Comparison of Estimates of Hardwood Bole Volume Using Importance Sampling, the Centroid Method, and Some Taper Equations

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ABSTRACT: *Various taper systems and the centroid method were compared to unbiased volume estimates made by importance sampling for 720 hardwood trees selected throughout the state of West Virginia. Only the centroid method consistently gave volumes estimates that did not differ significantly from those made by importance sampling, although some taper equations did well for most species. North. J. Appl. For. 19(3):141–142.*

Key Words: Centroid, importance sampling, taper equations.

Intensive forest management and wood utilization practices require accurate and versatile volume estimation techniques. Martin (1981) developed species-specific coefficients for Appalachian species using taper systems developed by Bruce et al. (1968), Demaerschalk (1972), Hilt (1980), Kozak et al. (1969), Max and Burkhart (1976), and Ormerod (1973) (consult those publications for formulae; some are rather complex). Max and Burkhart's model was found to be the most consistent.

Importance sampling, developed by Gregoire et al. (1986), estimates the volume of a tree bole using a proxy, or previously developed, taper function. The proxy volume is adjusted using the ratio of measured cross-sectional area over estimated area at height h, where h is selected proportional to estimated bole volume. Importance sampling provides unbiased estimates of bole volumes. The generic form used here assumes that the square of bole radius decreases linearly with height up the bole (Gray 1956), which in field tests has proven as good in the practical sense as using more complex proxy functions (Coble and Wiant 2000, Wiant et al. 1989). The equation is:

$$v = (k/2)(a/(H-h))$$

where

v = volume in cubic units,

k = $2H(m-s)+(s^2-m^2)$,

m = merchantable height,

s = stump height,

a = $0.005454d^2$, for English units,

d = diameter at h,

h = $H-((H-s)^2-nk)^{1/2}$,

H = total height, and

n = random number between 0 and 1.

Wood et al. (1990) and Wood and Wiant (1990) developed the centroid method, a variant of importance sampling, where h is selected with half of the estimated merchantable bole volume below and half above that point. The formula is the same as for importance sampling except n is replaced by 0.5, giving the centroid, the height of which is the expected mean height at which diameter measurements would be made in importance sampling if the sample were repeated many times (Van Deusen and Lynch 1987). The method has proved useful but is not unbiased. It has advantages as compared to importance sampling as the measurement point is rarely hidden by foliage and volumes are less variable than those found by

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Table 1. Percent bias between cubic-foot volumes estimates made by importance sampling (imp) and those made by the centroid method (cen) and taper systems by Hilt (hil), Ormerod (orm), Bruce (bru), Max-Burkhart (max), Demaerschalk (dem), Kozak (koz), and Gray (gra) for 11 hardwood species in West Virginia.

	No.	Mean	Percent bias[a]							
Species	trees	imp vol	Cen	Hil	Orm	Bru	Max	Dem	Koz	Gra
Red maple (*Acer rubrum*)	65	25.5	1.8	0.1	5.6	6.2[b]	11.8[b]	14.2[b]	15.3[b]	14.1[b]
Sugar maple (*A. saccharum*)	41	30.7	2.9	–5.8	–1.2	3.4	5.9	11.2[b]	10.0[b]	9.4[b]
Hickory (*Carya* sp.)	36	29.3	5.3	–3.8	5.0	9.3	13.0[b]	23.1[b]	24.2[b]	18.0[b]
Am. beech (*Fagus grandifolia*)	58	40.3	–4.0	–6.6	1.4	6.8	10.9[b]	11.4[b]	16.8[b]	11.6[b]
Yellow-poplar (*Liriodendron tulipifera*)	148	40.4	5.8	–17.5[b]	5.7	7.8[b]	12.6[b]	13.7[b]	15.2[b]	12.3[b]
White oak (*Quercus* alba)	99	30.4	–1.3	–4.6	–0.8	–1.4	7.7[b]	17.3[b]	22.2[b]	9.0[b]
Scarlet oak (*Q. coccinea*)	27	27.1	2.7	3.1	4.5	8.6	13.6[b]	29.5[b]	27.7[b]	16.3[b]
Chestnut oak (*Q. prinus*)	124	31.7	0.2	–6.6[b]	–1.3	0.8	3.8[b]	9.4[b]	9.5[b]	8.1[b]
North. red oak (*Q. rubra*)	58	33.7	1.7	2.5	11.0[b]	–2.3	17.9[b]	27.1[b]	28.2[b]	21.5[b]
Black oak (*Q. velutina*)	52	31.4	2.1	3.6	3.3	7.2	11.2[b]	19.8[b]	24.6[b]	20.3[b]
Black cherry (*Prunus serotina*)	12	34.4	1.9	–10.2	–0.6	–1.4	1.8	5.6	7.3	4.1
All trees	720	33.1	1.8	–6.8[b]	3.1[b]	4.1[b]	10.2[b]	15.5[b]	17.4[b]	12.7[b]

[a] ** is % bias = ((mean volume estimate by given method – mean imp volume)/mean imp volume)100.
[b] Significant at 0.05-level based on Dunnett's test.

importance sampling. See Wiant et al. (1992) for details concerning importance sampling and the centroid method.

Methods

Data were collected on 88 Forest Inventory Analysis (FIA) plots in 39 counties across West Virginia. All trees for 11 species (see Table 1) above 11 in. dbh were measured for stump height and diameter, dbh, total tree height, merchantable height to a 10 in. top or lower merchantable limit and diameter, importance height and diameter, and centroid height and diameter. Diameter measurements within reach of the ground were made with a d-tape, and upper stem diameters and heights were measured using a Criterion Survey Laser. Trees averaged 15.4 in, dbh, 78.4 ft in total height, 32.1 ft in merchantable height, and an importance sampling volume of 33.1 ft^3 (ranging from 5 to 353 ft^3).

Results

Table 1 shows the results of an analysis of variance and Dunnett's test for all species and by species. It is informative that in no comparison did volumes determined by the centroid method vary significantly from those obtained by importance sampling. We conclude that, although the centroid method is not unbiased, the bias introduced is negligible. The equations by Ormorod, Bruce, and Hilt performed well in most cases. Importance sampling and the related centroid method potentially eliminate the need for developing species-specific taper equations and coefficients, especially important in countries where mensurational research is limited. The greatest limiting factor, at present, is the development of accurate and affordable instruments for measuring upper-stem diameters.

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