when a limit on the number of selected sites is included. Relaxing these binary variable restrictions has been shown to lead to savings in solution time and effort. In all of our runs reported in Tables 2 and 3, the binary restrictions were relaxed. However, the effect of this variable designation has not been tested on the area-constrained maximal covering/minimal uncovering formulations. To test this, we repeated the runs of all 4 of our model specifications, now requiring the coverage variables to be binary. The results of this can be evaluated by comparing the appropriate columns in Tables 2 and 3 with those in Tables 4 and 5 for both the one and two-objective models. For both the maximal covering and minimal uncovering versions of the two-objective problem, average solution time was virtually the same whether the respective coverage/uncoverage variables were made binary or non-negative. Average solution time for all 4 sets of runs was around 3 s. However, dramatic and unexpected differences arise with the one-objective formulations. For the one-objective maximal covering formulation, average solution time was 3 times faster when the coverage variables were declared binary rather than non-negative and upper bounded at 1 (2294.08 versus 7962.79 s). For the one-objective minimal uncovering formulation, average solution time was an order of magnitude faster when the ‘uncoverage’ variables were declared binary rather than non-negative (633.89 versus 9297.92 s). While the variable designation seems to have no effect in the two-objective models, there is a pronounced effect with the one-objective models. So, despite theory that would suggest fewer binary decision variables would make for an easier

Table 4
Comparison of solution characteristics of the one and two objective, maximal covering formulation with the coverage variables declared binary

Solution number	One objective			Two objective		
	Time (s)	B&B nodes	Iterations	Time (s)	B&B nodes	Iterations
1	0	0	0	0.121	0	0
2	0.23	0	8	0.137	0	253
3	0.23	0	15	0.16	0	256
4	0.25	0	23	0.152	0	258
5	0.24	0	42	0.16	0	263
6	0.26	0	67	0.148	0	275
7	0.25	0	91	0.16	0	281
8	0.26	0	167	0.16	0	289
9	0.309	0	206	0.16	0	298
10	0.439	0	386	0.188	0	317
11	2.637	101	1603	0.219	7	343
12	28.309	1207	22,223	0.27	10	375
13	1.609	36	1164	0.262	8	445
14	91.859	1037	28,089	0.25	3	510
15	12.508	202	6618	0.309	4	569
16	129.98	1602	70,382	0.559	27	766
17	109.691	1037	38,498	0.539	14	939
18	463.117	13,427	298,020	1.367	87	2060
19	794.988	11,877	394,002	2.418	187	3065
20	400.84	4068	203,366	2.152	133	3259
21	449.559	6585	230,384	11.762	1169	10881
22	131.34	2571	83,227	19.859	2015	17522
23	70.061	1797	50,824	2.258	177	3341
24	19793.02	276,890	6,457,608	11.629	1206	8172
25	34870.031	917,903	17,066,480	24.852	2673	14491
	Average:					
	2294.081	49613.6	998139.7	3.21044	308.8	2769.12

problem to solve, we did not find that to be the case with our data when an area or budget constraint is included in maximal coverage/minimal uncovering formulations.

4. Discussion

We addressed the problem of selecting sites for protection with the objective of maximizing land type representation subject to a constraint on the total area of the selected sites. We formulated and compared computational efficiency of several models of this problem specification. Our results showed that for our data set and choice of optimization solver the two-objective formulations performed consistently better than one-objective formulations in terms of average solution time, number of iterations and nodes. Many instances of our one-objective problem took hours to over a day to solve, a prohibitive amount of time to make such models useful to planners often in need of real time feedback and information. However, all instances of our two-objective formulations could be solved in less than 24.85 s. Thus, while some reserve site selection coverage models have been readily able to be solved as single objective problems with area restrictions, we found the solvability of our models and data to be greatly enhanced by a multiobjective approach. The reader should also bear in mind that the solution performance of the one-objective models in this analysis was enhanced through the use of the optimality criteria discussed in Section 2. Without this intervention, the disparity of performance between the one and two-objective models would have been even greater.

We also found that when cast as a two-objective problem, both the maximal covering and minimal uncovering formulations solved equally fast. When cast as a single objective formulation, the average solution time of the maximal covering formulation was surprisingly faster than the uncovering formulation. Finally, even more surprising, was the finding that solution speed is enhanced by declaring the coverage variables as binary in the single objective problem specifications. We should point out, however, that our results are conditioned upon the use of our particular data set and solver, and as such, our findings might not be generalizable to other data sets. We suggest, however, that our results can be used to guide other researchers in their exploration of alternative model specifications if solution difficulties are encountered in solving area-constrained reserve site selection formulations. We found significant computational savings through the formulation and solution of alternative, equivalent models.

Table 5
Comparison of solution characteristics of the one and two objective, minimal uncovering formulation with the uncoverage variables declared binary

Solution number	One objective			Two objective		
	Time (s)	B&B nodes	Iterations	Time (s)	B&B nodes	Iterations
1	0.20	0	0	0.539	0	2013
2	0.441	0	15	0.172	0	453
3	0.438	0	27	0.402	0	1305
4	0.441	0	47	0.383	0	1213
5	0.441	0	58	0.48	0	1629
6	0.461	0	87	0.457	0	1851
7	0.488	0	187	0.363	0	1131
8	0.598	0	394	0.512	0	1679
9	3.191	0	614	0.449	0	1513
10	1.48	0	1800	0.5	0	1686
11	3.469	53	2748	0.508	7	1732
12	117.133	2955	67,394	0.609	10	1562
13	1.551	2	2013	0.621	8	1844
14	1.77	4	2288	0.449	3	1257
15	1.719	3	2247	0.59	4	1766
16	38.293	677	23,058	0.75	27	1549
17	95.562	1394	56,083	0.762	15	1672
18	670.379	15,677	392,672	1.328	78	1978
19	117.18	1113	47,957	2.5	206	2916
20	404.75	3835	168,923	2.031	125	2538
21	353.008	6120	218,773	8.488	865	7871
22	165.18	2928	76,898	16.051	1638	13,449
23	89.461	2107	47,251	1.18	94	930
24	10411.211	128,077	2,639,414	8.578	874	5434
25	3368.289	52117	1,117,568	22.398	2485	11,127
	Average:					
	633.8854	8682.48	194740.6	2.844	257.56	2883.92

Each of the models was solved 25 times, varying either the set of objective weights for the two-objective problem or the corresponding parameter, L, for the bound on total selected area for the single objective problem. The weights, total area parameter values and levels of coverage are contained in Table 1 and correspond to the 25 runs for each model. The trade-off curve that corresponds to the output of each model solved in this analysis is shown in Fig. 1. Each point on the curve represents one of the 25 solutions, illustrating the number of land systems that are (un)covered for each upper bound on total selected area.

Obviously, as the total area of selected sites increases, coverage increases as well. This is achieved in the two-objective models by incrementally increasing the value of w_1 while simultaneously decreasing the value of w_2. In the one-objective models, this is achieved by incrementally increasing the bound on total selected area. All 248 land systems can be covered by selecting sites whose total area is 12075.5 km, 3.7% of the total area of 1886 eligible sites. Thus, complete coverage, as coverage is defined here, can be achieved for a relatively small percentage of the landscape. Additionally, a decision-maker can see what levels of coverage are possible for lower investments in total area.

Using the weighting method we were quickly able to generate an estimate of the trade-off curve between the two objectives, shown in Fig. 1. The multiobjective weighting method, used to solve our two-objective problems, is an efficient and well-known means to generate an estimate of the trade-off curve for problems with binary decision variables. Our intent was not to enumerate every feasible solution on the trade-off curve. We suggest, moreover, that a decision-maker is unlikely to be interested in every possible feasible solution to a problem. Instead, the decision maker is likely to find specific regions on the trade-off curve to be of particular interest.

The particular shape of any trade-off curve, when generated through the weighting method, often reveals regions of the curve in which solutions are tightly packed and also regions where gaps between solutions indicate that further exploration might be warranted. The shape of the curve may also illustrate where the value of one objective is changing rapidly, and thus, where sharp trade-offs are occurring. It is only after a reasonable approximation of the trade-off curve has been generated that a decision maker is likely to focus attention on specific regions of the curve. Once a decision-maker has identified an area(s) of interest on the curve, then the modeler can go back and solve the problem via the multiobjective weighting method within a tighter range of weights. If gaps between solutions in the area of interest still remain, then solution techniques other than the multiobjective weighting method may be useful in more closely exploring the solutions in that region.

If gaps between solutions on the curve remain after having explored the region of interest with a series of weights, then the modeler may want to solve specifications of the corresponding single objective problem via the constraint method. With the constraint method, the upper bound on the total selected area parameter in our single objective model is systematically varied and the problem resolved with each new value. The reason for using the constraint method in conjunction with the weighting method is that when applied to a integer-programming problem, the multiobjective weighting method may not be able to find all possible non-inferior solutions due the presence of "gap points." Gap points are noninferior solutions to a multiobjective integer model that cannot be found by the weighting method. These can occur because the surface of the trade-off curve may not convex/concave due to the integrality of the variables. (Refer to Cohon, 1978 for a discussion of gap points.)

Despite the possible presence of gap points, however, an analyst can still likely generate a very good estimate of the non-inferior set of solutions using the multiobjective weighting method. This then allows the decision maker and analyst to locate the solution regions of greatest interest relatively quickly and efficiently, thus reducing the need to solve many instances of the less computationally efficient, single objective version of the problem via the constraint method.

In addition to formulating an area-constrained reserve selection model that can be quickly solved, we suggest our findings have additional significance. Specifically, since the area-constrained two-objective problem can readily be solved, additional constraints may be able to be added to this formulation allowing for even more realistic reserve selection model specifications. Additional work is planned to extend the concepts developed in this paper to a multi-period site selection application. Further, we plan to explore in relation to our findings the development of the much more difficult reserve site selection problem that requires a minimum area of each land type be included in selected reserve sites before the land type is considered covered.

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References

Arthur, J.A., Hachey, M., Sahr, K., Huso, M., Kiester, A.R., 1997. Finding all optimal solutions to the reserve site selection problem: formulation and computational analysis. Environmental and Ecological Statistics 4, 153–165.

Camm, J.D., Polasky, S., Solow, A., Csuti, B., 1996. A note on optimal algorithms for reserve site selection. Biological Conservation 78, 353–355.

Church, R.L., ReVelle, C.S., 1974. The maximal covering location problem. Papers of the Regional Science Association 71, 199–215.

Church, R.L., Stoms, D.M., Davis, F.W., 1996. Reserve selection as a maximal covering location problem. Biological Conservation 76, 105–112.

Csuti, B., Polasky, S., Williams, P.H., Pressey, R.L., Camm, J.D., Kershaw, M., Kiester, A.R., Downs, B., Hamilton, R., Huso, M., Sahr, K., 1997. A comparison of reserve selection algorithms using data on terrestrial vertebrates in Oregon. Biological Conservation 80, 83–97.

Cohon, J., 1978. Multiobjective Programming and Planning. Academic Press, New York.

GAMS Development Corporation, 1990. General Algebraic Modeling System. Version 2.25.090, Washington, DC.

Mabbutt, J.A., 1968. Review of Concepts of Land Classification. In: Stewart, G.A. (Ed.), Land Evaluation. MacMillan, Melbourne, pp. 11–28.

Önal, H., 2004. First-best, second-best, and heuristic solutions in conservation reserve site selection. Biological Conservation 115 (1), 55–62.

Pimm, S.L., Lawton, J.H., 1998. Planning for biodiversity. Science 279, 2068–2069.

Polasky, S., Camm, J.D., Garber-Yonts, B., 2001. Selecting biological reserves cost-effectively: an application to terrestrial vertebrate conservation in Oregon. Land Economics 77 (1), 68–78.

Pressey, R.L., Logan, V.S., 1995. Reserve coverage and requirements in relation to partitioning and generalization of land classes: analysis for Western New South Wales. Conservation Biology 9 (6), 1506–1517.

Pressey, R.L., Possingham, H.P., Logan, V.S., Day, J.R., Williams, P.H., 1999. Effects of data characteristics on the results of reserve selection algorithms. Journal of Biogeography 26 (1), 179–191.

ReVelle, C.S., 1993. Facility siting and integer friendly programming. European Journal of Operational Research 65, 147–158.

ReVelle, C.S., Williams, J.C., Boland, J.J., 2002. Counterpart models in facility location science and reserve selection science. Environmental Modeling and Assessment 7, 71–80.

Rodrigues, A., Gaston, K.J., 2002. Optimisation in reserve selection procedures – why not? Biological Conservation 107, 123–129.

Rothley, K.D., 1999. Designing bioreserve networks to satisfy multiple, conflicting demands. Ecological Applications 9 (3), 741–750.

Snyder, S.A., Tyrrell, L.E., Haight, R.G., 1999. An optimization approach to selecting research natural areas in National Forests. Forest Science 45 (3), 458–469.

Underhill, L., 1994. Optimal and suboptimal reserve selection algorithms. Biological Conservation 35, 85–87.

Walker, P.J., 1991. Land systems of Western New South Wales. Technical Report No. 25. Soil Conservation Service of New South Wales, Sydney.