

Correction for Slope in Point and Transect Relascope Sampling of Downed Coarse Woody Debris

Göran Ståhl, Anna Ringvall, Jeffrey H. Gove, and Mark J. Ducey

ABSTRACT. In this article, the effect of sloping terrain on estimates in point and transect relascope sampling (PRS and TRS, respectively) is studied. With these inventory methods, a wide angle relascope is used either from sample points (PRS) or along survey lines (TRS). Characteristics associated with line-shaped objects on the ground are assessed, e.g., the length or volume of downed logs. In their basic forms, the methods only work in flat terrain, and thus bias is incurred under sloping conditions.

Two different possibilities to correct for bias due to slope are presented. The first one involves using a slightly modified relascope device and making a measurement of the angle of inclination of each sampled object. The second involves applying correction factors based on the steepness of the terrain in the area surveyed. However, it is shown that moderate slopes cause only limited bias and in such cases there is little need to adjust the measurement procedures or apply correction factors. *For. Sci.* 48(1):85–92.

Key Words: Angle gauge sampling, bias, Bitterlich sampling, correction factor, forest inventory, line intercept sampling.

IN POINT AND TRANSECT RELASCOPE SAMPLING (PRS and TRS, respectively), a wide angle relascope is used for assessing characteristics associated with line-shaped objects on the ground. In PRS, the relascope is used at sample points, while in TRS it is used along survey lines. The instrument is operated horizontally, and all line-shaped objects that fill the gap of the relascope are included in the sample (Ståhl 1998, Gove et al. 1999). Because the instrument should have a wide angle, a standard Spiegel-relaskop (cf. Bitterlich 1984, p. 82–85) can generally not be used when applying these methods.

The primary use of PRS and TRS would probably be in connection with the estimation of parameters related to fallen trees and logging residue. The importance of these kinds of assessments has increased as today's forest management acknowledges the preservation of biodiversity as an important goal, and many organisms in the forest depend on dead decaying wood for their survival (e.g., Kruys et al. 1999).

PRS is closely related to traditional "angle count sampling" for stand basal area (Bitterlich 1984, p. 9–19), while TRS can be regarded as a special case of line intercept sampling (Warren and Olsen 1964, De Vries 1986, p. 242–257). Both PRS and TRS can be used for the estimation of any parameter that can be derived from the population of line-shaped objects studied. If the characteristic of interest is the sum of the objects' lengths, a TRS estimate can be obtained from a mere count of units included in the sample. Alternatively, the sum of lengths squared can be estimated by a simple count of objects in PRS.

Using either of the methods under horizontal conditions, the inclusion zone around an object is the union of two equally large intersecting circles. The size of the circles depends on the length of the object and the angle of the relascope. Figure 1 depicts the inclusion zone for each line-shaped object, which provides the basis for estimation in PRS and TRS.

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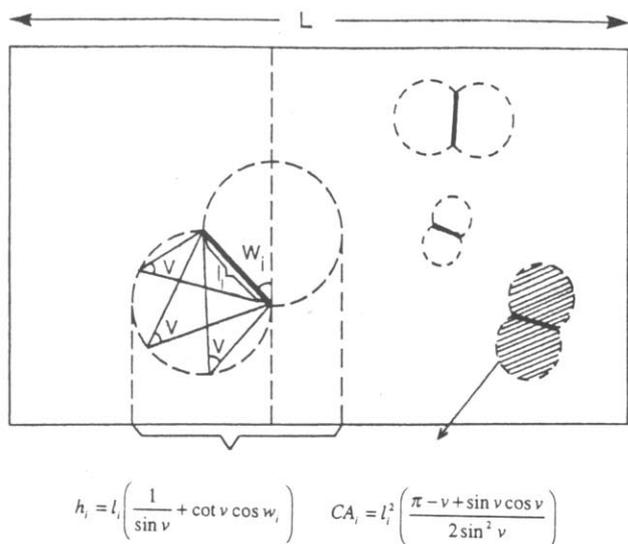


Figure 1. Line-shaped objects and their inclusion zones in PRS and TRS (flat terrain). The acute angle between the inventory line and the object is denoted w_i , l_i is the length of the object, v the fixed angle of the relascope, CA_i the area of the inclusion zone, h_i the width of the vertical projection of the inclusion zone, and L the width of the forest area perpendicular to the survey line.

If one PRS sample point is randomly laid out, the probability, u_i , of including a particular object in the sample is given as the area, CA_i , of the object's inclusion zone over the total area (ignoring boundary overlap problems):

$$u_i = \frac{CA_i}{T} = \frac{\pi - v + \sin v \cos v}{2 T \sin^2 v} l_i^2 \quad (1)$$

Here, v is the angle of the relascope (in radians), T the area of the forest surveyed, and l_i the length of the object. Details of the derivation of this formula, including the treatment of boundary overlap, are given in Gove et al. (1999).

If one TRS survey line is randomly laid out in a top-down direction somewhere along the baseline in Figure 1, again ignoring problems with boundary overlap, the probability of including a particular object in the sample is

$$u_i = \frac{h_i}{L} = \frac{1/\sin v + \cot v \cos w_i}{L} l_i \quad (2)$$

where h_i is the width of the projection of the inclusion zone, w_i the (acute) angle between the object and a survey line, and L the length of the baseline. Details are given in Ståhl (1998).

Knowing the probability of inclusion of objects, design-unbiased estimators of any population total, Y , can be established using the Horvitz-Thompson estimator (e.g., Cochran 1977, p. 259–261):

$$\hat{Y} = \sum_{i=1}^n \frac{y_i}{u_i} \quad (3)$$

Here, y_i is the value of the variable of interest on object i , and n is the number of units included in the sample. When more than one sample point or line is selected, the right-hand side of (3) must be divided by the actual number of points or lines (and the estimator is no longer a pure Horvitz-Thompson estimator). Also, the TRS estimator outlined can be improved on in the more

realistic case where the study area is not rectangular; however, for the purpose of this study, Formulas 2 and 3 are sufficient.

The above formulas hold in cases when the relascope is operated horizontally and the line-shaped objects extend in the horizontal plane. In case of hilly terrain, a pragmatic approach to avoid bias is to subjectively judge what would be the extension of an object in the horizontal plane, and then include or exclude the object accordingly. More dogmatic approaches would be either to use measurement procedures that automatically correct for slope, or to calculate correction factors that can be applied to estimates obtained from surveys in hilly areas. The aim of this article is to develop the methodology in these respects and also to provide results on the bias in PRS and TRS when no corrections are made. As a matter of simplification, hereafter all line-shaped objects considered will be referred to as downed logs or just logs.

Two different approaches to avoiding bias due to slope in PRS or TRS surveys will be described. The first is similar to the way this problem is handled in standard relascope sampling, where correction for slope is often built in to the relascope instrument (Bitterlich 1984, p. 80). However, in PRS and TRS, modifying the instrument alone is not sufficient, since an additional measurement on the logs is required (see below). The second approach is to compute correction factors that depend on the slope in the area surveyed. These correction factors also tell the magnitude of the bias incurred in the case where no corrections are made.

Adjusting the Measurement and Estimation Procedures

In Figure 2, the kind of instrument often used in PRS and TRS is shown. The ring is to be held close to the eye of the surveyor, and the angle of the relascope is determined by the delimiters at each side of the solid bar of the instrument. When the instrument is operated horizontally, probabilities of inclusion in PRS and TRS according to Formulas (1) and (2) are obtained. Problems arise when a log is situated at another elevation than the surveyor. However, by extending the delimiters both upwards and downwards (vertically), the surveyor will be able to judge whether or not logs located at another elevation should be included, even though the bar of the instrument is kept horizontally. As will be shown, when using extended delimiters it does not even matter whether the two endpoints of the log appear at different elevations. The principles behind this method are outlined below.

First, consider two imaginary vertical lines that extend both upwards and downwards from the two endpoints of a downed log. When addressing these (imaginary) lines with the relascope in the horizontal plane, the log should be counted as "in" when the lines appear further apart than the delimiters of the instrument. In three dimensions, the log's inclusion space can be described as the union of two cylinders (Figure 3). The surfaces of the two cylinders coincide along the two lines (see Figure 3). Thus, the two lines define a belt-shaped (unbounded) planar region between them, which has the same width, m_i , at any elevation. The width is related to the length of the log, l_i , as: $m_i = l_i \cos z_i$, where z_i is the log's angle of inclination from the horizontal plane.

If it were possible to make use of the imaginary lines during the inventory, a standard relascope operated in the horizontal

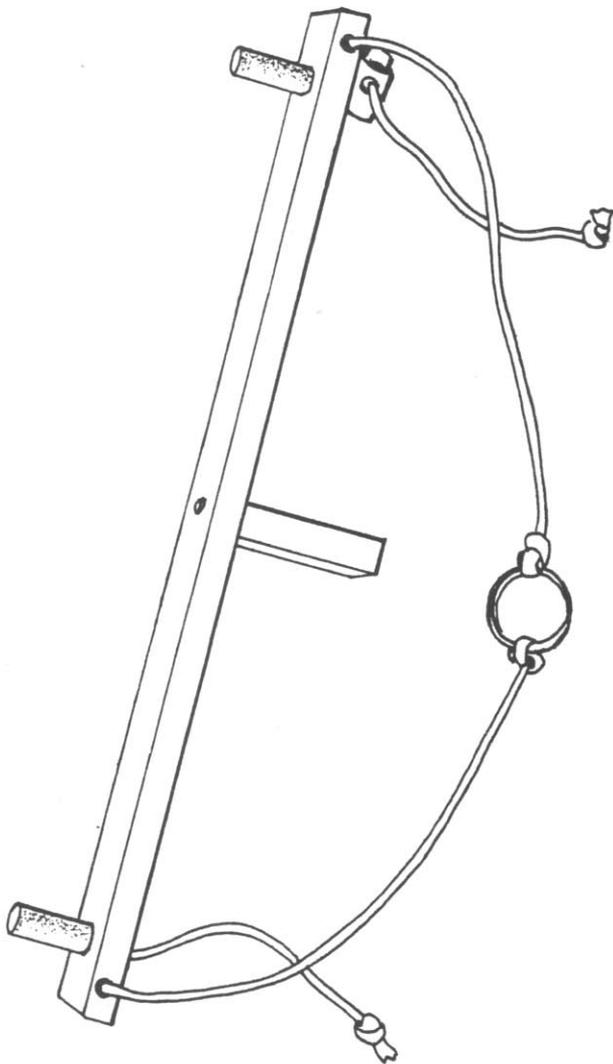


Figure 2. A wide-angle relascope that can be used in PRS and TRS surveys. This instrument is not adjusted for surveys in steep terrain.

plane would be sufficient in order to obtain unbiased estimates, provided m_i is used instead of l_i in Formulas (1) and (2). This requires that the angle of inclination of each sampled log be measured. However, in the field it would be very difficult to aim at imaginary lines, and this problem is avoided if the modified relascope is used. With this instrument, the extended delimiters of the scale also form a belt-shaped region analogous to the one obtained between the two imaginary lines extending from the end-points of the log (Figure 3), except that the two imaginary lines may be located at different distances from the surveyor. When standing at the perimeter of the inclusion zone of a log, the two belt-shaped regions will overlap exactly when looking through the relascope. Consequently, it does not matter whether the surveyor looks horizontally towards the imaginary lines or directly at the endpoints of the log. The judgment about whether or not to count the log will be the same in both cases with the modified relascope.

The conclusion is that design-unbiased estimates can always be obtained in PRS and TRS if an instrument with extended delimiters is used, and if the angle of inclination, z_i , or, alternatively, the projected length in the horizontal plane of each sampled log, is measured. Substituting m_i for

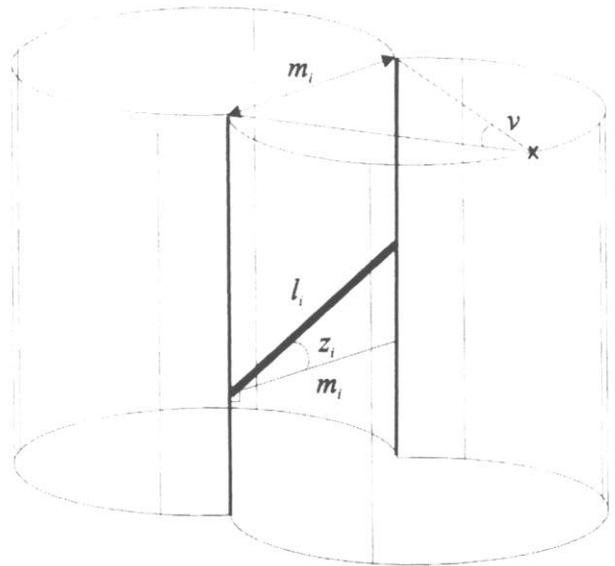


Figure 3. A log's inclusion space in three dimensions. If the surveyor aims, horizontally, at the imaginary lines extending from the endpoints of the log, unbiased estimates can be obtained if $m_i = l_i \cos \beta$ is used instead of l_i , the length of the log, in Formulas (1) and (2). The angle of the relascope is denoted v , and z_i is the angle of inclination of the log from the horizontal plane.

l_i in Formulas (1) and (2) is required for unbiasedness. Any other measurements based on length that are used to calculate the quantity of interest, y_i in (3), would use l_i as usual, however.

Computation of Correction Factors

Correcting for bias according to the above method requires using a slightly more awkward instrument and an additional measurement on each log sampled. Moreover, straightforward estimates of the sum of log lengths (in TRS) or the sum of lengths squared (in PRS) cannot be obtained by only counting the logs included as is otherwise the case because of the substitution of m_i for l_i in (2) and (3). These facts motivate studies on the extent of the bias incurred under sloping conditions in case no attempt is made to avoid it. If the resulting bias is mostly limited, it could be argued that there is no reason to abandon the standard methods, or that simple correction factors be used based on the average slope in the area surveyed.

Two cases will be discussed. The first is when a modified relascope is used (according to the principles given above) without any additional measurements of inclination angles or projected lengths on the logs. The second case is when a standard relascope is used (cf., Figure 2), although rather than being operated horizontally, in hilly terrain the instrument is tilted so that the bar of the device is always held parallel with the slope plane.

In both cases, the bias is derived for different slopes, given certain model assumptions regarding the population of logs. The model assumptions are introduced since the bias will depend on the orientation of the logs in relation to the slope. For a treatment of model-based approaches to inference in forest survey sampling, see Gregoire (1998). It is assumed that the acropetal orientations of the logs follow a uniform distribution in the interval 0 to 2π radians

in the horizontal plane. Regarding the length of logs (and the orientation of survey lines in TRS), no distributional assumptions need to be made, since they will not affect the results.

Ratios, R , were calculated for both PRS and TRS according to Formula (4). The expected value of the Horvitz-Thompson estimator, using noncorrected inclusion probabilities, appears in the numerator. In the denominator, the corresponding estimator with slope-adjusted inclusion probabilities is inserted. Principally, expectations are taken over the sample, the orientation of logs, the length of logs, and in TRS also over the orientation of survey lines. However, as will be shown later, simplifications can be made in the different cases where R is calculated, and the expectations need not consider all these factors.

$$R = \frac{E \sum_{i=1}^n \frac{y_i}{u_i}}{E \sum_{i=1}^n \frac{y_i}{t_i}} \quad (4)$$

The relative model bias of PRS and TRS under sloping conditions is given by $R - 1$. Consequently, if R is close to 1, it is safe to proceed in the standard way, without correcting for slope. However, when R deviates substantially from 1, an alternative to using modified measurement procedures would be to compute slope-corrected estimates by dividing the standard estimates with R .

In Formula (4), it can be seen that the main problem of deriving R consists of determining t_i , the slope-adjusted probability of inclusion of a log. Therefore, in the following sections the focus will be on deriving t_i for the different cases.

It is always assumed that the survey is conducted within an area of constant slope, the angle of inclination from the horizontal plane being denoted β . For many of the derivations, it is useful to conceive of coordinate systems in both the horizontal plane and the slope plane, each system having its x-axis parallel with the topographic lines of the slope. The angle between the x-axis and the vector corresponding to tree i is denoted α_i in the horizontal plane, and γ_i in the slope plane. The angle, in the horizontal plane, between the x-axis and any of the parallel TRS survey lines is denoted λ .

Case 1. A modified instrument held horizontally.—In this case, the slope-adjusted probability of inclusion will generally be smaller than the standard probability of inclusion. This follows from above, where it was shown how the measurement procedure should be adjusted in order to account for slope. Instead of using l_i in the formulas for the probability of inclusion, $l_i \cos z_i$ should be used. Consequently, the size of the potential bias depends on the orientation of the logs in relation to the slope. In the case where a log extends parallel with the topographic lines, z_i will be 0, and thus there will be no bias. On the other hand, if a log extends perpendicularly to the topographic lines $z_i = \beta$ and the bias will reach its maximum.

The slope-adjusted probabilities of inclusion in PRS and TRS will be [cf. Formulas (1) and (2)]:

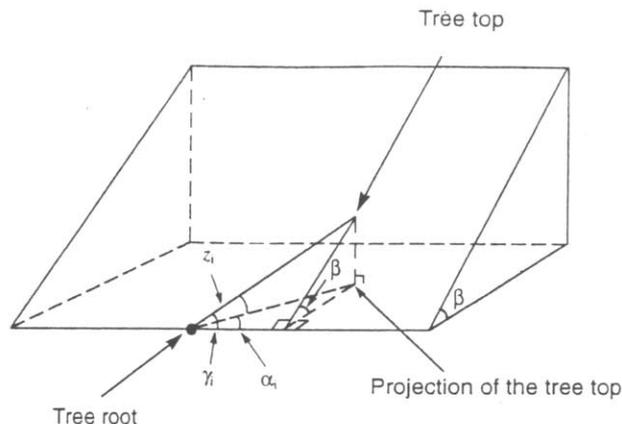


Figure 4. An illustration of the connection between a log's angle of inclination, z_p , the angle between the projection of the downed log and the x-axis in the horizontal plane, α_p , the corresponding angle in the slope plane, γ_p and the angle of inclination of the slope plane, β .

$$t_i^{PRS} = \frac{\pi - \nu + \sin \nu \cos \nu}{2 T \sin^2 \nu} (l_i \cos z_i)^2; \quad (5)$$

$$t_i^{TRS} = \frac{1/\sin \nu + \cot \nu \cos w_i}{L} l_i \cos z_i$$

The resulting ratios, derived according to the principle outlined in (4), can be obtained as (see Appendix 1):

$$R_{PRS} = E [\cos^2 z_i]; \quad (6)$$

$$R_{TRS} = E [\cos z_i]$$

The expectations only consider the orientation of the logs. The angle of inclination, z_i , of a log can be computed from the slope and the log's orientation. The angle is obtained as (cf. Figure 4):

$$z_i = \tan^{-1}(\sin \alpha_i \tan \beta) \quad (7)$$

Using (6), the R -values for different slope conditions, as well as the most extreme R -values from calculations with fixed α and λ , were computed. The results are presented in Table 1. A conclusion is that slope correction is more important for PRS than for TRS.

Table 1. R -values in PRS and TRS when a modified instrument is used together with non-adjusted estimators. The minimum and maximum R -values were derived in calculations with fixed orientations of logs (and survey lines).

Slope, β (radians and %)	Point relascope sampling (PRS)	Transect relascope sampling (TRS)
$\pi/18$ –18%	0.98	0.99
	0.97–1.00	0.98–1.00
$\pi/9$ –36%	0.94	0.97
	0.88–1.00	0.94–1.00
$\pi/6$ –58%	0.87	0.93
	0.75–1.00	0.87–1.00
$\pi/4$ –100%	0.71	0.83
	0.50–1.00	0.71–1.00

Case 2. The instrument held parallel with the sloping plane.— In this case, a standard wide-angle relascope is used (Figure 2). In hilly terrain, it is slightly tilted so that the bar of the instrument is kept parallel with the slope plane. It is assumed that the logs extend in the slope plane, and that the relascope is operated in the same plane; the surveyor is assumed to aim at the objects at a height above the ground corresponding to the height at which the instrument is held. (A modified instrument operated in the slope plane works just as well.)

The area of inclusion around an object is the union of two circles of equal size in the standard horizontal PRS and TRS cases (cf. Figure 1). This will also be the case in the slope plane under the above assumptions. By projecting the inclusion area vertically to the horizontal plane, the area of inclusion will be the union of two intersecting ellipses. This is illustrated in Figure 5.

Since the inventory points or lines are laid out on a map, their formal properties (length, direction, spacing) refer to their properties in the horizontal map plane. In this plane, however, the area of inclusion of an object is no longer the union of two circles, which was the basis for the derivation of Formulas (1) and (2). Consequently, use of Formula (3) leads to bias both for PRS and TRS. The size of the bias depends on the properties of the ellipses which, in turn, depend on the slope and the properties of the downed logs and the relascope.

To calculate the slope-adjusted probability of inclusion of an object in PRS, the area of the union of the two intersecting ellipses in the horizontal plane must be determined. For TRS, the width of the union of the two ellipses must be determined. While the procedure is quite simple in the PRS case, it requires slightly awkward derivations in the TRS case.

Correction Factors in the PRS Case

The area, CA_i , of the union of the two intersecting circles that constitute the inclusion zone in the slope plane is obtained from Formula (1), as u_i times T . To obtain the slope-adjusted probability of inclusion, the area of the inclusion zone projected onto the horizontal plane must be derived, and be divided with T . The area of the projection, denoted EA_i , is obtained simply as:

$$EA_i = CA_i \cos \beta \quad (8)$$

This follows since any area can be expressed as the definite integral

$$\int_{x_1}^{x_2} B(x) dx,$$

where $B(x)$ is the width of the area at point x along the x -axis. Now, if an area is defined in the slope plane, and we want to determine the area of its projection in the horizontal plane, note that regardless of the shape of the area the projection of $B(x)$ will be $B(x) \cos \beta$. Thus, since $\cos \beta$ is a constant that does not depend on x , it can be put outside the integration and Formula (8) follows. As a consequence, the R -values for the PRS case will always equal $\cos \beta$. Although possibly superfluous, the results are

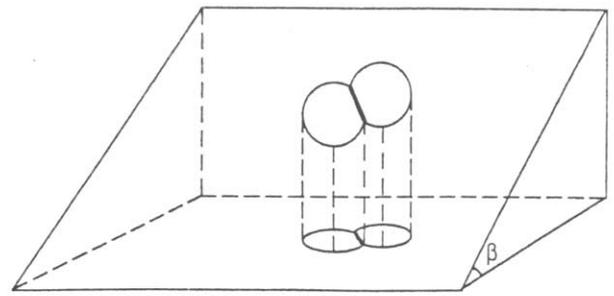


Figure 5. Projection of the inclusion zone of an object from the slope plane to the horizontal plane, leading to the inclusion zone being the union of two intersecting ellipses.

presented in Table 2 for the sake of completeness. The R -values turn out to be of similar size as the R -values in Case 1 (cf. Table 1). However, since they do not depend on any model assumptions about the log population, one could argue that this method is to be preferred if PRS is applied without any slope correction.

Correction Factors in the TRS Case

In the case of TRS, the derivation of slope-adjusted probabilities of inclusion turned out to be slightly complicated, and an algorithmic approach based on analytic geometry was adopted. Local coordinate systems are assumed to have their origins at the center of each of the ellipses (circles) forming the inclusion zone of a log. The ellipses are congruent, the length of their semi-major axes is denoted a , the length of their semi-minor axes b . Since the projection has no effect on distances along the x -axis, the length, a , of the semi-major axis will equal the radius of the circles: $a = l_i / (2 \sin \nu)$. The length of the semi-minor axis, on the other hand, will be affected by the projection, and thus it will be $b = a \cos \beta$.

Figure 6 illustrates what the projected inclusion zone looks like (above) and what the corresponding reference inclusion zone looks like (below). Unlike Figure 5, this illustration is not intended to show what happens through the projection, but rather what the adjusted and nonadjusted inclusion zones look like for a log with a certain length and orientation. In deriving a slope-adjusted probability of inclusion, s_i rather than h_i should be used in (2). Therefore, the determination of s_i will now be given attention. The steps of the algorithm developed for the purpose are briefly outlined in Figure 7.

The first step is to determine the coordinates of the points of tangency between the ellipses and imagined lines parallel with the survey lines. For this purpose, the ellipses are split into upper and lower parts in order to express them as one-to-

Table 2. R -values in PRS when a standard instrument is operated parallel with the slope plane, together with nonadjusted estimators.

Slope, β (radians and %)	R -value
$\pi/18$ –18%	0.98
$\pi/9$ –36%	0.94
$\pi/6$ –58%	0.87
$\pi/4$ –100%	0.71

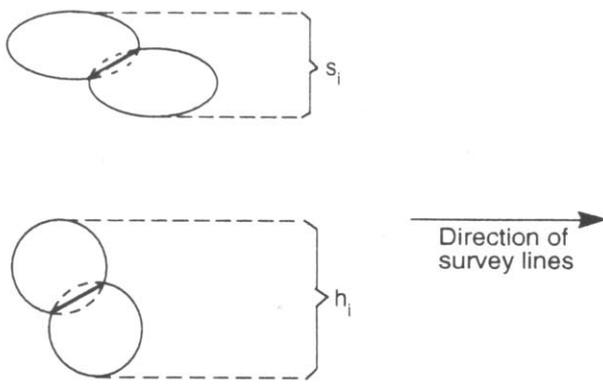


Figure 6. An illustration of the projected inclusion zone (above) and the reference inclusion zone (below) for a log with a certain length and orientation. The width of the prior zone is denoted s_p , the width of the latter h_p .

one functions. The upper part of an ellipse can, in a local coordinate system in the horizontal plane, be expressed as:

$$y = \frac{b}{a} \sqrt{a^2 - x^2} \quad (9)$$

To determine the points of tangency, the problem is to determine where a line with "slope coefficient" $\tan \lambda$ will be tangent to the ellipse. To do this, (9) is differentiated with respect to x , and the formula obtained set equal to $\tan \lambda$ (with due respect to $\tan \lambda$ having infinite value in some cases). The equation is solved for x , and the corresponding y calculated. The tangent point, in the upper part of an ellipse in the local coordinate system, is (with $k = \tan \lambda$):

$$(x, y) = \left(-\frac{ka^2}{\sqrt{k^2a^2 + b^2}}, \frac{b^2}{\sqrt{k^2a^2 + b^2}} \right) \quad (10)$$

The corresponding point of tangency in the lower part of an ellipse is easily found using symmetry relations (reflection through the local origin).

The next step is to set up a common coordinate system for the two ellipses (circles). To do this, the common coordinate system in the horizontal plane is assumed to have its origin at the middle point of the projection of the downed log. Consequently, if the coordinates of the ellipse centers in the common system can be determined, the new coordinates of the points of tangency can be determined by adding/subtracting the coordinates obtained in (10).

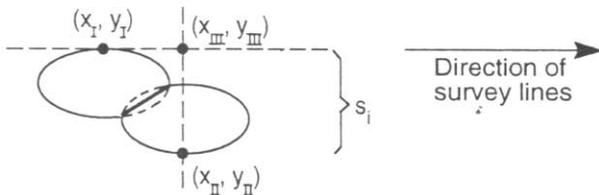


Figure 7. An overview of the procedure for determining s_p . The upper and lower tangent points of the ellipses and imagined lines parallel with the survey lines are first located (points I and II). A tangent line through point I and a perpendicular line passing through point II are then established. The coordinates for point III, in which the two lines intersect, are determined and the distance, s_p , between II and III calculated using the Pythagorean theorem.

The derivation of new coordinates will be made for the center of the right ellipse in Figure 7. The coordinate of the center of the left ellipse can be found from symmetry relations (reflection through the origin). In Figure 4 it can be seen that the angle γ_i can be derived from α_i and β as:

$$\gamma_i = \tan^{-1}(\tan \alpha_i / \cos \beta) \quad (11)$$

The angle γ_i is needed in order to determine the coordinate of the center of the ellipse in the common system in the horizontal plane. This coordinate is determined by first determining the coordinate of the center point of the corresponding circle in the sloping plane, and then projecting this point. Since the projection only affects the length along the y -axis, the new coordinate of the center of the right ellipse will be:

$$(x_0^{proj}, y_0^{proj}) = (l_i \cot v \sin \gamma_i / 2, -l_i \cot v \cos \gamma_i \cos \beta / 2) \quad (12)$$

Parts of the derivations are illustrated in Figure 8, with regard to the situation in the slope plane. To determine the points of tangency on the ellipses in the common system, the local coordinates of the tangency points are added to the coordinates of the ellipse centers.

When this is accomplished, the next step (see Figure 7) is to construct a tangent line through point I. This line has the equation:

$$y_I = k(x - x_I) + y_I \quad (13)$$

Then, a line perpendicular to the line obtained in (13) is derived. This line should pass through point II. The line's equation will be:

$$y_p = -\frac{1}{k}(x - x_{II}) + y_{II} \quad (14)$$

Next, Equations (13) and (14) are set equal, and the coordinate of their intersection determined (point III in Figure 7). Knowing the coordinates of points II and III, s_i can be obtained using the Pythagorean theorem. Thus, the

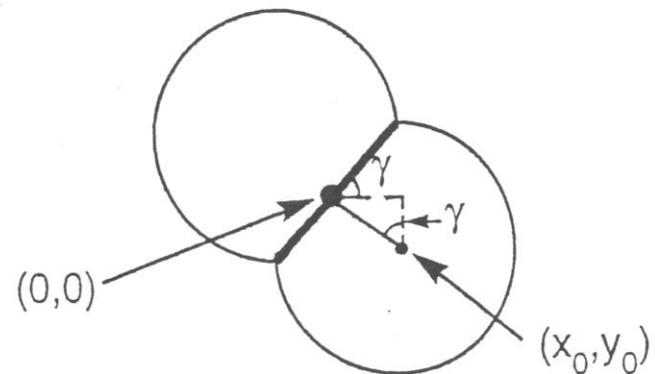


Figure 8. An illustration of parts of the determination of the global coordinate of the center point of the right ellipse. The coordinate of the center of the circle in the slope plane is first determined, then projected to the horizontal plane.

Table 3. *R*-values in TRS when a standard instrument is operated parallel with the slope plane, together with nonadjusted estimators. The minimum and maximum *R*-values were derived in calculations with fixed orientations of logs and survey lines.

Slope, β (radians and %)	Relascope angles, ν (radians)			
	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
$\pi/18$ –18%	0.99	0.99	0.99	0.99
	0.98–1.01	0.98–1.01	0.98–1.00	0.98–1.00
$\pi/9$ –36%	0.97	0.97	0.97	0.97
	0.91–1.03	0.92–1.02	0.93–1.01	0.94–1.00
$\pi/6$ –58%	0.94	0.94	0.94	0.93
	0.81–1.07	0.82–1.05	0.84–1.03	0.87–1.00
$\pi/4$ –100%	0.87	0.87	0.87	0.86
	0.60–1.17	0.63–1.13	0.65–1.08	0.71–1.00

slope-adjusted probability of inclusion can be obtained. In Table 3, the bias is presented in terms of *R*-values. Also, the largest and the smallest *R*-values, obtained from “worst case” orientations of logs and survey lines, are presented for different combinations of relascope angles and slopes. The principle used for deriving *R*-values was the same as shown in Appendix 1.

Discussion

The results indicate that moderate slopes cause only limited bias in PRS and TRS when standard instruments are operated parallel with the slope plane and standard estimation procedures are used. For example, a 36% slope results in a bias of –6% in PRS and –3% in TRS. However, in steep terrain the bias is more substantial. In such cases it is recommended that measurement procedures that automatically adjust for slope be applied. The procedure for this resembles the correction technique used in standard relascope sampling (Bitterlich 1984), although in the latter case, slope adjustments can be made without any additional measurements on the sampled trees. In the case of PRS and TRS, additional measurements of the angles of inclination or the projected lengths in the horizontal plane are required for all logs. Although the (model) bias is generally limited, it can be seen in Tables 1 and 3 that the worst cases, for certain log and survey line orientations, often lead to substantial bias. For example, using a modified relascope (extended delimiters) but a nonadjusted estimator in PRS, a 36% slope may lead to as much as –12% bias if all logs are oriented perpendicularly to the topographic lines. The bias of TRS for the corresponding case amounts to –6%, although with a standard instrument operated parallel with the slope plane it may amount to almost –10%. With a standard instrument in PRS, the bias will always be the same regardless of the orientation of the logs.

In some extreme cases, sloping terrain may even cause positive bias in TRS (Table 3), although standard instruments and estimation procedures are used. This follows since, for some combinations of log and survey line orientations, the width of the inclusion zone given by the union of the two ellipses is larger than the corresponding width given by the union of the two circles.

Finally, since the results concerning slope correction using *R*-values generally make distributional assumptions regarding the orientation of logs and survey lines, an alternative would be to make a minimax bias correction. In this case, the *R*-value used for the correction should be the average of the two extreme cases. Using the average of the extreme cases minimizes the maximum error in the correction. Minimax methods are not without their shortcomings (Berger 1985, p. 371–372), but they do represent a risk-averse and distribution-free approach to the problem. Generally (cf. Table 1 and Table 3), minimax corrections would be very similar to corrections based on standard *R*-values. The similarity is reassuring, in that the corrections based on standard *R*-values are nearly minimax, and pose little risk of extreme bias.

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APPENDIX 1

Below, the derivations of *R*-values in Formula (6) are presented. This concerns the case when a modified relascope is operated horizontally, although l_i is used rather than $l_i \cos z_i$ in the estimation. For both PRS and TRS, *R* is the ratio between two expected values. The expectations consider the random samples and the random population, following from

the model assumptions regarding the logs. First, the case of PRS is described. The notations are the same as in the main text. The indices s and pop denote the sample and the model population, respectively, whereas n and N denote the sample size and the fixed population size, respectively.

$$\begin{aligned}
 R_{PRS} &= \frac{E_{s,pop} \left[\sum_{i=1}^n \frac{y_i}{\frac{\pi - v + \sin v \cos v}{2 T \sin^2 v} l_i^2} \right]}{E_{s,pop} \left[\sum_{i=1}^n \frac{y_i}{\frac{\pi - v + \sin v \cos v}{2 T \sin^2 v} (l_i \cos z_i)^2} \right]} \\
 &= \frac{E_{pop} \left[E_{s|pop} \left[\sum_{i=1}^n \frac{y_i}{l_i^2} \right] \right]}{E_{pop} \left[E_{s|pop} \left[\sum_{i=1}^n \frac{y_i}{l_i^2 \cos^2 z_i} \right] \right]} = \\
 &= \frac{E_{pop} \left[E_{s|pop} \left[\sum_{i=1}^n \frac{y_i}{l_i^2 \cos^2 z_i} \cdot \cos^2 z_i \right] \right]}{E_{pop} \left[E_{s|pop} \left[\sum_{i=1}^n \frac{y_i}{l_i^2 \cos^2 z_i} \right] \right]} \\
 &= \frac{E_{pop} \left[\sum_{i=1}^N y_i \cdot \cos^2 z_i \right]}{E_{pop} \left[\sum_{i=1}^N y_i \right]} \cong E_{pop} [\cos z_i]
 \end{aligned}$$

The final result follows from an assumption of independence between the quantity of interest of a log, y_i , and the log's angle of inclination. It can be seen that all model assumptions, except the assumption regarding the orientation of logs, are unnecessary for the derivation of R .

The same procedure is used to derive the R -value for TRS.

$$\begin{aligned}
 R_{TRS} &= \frac{E_{s,pop} \left[\sum_{i=1}^n \frac{y_i}{\frac{1}{\sin v + \cot v \cdot \cos w_i} l_i} \right]}{E_{s,pop} \left[\sum_{i=1}^n \frac{y_i}{\frac{1}{\sin v + \cot v \cdot \cos w_i} l_i \cos z_i} \right]} \\
 &= \frac{E_{pop} \left[E_{s|pop} \left[\sum_{i=1}^n \frac{y_i}{(1 / \sin v + \cot v \cdot \cos w_i) l_i \cos z_i} \cos z_i \right] \right]}{E_{pop} \left[E_{s|pop} \left[\sum_{i=1}^n \frac{y_i}{(1 / \sin v + \cot v \cdot \cos w_i) l_i \cos z_i} \right] \right]} \\
 &= \frac{E_{pop} \left[\sum_{i=1}^N y_i \cdot \cos z_i \right]}{E_{pop} \left[\sum_{i=1}^N y_i \right]} \cong E_{pop} [\cos z_i]
 \end{aligned}$$

Although not shown explicitly, the same basic principle for the derivations was applied also in the case of TRS when a standard relascope was operated in the slope plane (the basis for the results in Table 3).