Diameter Growth Models for Inventory Applications

Ronald E. McRoberts
Christopher W. Woodall
Veronica C. Lessard
Margaret R. Holdaway

Abstract—Distant-independent, individual-tree, diameter growth models were constructed to update information for forest inventory plots measured in previous years. The models are nonlinear in the parameters and were calibrated using weighted nonlinear least squares techniques and forest inventory plot data. Analyses of residuals indicated that model predictions compare favorably to another regional diameter growth model and may be considered unbiased for their intended application. In addition, the models provide for estimation of model prediction uncertainties and may be easily recalibrated. The final model calibrations were by species within each of three ecological provinces in the North Central United States.

The U.S. Renewable Forest and Rangeland Resources Planning Act of 1978 requires that the Forest Service, an agency of the U.S. Department of Agriculture (USDA), conduct inventories of the nation's forest land to determine its extent and condition and the volume of standing timber, timber growth, and timber removals. The U.S. Agricultural Research Extension and Education Reform Act of 1998 further requires that in conducting these inventories, the Forest Service measure a proportion of the plots in each State annually. In response, the Forest Inventory and Analysis (FIA) program of the Forest Service has initiated the Enhanced FIA program that features a systematic sampling design with plots assigned to panels and individual panels selected for measurement on a rotating basis.

One objective of the FIA program at the North Central Research Station (NCRS), USDA Forest Service, has been to establish the capability of annually producing standard inventory estimates. Three approaches to calculating annual estimates using inventory data collected under the Enhanced FIA program have been proposed (McRoberts 1999). The simplest approach is to use the data from the most recently measured panel of plots. Although the resulting estimates would reflect current conditions, their low precision may be unacceptable for some variables because of the small sample size. A second approach is to use the most recent plot measurements for each panel, regardless of the date, and then employ a moving average estimator. The advantage of this approach is that precision is increased because data for all plots are used for estimation; the disadvantage is that the estimates reflect a moving average of conditions over past years and may lag current conditions in the presence of temporal trends. A third approach is to update to the current year data for plots measured in previous years and then base estimates on the data for all plots. If the updating procedure is sufficiently unbiased and precise, this approach provides nearly the same precision as using all plots but without the adverse effects of using out-of-date information.

Two models are currently available for predicting individual tree survival and growth in the North Central region of the United States: STEMS (Belcher and others 1982), and two variants of the Forest Vegetation Simulation (FVS). The FVS variant for the Lakes States (Bush and Brand 1995) and the variant for the Central States (Bush 1995) both use the TWIGS models (Miner and other 1998), which, in turn are based on the STEMS models. For over a decade, the FIA program at NCRS has used the STEMS (Belcher and others 1982) survival and diameter growth models for updating inventory plot information. A decision to construct new models was motivated by several limitations in the STEMS models:

1. Calibration data for the STEMS models were collected primarily from long-term research plots in selected locations and may not represent forest conditions across the North Central region as do FIA plots.
2. The mathematical form of the STEMS models includes a potential growth component that is difficult and cumbersome to calibrate, primarily because potential growth is unobservable.
3. The STEMS diameter growth models were calibrated using a two-step process in which a subset of the model parameters were estimated first and then held constant while the other parameters were estimated, a procedure that would be difficult to defend statistically today.
4. The STEMS calibrations did not use weighted regression, did not estimate model parameter covariances, and did not retain estimates of residual variances, thus precluding estimation of the precision of model predictions.

The discussion that follows outlines and reviews progress to date by FIA scientists at NCRS in constructing new individual tree, distant independent, diameter growth models for updating inventory plot information.

In: Crookston, Nicholas L.; Havis, Robert N., comps. 2002. Second Forest Vegetation Simulator Conference; 2002 February 12–14; Fort Collins, CO. Proc. RMRS-P-25. Ogden, UT: U.S. Department of Agriculture, Forest Service, Rocky Mountain Research Station.

Ronald E. McRoberts is a Mathematical Statistician, Christopher W. Woodall is a Research Forester, and Margaret R. Holdaway is a Mathematical Statistician (retired), North Central Research Station, USDA Forest Service, 1992 Folwell Ave., St. Paul, MN 55108. Veronica C. Lessard is a Statistician, Natural Resource Conservation Service, 1992 Folwell Ave., St. Paul, MN 55108.

Methods

Data

The models were calibrated using inventory data for trees with diameter-at-breast height (d.b.h.) (4.5 ft. above ground) greater than 5 inches for all ownership categories on lands capable of producing 20 ft^3/ac/yr of industrial wood (FIA timberland). For States in the North Central inventory region and some adjacent States, plot data were obtained for two consecutive periodic inventories: Illinois - 1985, 1998; Indiana - 1986, 1998; Iowa - 1974, 1990; Kansas - 1981, 1994; Kentucky -1974, 1987; Michigan - 1980, 1993; Minnesota - 1977, 1990; Missouri - 1972, 1989; Nebraska - 1983, 1994; North Dakota - 1980, 1994; Ohio - 1978, 1990; South Dakota - 1980, 1995; Tennessee - 1989, 1996; and Wisconsin - 1983, 1996. Data for a random selection of 75 percent of plots were assigned to a model calibration data set; the remaining data were assigned to a model validation data set and were excluded from use for initial model calibration.

Calibration analyses focused on the suite of plot- and tree-level variables identified in preliminary analyses reported by Holdaway (2000) and Lessard (2000a,b). Plot-level variables included latitude (LAT) and longitude (LON) of the plot center, plot basal area (BA), and physiographic class (PC). PC is a measure of site soil and water conditions that affect tree growth and is coded as follows: 3-xeric; 4-xeromesic; 5-mesic; 6-hydromesic; 7-hydric; and 8-bottomland (NCRS 1998). Tree-level variables included observed average annual d.b.h. growth, which was calculated as the ratio of the difference in d.b.h. measurements for the two inventories and the number of growing seasons between measurements and was used as a surrogate for observed annual d.b.h. growth. For the purpose of consistency in notation and model expression, DBH and ΔDBH are used to denote variables quantifying d.b.h and annual change or growth in d.b.h., respectively.

Other tree-level variables included crown ratio (CR), plot basal area in trees larger than the subject tree (BAL), and crown class (CC) at the time of the initial inventory. CC is a measure of a tree's dominance in relation to adjacent trees in the same stand and is coded as follows: 1-open grown; 2-dominant; 3-codominant; 4-intermediate; and 5-overtopped (NCRS 1998). BA and BAL are calculated as the sum of cross-sectional areas of live tree boles at breast height, are scaled to a per unit area basis, and when used to calibrate the models were calculated as the average value for the two inventories. Although current BA and BAL would be used as the value of these predictor variables when applying the models, average BA and BAL over the rather long (12 to 17 years) inventory remeasurement interval for the calibration data better reflect the growing conditions over the entire interval than do BA and BAL at the initial inventory. The values of CC and PC are categorical, but their coding is ordered with respect to their expected effects on growth, and they were treated as continuous predictor variables in the model formulation.

Although calibration data were selected from plots with no disturbance or minimal disturbance over the measurement interval, some plots had substantial mortality or harvesting before the measurement interval. Nevertheless, this disturbance is expected to have little impact on model prediction bias because predictor variables such as plot basal area partially account for disturbance and because the proportion of extensively disturbed plots was small.

Annual Forest Inventory System Models

The diameter growth models constructed for updating FIA plot information are designated the Annual Forest Inventory System (AFIS) models. Their mathematical form consists of the product of two components, an average component that predicts average ΔDBH with respect to DBH for the calibration area, and a modifier component that adjusts ΔDBH predictions in accordance with local plot and tree conditions. The average component is based on a two-parameter gamma function with a constant multiplier and uses DBH as the predictor variable. The modifier component consists of the product of exponential factors of which each expresses a multiplicative effect on growth predictions in terms of departures from the mean over the calibration area for a single predictor variable. The mathematical form of the AFIS growth models is

$$E(\Delta DBH) = Ave(DBH)Mod(X) \quad [1]$$

where E(.) denotes statistical expectation, ΔDBH is annual d.b.h. growth, and X represents the set of selected predictor variables. Further,

$$Ave(DBH) = \beta_1 \exp(\beta_2 DBH) DBH^{\beta_3} \quad [2]$$

and

$$Mod(X) = \exp[\beta_4(X_4-\gamma_4)]\exp[\beta_5(X_5-\gamma_5)]\ldots\exp[\beta_p(X_p-\gamma_p)]$$

$$= \exp\left[\sum_{j=4}^{p}\beta_j(x_j-\gamma_j)\right] \quad [3]$$

where the Xs are predictor variables, the γs are constants representing mean values for the associated predictor variables over the calibration area, the βs are parameters to be estimated, and p is the number of model parameters. After substituting [2] and [3] into [1], mathematically expanding the resulting expression, and combining constants where possible, the AFIS growth model form may be expressed as,

$$E(\Delta DBH) = \beta_1 \exp(\beta_2 DBH) DBH^{\beta_3} \exp\left(\sum_{j=4}^{p}\beta_j X_j\right) \quad [4]$$

where the βs are still parameters to be estimated, although not the same parameters as for [3]. The mathematical form of the d.b.h. growth models is generally similar to the forms used by STEMS and by Wykoff (1990) for modeling basal area increment in northern Idaho.

Analyses

Three sets of analyses were performed to evaluate the AFIS models: (1) the AFIS and STEMS model predictions were compared; (2) the AFIS models were calibrated by species for three ecological provinces in the North Central United States, and an analysis of the residuals was conducted; and (3) the utility of the AFIS models for updating FIA plot information was evaluated.

AFIS/STEMS Comparisons—The objectives of the first set of analyses were to compare both the forms and calibrations of the STEMS and AFIS models using their predictions for trees on a subset of a plots in northeastern Minnesota (Holdaway 2000, Lessard 2000a,b, McRoberts 2000).

These analyses were conducted early in the model construction process and used the mathematical form of the models as expressed in [1], [2], and [3]. Variables were selected for inclusion in the model using the criterion, C, proposed by Linhart and Zucchini (1986),

$$C = SS_e + 2p^*MS_e, \qquad [5]$$

where p is the number of model parameters, and SS_e and MS_e are the sum of squared residuals and mean square residual, respectively. This criterion simultaneously accounts for unexplained residual variability and the number of model parameters. Models with lower values of C are judged to provide better fits to data. The final set of predictor variables considered for inclusion in the models was restricted to DBH, CR, BAL, BA, CC, PC, LAT, and LON. If the estimate of a parameter corresponding to a variable included zero in its asymptotic 95 percent confidence interval, the parameter estimate was set to zero in the model, effectively excluding any effect of the predictor variable.

Heterogeneity of residual variance was accommodated by weighting each observation by the inverse of the estimated residual standard deviation, $\hat{\sigma}_{res}$, which was adequately described by,

$$E\left[\ln\left(\hat{\sigma}_{res}\right)\right] = \alpha_1 + \alpha_2 \ln\left(\Delta D\hat{B}H\right) \qquad [6]$$

where E(.) denotes statistical expectation, $\Delta D\hat{B}H$ is predicted diameter growth from the models, and the αs are parameters to be estimated (McRoberts and others 2000).

To compare the form of the STEMS models to the form of the AFIS models, the STEMS models were first recalibrated using FIA data. Because of the complexity and time-consuming nature of this task, the recalibration was restricted to four commonly occurring species in northeastern Minnesota: two conifers, red pine and balsam fir, and two hardwoods, quaking aspen and paper birch. The recalibrated STEMS models were designated the STEMS-FIA models and were calibrated with data for 37,550 trees on 2,434 plots. The analyses consisted of three-way comparisons of the ΔDBH predictions on the selected plots using the existing STEMS models, the STEMS-FIA models, and the AFIS models. The comparisons were based on median residuals and correlations, r^2, between observed and predicted ΔDBH.

AFIS Model Calibrations for Ecosystem Provinces—The AFIS models were calibrated for each of the three largest ecosystem provinces defined by Bailey that occur in the North Central region: (1) Province 212, Laurentian mixed forest, characterized as a transitional zone between the boreal and broadleaf deciduous forests; (2) Province 222, Eastern broadleaf forest (continental), characterized by dominance of broadleaf deciduous species favoring drought-resistant oak-hickory associations; and (3) Province 251, Prairie parkland (temperate), characterized by intermingled prairie, groves, and strips of deciduous species (Bailey 1995) (fig. 1).

The model fitting, verification, and validation procedures were similar to those for the first analyses and as reported by Lessard and others (2001) with two exceptions: first, [4] was used as the mathematical form of the model rather than [1], [2], and [3], and second, the parameters were tested for statistical significance using an F test based on the extra sum of squares principle (Ratkowsky 1983). Validation analyses were conducted using the independent validation data sets and consisted of analyses of standardized residuals, ε, calculated as,

$$\varepsilon = \frac{\Delta DBH - \Delta D\hat{B}H}{\hat{\sigma}_{res}} \qquad [7]$$

where ΔDBH is observed average annual d.b.h. growth, $\Delta D\hat{B}H$ is the corresponding model prediction, and $\hat{\sigma}_{res}$ is the estimate of residual standard deviation obtained from [6]. The standardized residuals were analyzed by evaluating their distributions by species within ecosystem provinces for the validation data sets. Following the validation analyses, the calibration and validation data were pooled to form final calibration data sets, the models were calibrated again, and the residuals were evaluated using the same procedures.

Updating—The AFIS models were evaluated with respect to bias and precision for updating data for FIA plots measured in previous years. Because this task required additional analyses to estimate the uncertainty in model parameters for each species, the evaluations were restricted to data for Province 212 plots that included only trees of the four species, red pine, jack pine, quaking aspen, and balsam fir.

An annualized 11-year database of plot and tree variables was constructed using data for the selected inventory plots. The year of first inventory measurement for each plot was designated year 0, growth for individual trees was distributed equally over the observed remeasurement intervals,

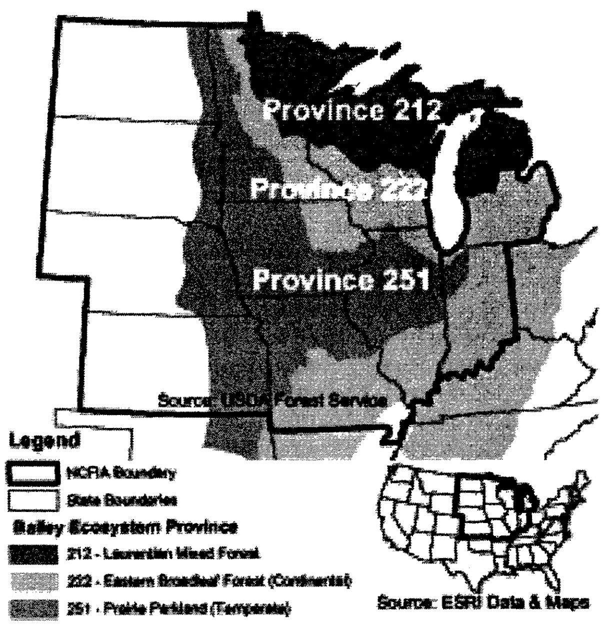

Figure 1—Ecosystem provinces for the North Central region of the United States.

and years of mortality, harvest, and in-growth for individual trees were randomly selected using a uniform distribution over the remeasurement intervals. The resulting annual database included data for 2,900 trees on 185 plots for years 0 to 10. Details regarding construction of the annualized database are documented in McRoberts (2001).

Monte Carlo simulations were used to obtain estimates of uncertainty for model parameter estimates, annual mean plot BA estimates, and annual inventory estimates of mean plot BA and the standard error of the mean. Before the simulations could be implemented, uncertainty had to be quantified for all the relevant components. Measurement error and sampling variability in predictor variables were ignored and assumed to be negligible. Estimates of residual variability were obtained from [6] as by-products of calibrating the models.

Model parameter covariances reflect uncertainty in the parameter estimates and must be included as a component of total uncertainty whenever the precision of model predictions is to be estimated. When the models are relatively simple (for example, linear), parameter covariance estimates may be easily calculated using analytical methods. However, when the models are complex and nonlinear, then Monte Carlo simulations are appropriate for reliably estimating these covariances. Distributions of model parameter estimates from which covariances may be derived were obtained using a three-step Monte Carlo procedure:

1. Simulated d.b.h. growth observations were calculated as the sums of two components: first, ΔDBH predictions obtained using the models with the original parameter estimates and observations of the predictor variables, and second, simulated residuals randomly selected from a Gaussian distribution with zero mean and standard deviations obtained from [6].
2. Model parameter estimates were obtained by fitting the models to data sets consisting of the simulated ΔDBH observations obtained from step 1 and the observed values of the predictor variables.
3. Simulated distributions of model parameter estimates were obtained via 250 repetitions of steps 1 and 2.

Estimates of the uncertainty in ΔDBH predictions and in estimates of derived BA variables were obtained using Monte Carlo simulations. The simulation approach was designed to mimic the sampling procedures of the Enhanced FIA program. The selected plots were ordered with respect to their plot numbers for each State and distributed among five equal-sized panels by systematically assigning every fifth plot to the same panel. Because FIA plot numbers had been assigned sequentially on the basis of their geographic locations, the panel assignments approximated the systematic, interpenetrating feature of the sampling design. Annual inventory estimates of mean plot BA and the standard error of the mean were calculated with three methods: (1) the SAMPLE20 estimates were based on measurements for plots in the current year's 20 percent panel; (2) the MOVING estimates were based on the most recent measurements for all plots; and (3) the UPDATE estimates were based on measurements for plots in the current year's 20 percent panel of plots and updated data obtained using the AFIS growth models for plots in the four panels measured in previous years.

Annual estimates of mean plot BA and the standard error of the mean were obtained using a four-step Monte Carlo procedure:

1. Year 0:
   a. Measurement of all plots was simulated by selecting the year 0 values of DBH, CR, CC, PC, LAT, and LON from the annualized database.
   b. Simulated values of BA and BAL were obtained from the simulated d.b.h. observations by calculating BA for each plot and BAL for each tree on each plot; mean plot BA and the standard error of the mean were calculated and recorded.
   c. A set of model parameter estimates for each species was randomly selected without replacement from the simulated distributions previously constructed.
2. Subsequent years:
   a. For panels selected for measurement, plot field measurement was simulated by replacing values for each tree with values from the annualized database for the appropriate year.
   b. For panels not selected for measurement, an updated value of DBH for each tree was calculated as the sum of previous year's DBH, predicted ΔDBH, and a simulated residual randomly selected from a Gaussian distribution with zero mean and standard deviation obtained from [6].
   c. BA was calculated for each plot, and BAL was calculated for each tree on each plot; mean plot BA and the standard error of the mean were calculated and recorded for all three methods.
3. Step 2 was repeated 10 times to obtain estimates for years 1 through 10.
4. Steps 1 through 3 were repeated 250 times to obtain distributions of mean plot BA and the standard error of the mean for each method for each year.

The standards of comparison for evaluating bias and the contribution of uncertainty in model predictions to the uncertainty in estimates of mean plot BA were the annual estimates of mean plot BA and the standard errors of the means calculated from the annualized database values. For comparison purposes, the latter estimates represent a current year sample of the entire area of interest and are regarded as being without measurement error. These estimates use 100 percent of the sample plots and are designated the SAMPLE100 estimates.

Bias and uncertainty in the annual inventory estimates of mean plot BA and standard errors of the means were evaluated using the medians of the distributions of simulated estimates. Comparisons of median estimates of mean plot BA for the SAMPLE20, MOVING, and UPDATE methods to the annual SAMPLE100 estimates of mean plot BA provide the bias check. Comparisons of the medians of distributions of estimates of the standard error of mean plot BA for the UPDATE method with the SAMPLE100 estimates reveals the effects of uncertainty in model predictions on annual inventory estimates of mean plot BA.

Results

AFIS/STEMS Comparisons

The distributions of residuals for the STEMS, STEMS-FIA, and AFIS predictions were not substantially different (table 1). Median residuals for all species were less in absolute value than 0.015 inch. Values of r^2 were also comparable for the three models, although values for the AFIS models were largest for three of four species, while values for the STEMS models were smallest for three of four species.

Table 1—STEMS versus AFIS comparisons for validation data sets.

Species	Model	No. trees*	Median residual	r^2
Red pine	STEMS	501	0.004	0.480
	STEMS-FIA	496	–0.002	0.462
	AFIS	496	0.005	0.469
Balsam fir	STEMS	1,924	–0.012	0.201
	STEMS-FIA	1,922	–0.012	0.324
	AFIS	1,922	–0.010	0.363
Quaking aspen	STEMS	5,348	–0.005	0.133
	STEMS-FIA	5,345	–0.001	0.197
	AFIS	5,345	–0.009	0.228
Paper birch	STEMS	1,708	–0.005	0.066
	STEMS-FIA	1,704	–0.006	0.123
	AFIS	1,704	–0.005	0.144

*The number of trees represents only the 25 percent of the data randomly selected for the validation data sets; slight differences in number of trees by model for a particular species reflects analyses of residuals conducted at different times.

Ecosystem Province Models

The distributions of standardized residuals were consistently slightly skewed with longer tails in the positive direction and with slightly negative medians. These results are attributed to the combination of a lower bound, ΔDBH=0, for observed average annual d.b.h. growth and the lack of an analytical (not a physiological) upper bound for average annual d.b.h. growth. In no case, however, was this skewness considered severe enough to affect the robustness of least squares techniques to slight violations of the normality assumption.

The analyses of residuals focused on median and mean residuals for the validation and final calibration data sets. Standards of 0.20 inch for absolute values of medians and 0.05 inch for absolute values of means were arbitrarily selected. The analyses sought to identify any species group with residual statistics that consistently failed to satisfy these standards for reasons that could not be attributed to small sample sizes or skewness in the distributions as previously discussed. For Province 212, absolute values of mean residuals were less than 0.05 inch for 17 of 25 species groups for the validation analyses and for all 25 species groups for the final calibration analyses. Only yellow birch exhibited any consistent failures, although the mean residual for the final calibration data set was less than 0.01 inch For Province 222, absolute values of mean residuals were less than 0.05 inch for the validation data sets for only 10 of 26 species groups, a result attributed primarily to small validation sample sizes. For the final calibration data sets, absolute values of mean residuals were less than 0.01 inch for all species groups. For Province 251, absolute values of mean residuals were less than 0.05 inch for five of nine species groups for the validation data sets. These results are partially attributed to overall small sample sizes and the necessity of pooling data over larger geographic areas. Absolute values of mean residuals were less than 0.01 inch for all species groups for the final calibration data sets. Parameter estimates by species within ecosystem provinces are reported by McRoberts and others (submitted).

Updating

The 250 simulations were deemed adequate based on the observation that coefficients of variation for estimates had stabilized by 100 to 150 simulations and were virtually unchanged for the final 50 simulations. Bias in the annual inventory estimates of mean plot BA was evaluated by comparing the medians of the distributions of the SAMPLE20, MOVING, and UPDATE estimates to the SAMPLE100 estimates (fig. 2). The medians of the SAMPLE20 estimates deviated considerably from the SAMPLE100 estimates due to the SAMPLE20 small sample size, while the medians of the MOVING estimates exhibited consistent bias due to the trend in the SAMPLE100 estimates. The medians of the distributions of the UPDATE estimates tracked the SAMPLE100 estimates quite closely, a result confirmed by the failure of the Wilcoxon Signed Ranks test to detect

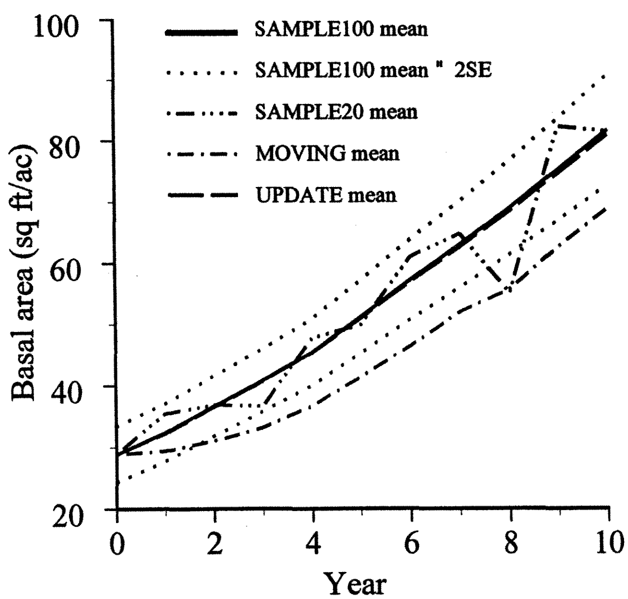

Figure 2—Annual inventory estimates of mean plot basal area.

statistically significant differences (α=0.05). The medians of the distributions of the UPDATE estimates of the standard errors of the means were only slightly larger than the SAMPLE100 estimates, indicating that uncertainty in model predictions of d.b.h. growth has only a slight negative impact on the uncertainty of annual inventory estimates of mean plot BA. This result is attributed primarily to the observation that variability among plots is much greater than model prediction uncertainty.

Summary

The AFIS models were constructed to update information for FIA plots measured in previous years. Predictions using the AFIS and STEMS models were comparable, but the AFIS models remedy the noted limitations in the STEMS models. Based on the general observation that average annual d.b.h. growth for the North Central region is approximately 0.1 inch and that time intervals between inventories for the calibration and validation data ranged between 12 and 17 years, the small mean and median residuals obtained with the AFIS models indicate that they may be regarded as unbiased for updating d.b.h. for trees measured in previous years. In addition, annual inventory estimates of mean plot BA obtained using the AFIS models to update information for plots measured in previous years were superior to estimates using only the current year's panel of data and estimates using a five-panel moving average. Finally, the AFIS models may be easily recalibrated as new data becomes available or as regional growth conditions change due to climatic changes, management practices, or forest succession.

References

Bailey, R.G. 1995. Description of the ecoregions of the United States. Ed. 2. Revised and expanded (1st ed. 1980). USDA For. Serv. Misc. Publ. No. 1391 (rev.), Washington, DC. 108 p. with separate map.

Belcher, David W., Margaret R. Holdaway, and Gary J. Brand. 1982. A description of STEMS-the stand and tree evaluation and modeling system. General Technical Report NC-79. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Research Station. 18 p.

Bush, Renate. 1995. The Central States TWIGS variant of the Forest Vegetation Simulator. Internal Report. Fort Collins, CO: U.S. Department of Agriculture, Forest Service, Forest Management Service Center. 28 p.

Bush, Renate, and Gary J. Brand. 1995. The Lakes States TWIGS variant of the Forest Vegetation simulator. Internal Report. Fort Collins, CO: U.S. Department of Agriculture, Forest Service, Forest Management Service Center. 30 p.

Holdaway, Margaret R. 2000. The AFIS tree growth model for updating annual forest inventories in Minnesota. In: Proceedings of the IUFRO conference, 1998 August 16-20, Boise, ID. Hansen, M.H. and T.E. Burk (eds.). General Technical Report NC-212. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Research Station. pp. 507-514.

Lessard, Veronica C. 2000a. Calibration of the STEMS diameter growth models using FIA data. In: Proceedings of the IUFRO conference, 1998 August 16-20, Boise, ID. Hansen, M.H. and T.E. Burk (eds.). General Technical Report NC-212. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Research Station. pp. 525-532.

Lessard, Veronica C., 2000b. Diameter growth models using FIA data from the Northeastern, Southern, and North Central Research Stations. In: Proceedings of the First Annual Forest Inventory and Analysis Symposium, 1999 November 2-3, San Antonio, TX. McRoberts, R.E., G.A. Reams, and P.C. Van Deusen (eds.). General Technical Report NC-213. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Research Station. pp. 37-42.

Lessard, Veronica C., Ronald E. McRoberts, and Margaret R. Holdaway. 2001. Diameter growth models using Minnesota forest inventory data. Forest Science 47(3):301-310.

Linhart, Heinz, and Walter Zucchini. 1986. Model selection. John Wiley and Sons, Inc. New York. 301 p.

McRoberts, Ronald E. 2001. Imputation and model-based updating techniques for annual forest inventories. Forest Science 47(3):322-330.

McRoberts, Ronald E. 1999. Joint annual forest inventory and monitoring system: the North Central perspective. Journal of Forestry 97(12):27-31.

McRoberts, Ronald E., Margaret R. Holdaway, and Veronica C. Lessard. 2000. Comparing the STEMS and AFIS growth models with respect to the uncertainty of predictions. In: Proceedings of the IUFRO conference, 1998 August 16-20, Boise, ID. Hansen, M.H. and T.E. Burk (eds.). General Technical Report NC-212. General Technical Report NC-212. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Research Station. pp. 539-548.

McRoberts, Ronald E., Christopher W. Woodall, and Veronica C. Lessard. (submitted). Individual-tree diameter growth models for three ecological provinces in the North Central region of the United States. Forest Science.

Miner, Cynthia L., Nancy R. Walters, and Monique L. Belli. 1988. A guide to the TWIGS program for the North Central United States. General Technical Report NC-125. St. Paul, MN: U.S. Department of Agriculture, Forest Service, North Central Research Station. 105 p.

North Central Research Station (NCRS). 1998. North Central Region Forest Inventory and Analysis Field Instructions, First Edition. North Central Research Station, USDA Forest Service, St. Paul, MN.

Ratkowsky, David A. 1983. Nonlinear regression modeling. Marcel Dekker, New York. 276 p.

Wykoff, William R. 1990. A basal area increment model for individual conifers in the northern Rocky Mountains. Forest Science 36(4): 1077-1103.