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Multistage point relascope and randomized branch sampling for downed coarse woody debris estimation

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Abstract

New sampling methods have recently been introduced that allow estimation of downed coarse woody debris using an angle gauge, or relascope. The theory behind these methods is based on sampling straight pieces of downed coarse woody debris. When pieces deviate from this ideal situation, auxillary methods must be employed. We describe a two-stage procedure where the relascope is used to select pieces of downed coarse woody debris in the first stage. If the pieces so chosen on the first stage have multiple branches and detailed estimates are required for the entire piece, then a second stage sample is advocated using the randomized branch technique. Both techniques are reviewed and an example is given examining possible surrogate variables for the second stage.

Keywords: Angle gauge sampling; Inventory methods; Probability proportional to size sampling

1. Introduction

Two new methods were recently introduced for sampling downed coarse woody debris (CWD) in forests. Both methods use an angle gauge, or relascope, with a wide critical angle, to select pieces of downed CWD with probability proportional to size (PPS). However, the two techniques differ in their implementation: in transect relascope sampling (TRS), pieces are selected along transect lines with probability proportional to length (Ståhl, 1998); by contrast, in point relascope sampling (PRS), pieces of CWD are selected from point locations with probability proportional to the square of their length (Gove et al., 1999). Field and simulation studies for both TRS (Ringvall and Ståhl, 1999) and PRS (Brissette et al., 2001) are ongoing. While it is too early to make definitive statements about the efficacy of these techniques, the preliminary results suggest that they hold some promise for the unbiased quantification of downed CWD in forest ecosystems.

The foundational papers on transect and point relascope sampling laid out the general framework for estimation and field implementation. In addition, Ståhl et al. (2001) discusses how to use both techniques on sloping terrain. Other issues still need to be addressed, however, in the everyday use of these techniques. One such issue is the case of multiple-stemmed (or significantly crooked) pieces of downed CWD. Both theoretical developments have made the assumption that the population to be sampled is composed of straight pieces of downed CWD (hereinafter termed "logs") with little or no branching of consequence. This assumption may be fine in coniferous forests where the branching habit of the the trees is largely excurrent. However, in deciduous

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woodlands—where branching habits tend more towards the deliquescent form—blowdowns, and storm-damaged or normal mortality contribute pieces to the pool of CWD that are multiple-stemmed (hereafter simply “branched”). As will be shown shortly, failure to correctly account for branching in such cases can introduce substantial biases into the final estimates.

In line intersect sampling (LIS) for downed CWD, the general estimator allows for multiple intersections on the same piece and is a convenient way to deal with estimates of quantities based on total length for branched stems (de Vries, 1986, p. 273). However, this convenience does not necessarily extend to estimation of certain other quantities in LIS, nor does it translate to either fixed area (e.g. plot or strip) designs, or to the angle gauge methods described above. In this paper, we propose a two-stage design which employs PRS as the first stage to select pieces of downed CWD with PPS. The second stage, which is employed only on pieces of downed CWD that are branched, uses randomized branch sampling (RBS) (Jessen, 1955; Valentine and Hilton, 1977) to further subsample for the characteristic of interest. The second stage sampling procedure is independent of how the individual pieces were selected on the first stage sample. Thus, while not necessary for the general LIS estimator, the RBS subsampling can be applied equally as well to TRS or fixed-area designs.

2. Methods

2.1. Point relascope sampling

In PRS, a critical angle, $0 < \nu \leq 90°$, is projected using an angle gauge and individual logs are selected with probability proportional to their length-square. On a typical point, one swings a complete 360° arc and views the length of each candidate log with the angle gauge. If the log’s length appears greater than the projected angle, then the log is included in the sample for that point, otherwise it is ignored. PRS may therefore be thought of as a “plotless” technique, akin to horizontal point sampling (HPS) for standing trees (Grosenbaugh, 1958).

The details of the development of PRS, including the formulas used in estimation, methods for handling borderline logs and points falling near to the tract boundary are given by Gove et al. (1999). To summarize, an unbiased estimate of some quantity Y for the entire tract based on a single sample point is given as

$$\hat{Y}_k = A\mathscr{L}\sum_{i=1}^{m_k}\frac{y_{ki}}{l_{ki}^2} \tag{1}$$

where A is the tract area in hectares, m_k the number of logs tallied on the kth point, y_{ki} the quantity of interest measured on the ith log on the kth point, and l_{ki} its length. The constant $\mathscr{L}$ is the squared length factor in units $\mathrm{m}^2\,\mathrm{ha}^{-1}$ and depends only on the relascope angle ν.

If n sample points are randomly chosen within the tract boundaries, then an unbiased estimator for the tract total is simply the average of the individual point estimates; viz

$$\hat{Y} = \frac{1}{n}\sum_{k=1}^{n}\hat{Y}_k. \tag{2}$$

Additionally, an unbiased estimator for the variance is given as

$$\mathrm{var}(\hat{Y}) = \frac{1}{n(n-1)}\sum_{k=1}^{n}(\hat{Y}_k - \hat{Y})^2. \tag{3}$$

Fig. 1 presents the typical situation for sample selection of individual straight logs. Consider the case

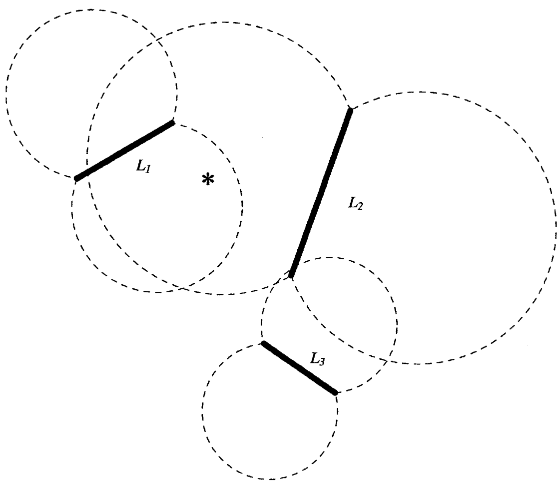

Fig. 1. The inclusion zones (dashed) for three straight logs of differing length using a fixed relascope angle of $\nu = 40°$. Logs L_1 and L_2 would be selected from the arbitrary sample point (∗).

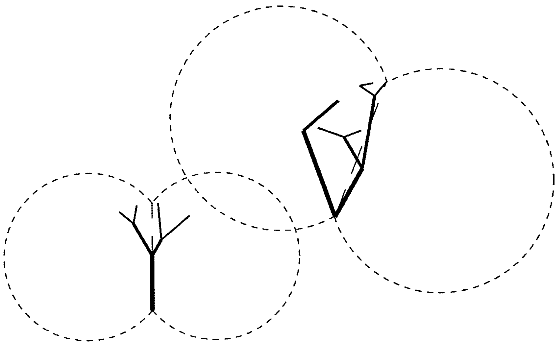

Fig. 2. Two multiple-branched pieces of downed CWD with their projected lengths shown as the dashed lines; the inclusion zones for PRS with $\nu = 40^\circ$ are also shown.

where the kth sample point falls anywhere within the inclusion area for a log (dashed lines). Then that log's length will clearly appear longer than the projected angle and the log will be sampled. Any measurements subsequently desired on that log, including its length, can be measured directly. This procedure is straightforward for straight logs that have little or no significant branching. However, in Fig. 2, we have a situation where a tree and a top are pictured. In either case, because the true length of the piece is as yet unknown and is not linear, one must resort to use of a "projected" length to determine whether the subject piece is sampled or not. The projected length may be defined as the horizontal, straight, mid-line distance between the butt end of the piece and the terminal end of the most distal branch segment that meets the minimum criterion for CWD used in the survey. The projected length should be measured either along a straight line extending through the longitudinal midsection of the piece in question, or directly from the butt to the end of the most distal branch segment, depending upon what is most appropriate. Thus, the distance between the butt and the terminal end is "projected" onto this imaginary line for the sake of PRS determination. This distance is shown as the dashed extension line on the two pieces pictured in Fig. 2. As will be shown presently, the projected length must be measured for all quantities to be calculated for the subject piece.

2.2. Randomized branch sampling

Randomized branch sampling (RBS) is a special method of multi-stage probability sampling, which was introduced by Jessen (1955) to estimate fruit counts on orchard trees. Valentine et al. (1984) showed that the method also can be used to estimate the size or mass of tree-shaped objects. Gregoire et al. (1995) recently reviewed published applications of RBS, while Gregoire and Valentine (1996) proposed additional applications. In this section, we outline how RBS can be used to estimate a measure of size of a tree-shaped piece of CWD, which is selected in the first stage of sampling by PRS.

The piece of CWD, from the first-stage sample, could be an entire fallen tree or a fallen branch of a tree. For the purpose of explanation, we define the structure of a tree or branch entirely in terms of branch nodes and branch segments. A branch segment naturally occurs between two branch nodes. Under this definition, the basal end of a tree or branch and terminus of a terminal shoot are considered to be branch nodes. It often is convenient to refer to boles as basal branch segments and terminal shoots as terminal branch segments.

The RBS is used to select a "path" consisting of connected branch segments, extending from the basal branch segment to a terminal branch segment. Thus, there are as many possible paths in a tree or branch as there are terminal branch segments. The branch segments of a path—when selected by RBS—constitute a probability sample from which attributes for the entire tree can be estimated. In this section, we assume that the attribute of interest is the sum of the lengths of all of the branch segments that comprise the fallen tree or branch. However, mass, volume, or surface area may also be of interest in studies of CWD.

Fig. 3 presents a diagram of a multiple-forked branch that was measured as part of a CWD study. For illustration, each fork, or node, is uniquely numbered to facilitate explanation, although this is not required to apply RBS in general. The individual branch segments can be indentified by the numbers of their acropetal nodes; for example, the basal segment extends from node 1.0 to node 1.1 (or 1.0 $\rightarrow$ 1.1), so we can identify the basal branch segment by the number 1.1; in general, any branch segment may be identified by its node number in the form $(i.j)$. Thus, the large- and small-end diameters, and the length of the $(i.j)$th branch segment are denoted by $D_{i.j}, d_{i.j}$ and $l_{i.j}$, respectively. Diameter and length measurements of all the branch segments depicted in

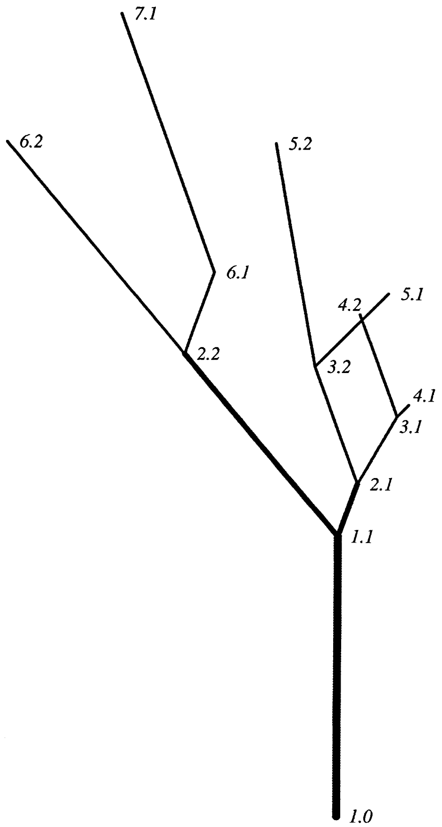

Fig. 3. A multiple-branched top showing the branching pattern in two dimensions with node numbers presented for cross-reference with Table 1; branch segment lengths are to scale, but diameters are not.

Fig. 3 are given in Table 1. Paths can also be identified in terms of these nodal numbers. For example, one path extends from the basal node 1.0 to node 1.1 to node 2.2 to node 6.1 to the terminal node 7.1 (or $1.0 \rightarrow 1.1 \rightarrow 2.2 \rightarrow 6.1 \rightarrow 7.1$). The six possible paths associated with the subject branch in Fig. 3 are enumerated in Table 2.

The selection of a sample path is achieved through repeated application of probability sampling. Referring again to Fig. 3, we begin at the basal node 1.0. Because there is only one way to proceed, no choice is involved in the selection of branch segment $1.0 \rightarrow 1.1$; therefore, its probability of selection is $q_{1.1} = 1$. Now, at node 1.1, we can select either branch segment 2.1 or 2.2. A conditional selection probabi-lity is assigned to either segment, namely, $q_{2.1}$ and $q_{2.2} = 1 - q_{2.1}$. Suppose that $q_{2.1} = 0.47$ and $q_{2.2} = 0.53$. A random number, u, is drawn between 0 and 1 to determine the selection. If $u \in [0, 0.47]$, then segment 2.1 is selected; otherwise, segment 2.2 is selected.

Assume that segment 2.2 was selected. Our first path now has evolved to $1.0 \rightarrow 1.1 \rightarrow 2.2$. At node 2.2, we must repeat the procedure, selecting either segment 6.1 or 6.2. Let us assume that segment 6.1 is selected with probability $q_{6.1} = 0.6$ so the path is now $1.0 \rightarrow 1.1 \rightarrow 2.2 \rightarrow 6.1$. Finally, at node 6.1 there is only one possible choice, so segment 7.1 is selected with probability $q_{7.1} = 1$. The entire path has now become $1.0 \rightarrow 1.1 \rightarrow 2.2 \rightarrow 6.1 \rightarrow 7.1$, which corresponds to path #6 in Table 2.

The total length of all of the segments of the fallen subject branch in Fig. 3 is unbiasedly estimated from the measured lengths of the branch segments in the path and their respective unconditional selection probabilities, i.e.

$$\hat{l} = \frac{l_{1.1}}{Q_{1.1}} + \frac{l_{2.2}}{Q_{2.2}} + \frac{l_{6.1}}{Q_{6.1}} + \frac{l_{7.1}}{Q_{7.1}}$$

where the unconditional selection probabilities are

$$Q_{1.1} = q_{1.1}, \qquad Q_{2.2} = q_{1.1} \times q_{2.2},$$
$$Q_{6.1} = q_{1.1} \times q_{2.2} \times q_{6.1},$$
$$Q_{7.1} = q_{1.1} \times q_{2.2} \times q_{6.1} \times q_{7.1}$$

Thus, for path #6

$$Q_{1.1} = 1, \qquad Q_{2.2} = 1 \times 0.53 = 0.53,$$
$$Q_{6.1} = 1 \times 0.53 \times 0.6 = 0.318,$$
$$Q_{7.1} = 1 \times 0.53 \times 0.6 \times 1 = 0.318$$

Finally, using the actual length measurements for the segments in path #6 (Table 1), the unbiased estimate of the total length of all of branch segments in the subject branch by RBS is

$$\hat{l} = \frac{1.54}{1} + \frac{1.32}{0.53} + \frac{0.48}{0.318} + \frac{1.52}{0.318} = 10.3\,\text{m}.$$

In the course of RBS, the conditional selection probabilities assigned to the branch segments emanating from any node must sum to 1; beyond that, there are no restrictions. However, it is desirable to pursue a strategy of assignment that provides accurate estimates. Returning to our example, suppose that we are at node 1.1 and must select either segment 2.1 or 2.2

Table 1
Measurements for the application of RBS to the CWD subject branch shown in Fig. 3

Branch segment	Segment length (m)	Large-end diameter[a] (cm)	Small-end diameter[a] (cm)	Branch order	Estimated length[b]
1.0 → 1.1	1.54	9.0	8.0	1	2
1.1 → 2.1	0.31	6.5	6.5	2	2/5
1.1 → 2.2	1.32	5.5	5.0	2	2
2.1 → 3.1	0.43	4.0	4.0	3	3/5
2.1 → 3.2	0.69	5.0	5.0	3	1
3.1 → 4.1	0.09	2.0	2.0	4	1/5
3.1 → 4.2	0.60	3.5	3.0	4	1
3.2 → 5.1	0.57	2.0	2.0	4	1
3.2 → 5.2	1.25	4.5	4.0	4	2
2.2 → 6.1	0.48	4.5	4.5	3	1
2.2 → 6.2	1.53	2.5	2.0	3	2
6.1 → 7.1	1.52	3.0	2.0	4	2

[a] Large-end diameters are from the basal end of the segment while small-end diameters are at the terminal end.
[b] These lengths were visually estimated from Fig. 3 to be approximately proportional to the actual measured lengths and do not necessarily have units meters.

as the next segment of our path. In a sense we are not choosing segment 2.1 or 2.2, rather we are choosing either a subpopulation of branch segments consisting of segments $\{2.1, 3.1, 3.2, 4.1, 4.2, 5.1, 5.2\}$, or a second subpopulation consisting of segments $\{2.2, 6.1, 6.2, 7.1\}$ (see Fig. 3). The total length of the segments in the first subpopulation is $L_{2.1} = l_{2.1} + l_{3.1} + \cdots + l_{5.2}$ and the total length of the segments in the second subpopulation is $L_{2.2} = l_{2.2} + l_{6.1} + l_{6.2} + l_{7.1}$. This suggests that we might use selection probabilities $q_{2.1} = L_{2.1}/(L_{2.1} + L_{2.2})$ and $q_{2.2} = L_{2.2}/(L_{2.1} + L_{2.2})$. Of course, these assignments are implausible because we do not know the individual segments lengths, and if we did, there would be no need for the sampling. One alternative is to calculate the selection probabilities with ocular estimates of the segments lengths; such ocular estimates are given in Table 1 and provide $q_{2.1} = 0.47$ and $q_{2.2} = 0.53$, the probabilities used in our example (see Table 3). Of course, even this method is too tedious and requires too much calculation for use in the field. Ordinarily one would assign the probabilities on the basis of quick ocular estimates of the total length of segments in each subpopulation. Alternatively one might calculate probabilities on the basis of measurements of diameter ($q_{2.1} = D_{2.1}/(D_{2.1} + D_{2.2})$ and $q_{2.2} = 1 - q_{2.1}$) or diameter-square ($q_{2.1} = D^2_{2.1}/(D^2_{2.1} + D^2_{2.2})$) under the assumption that differences in these measurements reflect the differences in the total length of the branch segments comprising the two subpopulations. Suffice it to say that there is no known optimal procedure for making the assignments of selection probabilities. Fortunately, the unbiasedness of the method is not affected by these assignments at any node unless, of course, they do not sum to one.

Table 2
All possible RBS paths running acropetally and corresponding to the CWD subject branch in Fig. 3

Path number	Path nodes[a]
1	1.0 → 1.1 → 2.1 → 3.1 → 4.1
2	1.0 → 1.1 → 2.1 → 3.1 → 4.2
3	1.0 → 1.1 → 2.1 → 3.2 → 5.1
4	1.0 → 1.1 → 2.1 → 3.2 → 5.2
5	1.0 → 1.1 → 2.2 → 6.2
6	1.0 → 1.1 → 2.2 → 6.1 → 7.1

[a] The "→" notation between nodes signifies that the path travels along the connecting branch segment in each case.

In general, it is convenient to denote the conditional selection probability of the rth segment of the pth path by q_{pr}. Then, the unconditional selection probability for the rth segment of the pth path is given as

$$Q_{pr} = \prod_{j=1}^{r} q_{pj}, \qquad r = 1, \ldots, R$$

Table 3
Selection probabilities based on ocular length estimates (Table 1) for the application of RBS to the CWD subject branch in Fig. 3

Path (p)	Segment (r)				RBS estimate of length ($\hat{l}_p$)
	1	2	3	4	
Conditional selection probabilities (q_{pr})					
1	1.0	0.4697	0.3103	0.1667	–
2	1.0	0.4697	0.3103	0.8333	–
3	1.0	0.4697	0.6897	0.3333	–
4	1.0	0.4697	0.6897	0.6667	–
5	1.0	0.5303	0.4	–	–
6	1.0	0.5303	0.6	1.0	–
Unconditional selection probabilities (Q_{pr})					
1	1.0	0.4697	0.1458	0.0243	8.85
2	1.0	0.4697	0.1458	0.1215	10.09
3	1.0	0.4697	0.3239	0.1080	9.61
4	1.0	0.4697	0.3239	0.2160	10.12
5	1.0	0.5303	0.2121	–	11.24
6	1.0	0.5303	0.3182	0.3182	10.31

where R is the number of segments in the path. The estimate of total length for the subject branch provided by the segments of the the pth path is

$$\hat{l}_p = \sum_{r=1}^{R} \frac{l_{pr}}{Q_{pr}}$$

where l_{pr} is the measured length of the rth branch segment of the pth path. Notice from Table 3 that the path depth, R, need not be the same for every path. Additionally, more than one path can be sampled on any one subject branch with RBS. If m_p such paths are sampled, an unbiased estimator for the total length is

$$\hat{l} = \frac{1}{m_p} \sum_{p=1}^{m_p} \hat{l}_p.$$

If more than one path is sampled, then the variance of $\hat{l}$ may be estimated by

$$\mathrm{var}(\hat{l}) = \frac{1}{m_p(m_p - 1)} \sum_{p=1}^{m_p} (\hat{l}_p - \hat{l})^2.$$

2.3. Combined PR and RB sampling

The preceeding discussions of PRS and RBS have presented each technique seperately. The purpose of this section is to show how these two techniques can be combined in a useful manner to allow the proper selection and estimation for quantities of interest when branched pieces of downed CWD occur in a sample. For simplicity, total branch (log) length is considered the primary quantity of interest in the illustrations, though volume is also mentioned. In practice, the techniques described are completely general and apply to any quantity that can be measured on the subject piece of CWD such as biomass or carbon content.

At this point, a short digression is in order. Assume that all the logs sampled on a given PRS point are straight, as in Fig. 1. Then, quantities to be calculated that are simple functions of length may require only l_{ki} to be measured. Furthermore, formula (1) tends to simplify in certain cases. For example, if total length for the entire tract is to be estimated, (1) becomes

$$\hat{Y}_k = A\mathscr{L} \sum_{i=1}^{m_k} \frac{1}{l_{ki}}$$

and if length squared for the entire tract is desired it reduces even further to $A\mathscr{L}m_k$ because of simple cancellations. These cancellations are possible because with straight (or mildy crooked) logs, the projected length and actual length are equivalent; thus, the length terms in the numerator and denominator of (1) measure the same quantity, yielding the simplifications. Formulas (2) and (3) remain unchanged in all cases when more than one sample point is taken.

Assume now that the branch in Fig. 3 has been selected on an individual sample point with PRS along with several other logs. To this point, no measurements have been required because the branch was determined to be "in" on the sample point with PRS simply by viewing its projected length as described earlier. For estimation purposes, however, the projected length is now required for all quantities because the branch has been selected with probability proportional to its projected length (as illustrated in Fig. 2), which now appears in the denominator of (1). For the branch in question, this length is measured to be $l_{ki} = 4.43$ m. Thus, for example, the contribution of this branch to an estimate of the total number of pieces of downed CWD is straightforward and is given as $A\mathscr{L} \times 4.43^{-2}$.

When considering estimating the contribution of even slightly more complicated quantities of our subject branch, the total length of the piece will also presumably be required. For example, if an estimate of total length for the entire tract is required, then the contribution of the subject branch to (1) becomes

$$A\mathscr{L}\frac{\tilde{l}_{ki}}{l_{ki}^2}$$

where $\tilde{l}_{ki}$ is an estimate of total branch length on the kth piece for the ith point. At this point, it should be apparent that one way to estimate $\tilde{l}_{ki}$ unbiasedly is with RBS. Thus, RBS can be applied to the subject branch with either one path (in which case $\tilde{l}_{ki} \equiv \hat{l}_p$), or several paths ($\tilde{l}_{ki} \equiv \hat{l}$). Because the RBS estimate for total length is unbiased, it follows that the PRS estimate will also be unbiased providing that PRS is applied correctly (Gove et al., 1999).

This same methodology holds when estimates for more complicated quantities are desired. For example, Smalian's formula (Avery and Burkhart, 1994, p. 55) can be used with the measurements in Table 1 to calculate segment volumes for the subject branch. However, in such cases, the question arises as to whether subsampling using RBS with selection probabilities proportional to cumulative length, will yield estimates for path volume as good as some other variable more highly correlated with volume. To this end, three surrogate quantities were used to determine selection probabilities, all of which can be easily calculated from the information in Tables 1 and 2.

1. An ocular estimate of the cumulative length above each fork at a node as described earlier.
2. The basal, or large-end diameter-square which is proportional to cross-sectional area and thus correlated with volume above the point of measurement (Valentine et al., 1984).
3. The product of basal diameter-squared and ocularly estimated cumulative length above the fork; i.e. the product of the first two surrogate variables.

All three of these surrogates were computed on the subject branch in Fig. 3 and were used to estimate total branch lengths and volumes for all six possible paths. This was done to determine which surrogate might be most useful for the estimation of each individual quantity of interest. An ancillary objective, however, is to determine if a common surrogate could be used for both quantities, perhaps with the trade-off of accepting slightly less precision in the RBS path estimates while having only to perform RBS once on a subject branch. Some recommendations on the choice of surrogates appear in the literature, but these are generally with regard to estimation of only one quantity of interest.

The results of the surrogate comparisons are presented in Table 4. When length and volume are considered separately, it appears that each variable has its own optimal surrogate for the subject branch considered here. The best surrogate for total length is the ocular estimate of length, while the best for volume is the combined surrogate. This is easily seen by comparing the relative and standard errors of the mean estimate over all paths for each surrogate. Surprisingly, all three surrogates performed well for volume estimation, while the combined surrogate was poor when used for the estimation of total length. It appears that the simple ocular estimate of cumulative length could be used with little loss of precision on this subject branch for the estimation of both quantities.

Incidently, given the exposition of all possible RBS paths in Table 3, it is straightforward to show that the technique is design unbiased. Because RBS is a probability sampling technique, the probability-weighted average of all possible estimates is required to show this; viz

$$L = \sum_{p=1}^{P} Q_{pR}\hat{l}_p$$

Table 4
Comparison of surrogates for estimation of total length and volume using RBS on the CWD subject branch in Fig. 3

Surrogate variable	Path number	Length			Volume		
		m	m ha^{-1} [a]	Error (%)	m^3 × 10^{-2}	m^3 ha^{-1}	Error (%)
Measured total[b]	–	10.33	760.0	–	1.91	1.41	–
Cumulative length	1	8.85	651.4	−14.3	1.58	1.16	−17.4
	2	10.09	742.3	−2.3	1.88	1.38	−2.0
	3	9.61	706.9	−7.0	1.68	1.23	−12.4
	4	10.12	744.4	−2.0	2.33	1.72	21.9
	5	11.24	827.1	8.8	1.70	1.25	−11.1
	6	10.31	758.9	−0.2	1.89	1.39	−1.2
	Mean	10.04	738.5	−2.8	1.84	1.36	−3.7
	S.E.	0.32	23.7	–	0.11	0.08	–
Diameter-square	1	5.57	409.8	−46.1	1.34	0.98	−30.0
	2	7.46	549.0	−27.8	1.58	1.16	−17.5
	3	13.74	1010.8	33.0	1.74	1.28	−9.2
	4	8.23	605.2	−20.4	2.03	1.49	6.1
	5	20.25	1489.9	96.0	2.18	1.60	13.9
	6	10.98	807.5	6.3	2.04	1.50	6.4
	Mean	11.04	812.0	6.8	1.82	1.34	−5.0
	S.E.	2.18	160.5	–	0.13	0.10	–
Diameter-square × cumulative length	1	17.45	1283.9	68.9	1.87	1.38	−2.3
	2	10.75	790.7	4.0	1.93	1.42	0.7
	3	18.48	1359.4	78.9	1.84	1.35	−3.9
	4	6.91	508.1	−33.1	1.83	1.35	−4.4
	5	24.55	1806.2	137.7	2.32	1.70	20.9
	6	9.89	727.4	−4.3	1.92	1.42	0.5
	Mean	14.67	1079.3	42.0	1.95	1.44	1.9
	S.E.	2.70	198.5	–	0.07	0.05	–

[a] The quantities expressed on a per-hectare basis have been expanded by assuming that the subject branch was sampled using PRS with a relascope angle of $\nu = 28.07°$, yielding a squared length factor of $\mathscr{L} = 1444.17$ (Gove et al., 1999).

[b] The actual measured totals for each variable on the subject branch.

where L is the true total length of the subject branch. For the data in Table 3, it is easy to calculate from the above that $L = 10.33$, which is indeed the true total length of the subject branch.

3. Discussion

It can not be stressed enough that the results on surrogate variables for RBS in the previous section are based on only one subject branch; thus, these results should be taken only as general guidelines which *may* hold for other pieces of downed CWD. The myriad possibilities of sizes, branching habits, breakage, species-specific characteristics, etc. all combined make anything other than simple recommendations unrealistic without resorting to detailed simulation studies. In addition, the subject branch chosen to use as an illustration was a relatively small piece as witnessed by its dimensions in Table 1. This choice was made to expedite discussion of the methods—a larger subject branch quickly becomes unwieldy for total enumeration. Indeed, measuring the entire subject branch would probably be more efficient than RBS in this case because of the small size involved. Relatedly, the volumes on such a small piece are also small and thus the surrogate relationships found here must be carefully considered when applied to larger pieces.

The above qualification notwithstanding, the combined methods of PRS and RBS described herein may be generally applied. However, it is important not to confuse the roles of total and projected length. If one of the objectives of the inventory is to estimated total length per hectare, then there is no substitute for either measuring the length of all branches or employing an unbiased subsampling scheme like RBS. That is, projected length should never be used as a substitute for total length, because what one is really estimating then is projected length per hectare, not total length. If this substitution is made, significant biases will result both in the estimate of total length per hectare and of any quantities that are based on it. For example, assume that total length per hectare is desired from the survey. Then, if path #6 were chosen and a relascope angle of $v = 28.07^{\circ}$ was used to select the subject branch in Table 1, it is easy to calculate that the contribution of this branch to total length per hectare is 759 m ha^{-1} (Table 4). If the RBS subsampling (or direct measurement) for length of the subject branch is ignored, then the result is $1444.17 \times 4.43^{-1} = 326$ m ha^{-1}. The error associated with this difference is significant and inflating to the tract total exacerbates this even more. Furthermore, ad hoc estimates of other quantities like volume when based on projected length further compound the error. For example, one might argue that basal cross-sectional area multiplied by projected length could provide an ad hoc estimate of overall volume for the subject branch. In this case, it is straightforward to calculate that such an ad hoc estimate will result in a significant overestimate of the contribution to volume per hectare by subject branch.

Having pointed out some possible errors inherent in using incorrect quantities for multiple-stemmed pieces of downed CWD, one other consideration is the factor of inventory cost. Downed CWD may have little economic value and, indeed, be only a minor component to a larger overall inventory effort. If this is the case, it might be difficult to justify spending too much time sampling any one piece with very detailed measurements. In fact, the procedures advocated here possibly apply best to research studies where detailed measurements may be a necessity. If costs are an overriding factor, one might opt for taking only one RBS path per piece rather than multiple paths, with the full realization of the resulting effect on precision of the final estimates (e.g. see Table 4). Additionally, if estimates of multiple quantities (e.g. length, volume, biomass) are desired under cost constraints, it makes the question of surrogate variable selection even more important as one would like to sample a given path only once for all variables concerned. On the other hand, because subject trees or branches are already on the ground, much of the work involved in the traditional application of RBS subsampling to the standing crop is eliminated.

Neglecting to implement procedures such as those described here that correctly account for branched downed CWD not only affects simple estimates of length, volume or biomass, but it will also affect the distributions of these variables as well. Since PRS is a PPS design, its associated sampling distribution is size-biased. Thus, errors in field procedure carry over into the estimation of the inflated stand distribution and will also tend to perturb the theoretical relationship that exists between the size-biased sampling distribution and the associated stand distribution.

Finally, in lieu of using a formula like Smalian's to arrive at branch segment volumes, importance sampling can be used to some advantage. Importance sampling adds another stage to the sampling procedure but is unbiased—an attribute lacking in typical conic formulas. In addition, importance sampling can be used for other quantities like biomass where usually no one formula applies for estimation purposes (Valentine et al., 1984).

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