

Identifying Influences on Model Uncertainty: An Application Using a Forest Carbon Budget Model

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ABSTRACT / Uncertainty is an important consideration for both developers and users of environmental simulation mod-

els. Establishing quantitative estimates of uncertainty for deterministic models can be difficult when the underlying bases for such information are scarce. We demonstrate an application of probabilistic uncertainty analysis that provides for refinements in quantifying input uncertainty even with little information. Uncertainties in forest carbon budget projections were examined with Monte Carlo analyses of the model FORCARB. We identified model sensitivity to range, shape, and covariability among model probability density functions, even under conditions of limited initial information. Distributional forms of probabilities were not as important as covariability or ranges of values. Covariability among FORCARB model parameters emerged as a very influential component of uncertainty, especially for estimates of average annual carbon flux.

Environmental decision-making commonly relies on simulation modeling, yet deterministic models, which are often first-pass attempts at assessment modeling, usually lack the quantitative descriptions of uncertainty necessary for decision-making. This is the case with assessments of carbon storage in forests, or forest carbon budgets, which contribute information relevant to managing national greenhouse gas inventories (Birdsey and Heath 1995, IPCC/OECD/IEA 1997). Expected effects of anthropogenic and climate influences are often based on simulation modeling of alternate scenarios. Data limitations for modeling are reflected in the scarcity of quantitative descriptions of uncertainty in large-scale forest carbon budgets (Birdsey and Heath 1995, Turner and others 1995, Heath and others 1996, Kurz and Apps 1996, Hamburg and others 1997). Unfortunately, simulation models lacking adequate representation of uncertainty have limited value for environmental assessments (Rowe 1994, Morgan and Dowlatabadi 1996). The number of model variables affected by uncertainty about their value can become large; complete characterization of uncertainties can be data-intensive and costly. Our objective is to demonstrate how a relatively simple and flexible approach to

quantitative uncertainty analysis can reduce the need for all model variables to include accurate definitions of uncertainty. This is demonstrated with a modified deterministic forest carbon budget model and data for Northeastern maple-beech-birch forests.

The increasing prominence of forest carbon budget models in management or policy decision-making reflects the importance of forests as the largest terrestrial biotic sink of an important greenhouse gas: carbon dioxide (IPCC/OECD/IEA 1997). Estimates of carbon stored in forests depend on the complex system of biophysical and socioeconomic influences that ultimately determines forest productivity. This complexity contributes to uncertainty in such estimates. However, forest carbon budget models ranging from landscape to national scales generally do not present estimates of uncertainty (Birdsey and Heath 1995, Turner and others 1995, Heath and others 1996, Kurz and Apps 1996, Hamburg and others 1997). The models are pragmatic approaches to applying sparse data for understanding processes and providing national assessments of greenhouse gas inventories. Expressions of uncertainty play important roles in development and analysis of model behavior. Indeed, most of the forest carbon budget publications cited above discuss aspects of uncertainty within the systems modeled.

Deterministic simulation models contain uncertainty, even if not explicitly represented. Model results expressed as single numerical values fail to reflect useful information about uncertainty. It is possible, however, to reformat a deterministic simulation to include

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uncertainty and produce model results that reflect uncertainties in the model. A numerical approach to this process involves probabilistic definitions of uncertainty about specific model elements and Monte Carlo simulation (Morgan and Henrion 1990, Vose 1996, Cullen and Frey 1999). This method is most commonly applied either to (1) propagate uncertainty to a total for results or (2) identify important influences on model uncertainty. Two caveats are immediately clear. First, quality of results is a function of quality of inputs. Second, establishing quantitative probabilistic definitions for initial conditions or model inputs can be difficult when information is scarce. Without adequate objective information, initial analyses thus must rely heavily on preliminary valuation of input uncertainty, which can, in turn, reduce confidence in model results.

We examine steps to refine initial values of input uncertainty for a model originally developed in a deterministic form. The forest carbon budget model FORCARB (Birdsey 1992, Plantinga and Birdsey 1993) makes empirical estimates of carbon within discrete pools. Model parameters were developed to estimate individual carbon pools, which sum to total forest carbon per unit area. For example, carbon in above-ground portions of trees is estimated from merchantable volume per unit area in older stands. To estimate carbon budget uncertainty, we are interested in quantifying uncertainties about the expected values of these FORCARB model parameters from the deterministic model. Unfortunately, uncertainties about model parameters are not well defined, and they are likely to represent an aggregation of many uncertainties, such as errors in both the independent and dependent variables used in establishing a model parameter. Additionally, sampling error within a forest stratum and error in extrapolating to other forest lands must be included in parameter uncertainty. Uncertainty also extends beyond simple accumulation of statistical errors in measurement, regression, and sampling. For example, tree carbon parameters also must account for nontimber trees, not a part of the sampled population (Birdsey 1992), and soil carbon estimates are further modified by assumptions about soil carbon dynamics following disturbance or land-use change. Thus, complete descriptions of all uncertainties in each FORCARB model parameter are likely to be computationally and data intense.

We present an analysis that begins with a number of preliminary definitions of parameter uncertainty and reduces the number of candidate uncertainties. Our approach is based on the idea that for most simulation models, not all uncertain values are equally likely to influence model results (Morgan and Henrion 1990). A

series of "what if" analyses may identify those quantitative definitions of uncertainty that most affect FORCARB results. Thus, useful uncertainty analyses may be possible even under conditions where very little is known about the underlying uncertainty. These results are then an initial step in the iterative process of data collection and model refinement. Results are presented in three parts: basic Monte Carlo simulation results, a comparison of two indexes used to identify the influence of input parameter uncertainties, and an extension of the uncertainty analysis that describes sensitivity of results to components of parameter uncertainty. We end with a brief discussion of the usefulness of this approach.

Methods

An uncertainty analysis identifies effects of uncertainty within a given system (Morgan and Henrion 1990, Rowe 1994, Cullen and Frey 1999). The goal here is to provide a useful analysis where only preliminary values are known for uncertainties. The form of an uncertainty analysis partly depends on data available, model used, and the appropriate definition of uncertainty. As such, a presentation of methods necessarily includes some brief discussion of the rationale for the approach chosen.

Carbon Accounting and the Model FORCARB

The forest carbon accounting model FORCARB was developed to estimate carbon budgets of US timberlands (Plantinga and Birdsey 1993) and is an essential element in USDA Forest Service projections of carbon for US forests. The model is linked with the forest sector model TAMM/ATLAS, which provides periodic estimates of area and inventory volume according to classifications of forest type, age, and management influences (Adams and Haynes 1980, Mills and Kincaid 1992). Periodic model revisions reflect changes in available information, and work is underway to link to a larger system of models that represent external influences and sources of uncertainty. The basic FORCARB model as well as a number of regional-scale carbon budgets are detailed elsewhere (Plantinga and Birdsey 1993, Birdsey and Heath 1995, Heath and others 1996).

Forest carbon is modeled as an aggregate of discrete pools of carbon, each estimated according to empirical relationships. Simulations by FORCARB associate parameterized estimators for each carbon pool with appropriate subsets of forest projected by TAMM/ATLAS. Briefly, the model estimates pool sizes, or carbon inventories, for large-scale areas (10^5 – 10^7 ha) of forest; these include above-ground portions of hardwood and

Table 1 Parameter values and associated uncertainties used in FORCARB model estimates of carbon in maple–beech–birch forests in northeastern United States^a

Parameter	Expected value	Range of uncertainty in 1990
Softwood carbon in merchantable volume (kg C) estimated from merchantable volume ^b (V_s)	$199.8 \times V_s$	$\pm 7\%$
Total above-ground carbon in softwood trees (kg C) estimated from carbon in merchantable volume (C_s)	$2.193 \times C_s$	$\pm 13\%$
Hardwood carbon in merchantable volume (kg C) estimated from merchantable volume (V_h)	$298.6 \times V_h$	$\pm 7\%$
Total above-ground carbon in hardwood trees (kg C) estimated from carbon in merchantable volume (C_h)	$2.14 \times C_h$	$\pm 10\%$
Understory carbon (metric tons ^b /ha) for stands up through 15 years estimated from stand age (Y)	$4.98 + 0.249Y$	$\pm 25\%$ for youngest stands, linearly adjusted to $\pm 10\%$ at age 50 years
Understory carbon (metric tons/ha) for stands older than 15 years estimated from total merchantable volume (V)	$6.15 + 0.275V - 0.000531V^2$	$\pm 25\%$ for youngest stands, linearly adjusted to $\pm 10\%$ at age 50 years, and constant at that value above 50 years
Floor carbon (metric tons/ha) for stands up through 15 years estimated from stand age (Y)	$2.07 - 0.00997Y$	$\pm 50\%$ of youngest stands
Floor carbon (metric tons/ha) for stands older than 15 years estimated from total merchantable volume (V)	$2.06 - 0.0216V + 0.000310V^2$	$\pm 50\%$ of youngest stands
Soil carbon ^c (metric tons/ha) for stands up through 15 years estimated from stand age (Y)	$145 - 1.43Y$	$\pm 15\%$ for youngest stands and linearly adjusted to $\pm 25\%$ at age 15 years
Soil carbon ^c (metric tons/ha) for stands older than 15 years estimated from total merchantable volume (V)	$113 + 0.979V - 0.000824V^2$	$\pm 25\%$ at age 15 years, linearly adjusted to $\pm 10\%$ at age 50 years, and constant at that value above 50 years

^aUncertainty in a parameter value was defined as a range of values about an expected value (\pm percent of mean). Uncertainty was modeled in this table as a normal distribution with the 5th and 95th percentiles corresponding to this range. Standard deviations were set to increase 0.5% per year. Probability distributions were independently sampled across the 10 parameters and identically sampled across the simulated years.

^bMerchantable volume (m^3), metric ton (10^3 kg).

^cIntercept values for soil carbon are for 1990 only; estimates in subsequent years are adjusted to reflect effects of subsequent growth and harvest intervals.

softwood trees, understory species, forest floor, and soil carbon. Estimates for each of these components of total carbon are based on parameterized relations between a composite measure of a subset of forest (for example, merchantable volume) and the expected size of each carbon pool (for example, total carbon corresponding to the given volume).

Separate parameterizations are made by forest type by region. The identities of the ten model parameters assigned uncertain values for the maple–beech–birch example presented here are listed in the first column of

Table 1. Expected values for the parameterized relations are listed in the second column. Carbon content of trees is determined in two consecutive steps: biomass is estimated from merchantable volume, followed by an estimate of total tree carbon. Pools for understory, floor, and soil carbon are each formed from separate estimators based on age or volume, depending on stand age class. These are then summed according to total area for each age class to provide an estimate of inventory. Periodic average annual net flux is the difference between two successive estimates of inventory divided

by the length (in years) of the intervening period [for example, $\text{carbon flux}_{2010} = (\text{inventory}_{2010} - \text{inventory}_{2000})/10$].

FORCARB Uncertainty Defined for This Example

Uncertainty usually implies a lack of knowledge or the absence of information that may or may not be obtainable (Morgan and Henrion 1990, Rowe 1994). Further refinement of the definition often depends on the particular applications or objectives (Hattis and Burmaster 1994). For this reason, model-specific definitions generally are more useful than a comprehensive definition. We employ the simple definition of uncertainty as the inability to confidently specify single-valued quantities. Carbon content of a spatially and temporally defined portion of a forest is such a single value; uncertainty in a simulation model is a function of the information available to specify a variable with a single value.

We employ probability density functions (PDFs) to quantify uncertainty about model parameter values. Alternate methods are available to quantify uncertainty about values within a model, including the use of probability bounds and fuzzy numbers (Morgan and Henrion 1990, Kosko 1991, Ferson and Long 1995). Choice of a numerical representation for uncertainty can largely depend on both appropriate qualitative characteristics assigned to a definition of uncertainty and available data. We assume uncertainty about expected values for parameters; this implies greater information than simply placing bounds on possible values. We further assume that uncertainty is inversely proportional to available information. Probabilistic definitions of uncertainty may be more appropriate than fuzzy-number sets where uncertainties are thought to decrease with additional information (Kosko 1991). Thus, we use PDFs, which describe both ranges of possible values and relative expectation that those values may occur.

FORCARB was initially developed as a deterministic model, and we retain the basic characteristics of that model. Uncertainty for each of the ten parameters is characterized by assigning a PDF of likely values around the expected value of that parameter. Initial values for parameter uncertainties are given in Table 1. They were based on a subjective evaluation of available data and represent preliminary estimates. The model and analyses presented here address internal FORCARB uncertainties on modeled carbon budgets. This is preliminary to incorporating exogenous uncertainties through links with associated models (Birdsey and Heath 1995). Examples of external uncertainties include projections of inventory, growth, and harvests.

Although exact numerical values for parameter uncertainty differ with forest type, results presented here are not qualitatively different among types. Thus, results from a single forest type are sufficient for presenting these methods. Simulations used data for forest industry maple-beech-birch forests in the northeastern United States from the base model of the 1993 Resources Planning Act (RPA) assessment timber update (Birdsey and Heath 1995, Haynes and others 1995).

Monte Carlo Simulation of Uncertainty

Concerns about model development, system optimization, and decision-making have been principal motivations for incorporating estimates of uncertainty in forest simulation models (Gertner 1987, Dale and others 1988, Mowrer 1988, Luxmoore 1992, van der Voet and Mohren 1994, Gertner and others 1996, Pacala and others 1996). Monte Carlo simulation is a numerical approach to propagating model uncertainty. It is characterized by two distinct advantages for the analysis of interest: identification of influences and minimal imposition of assumptions. Model results in the form of PDFs are produced through Monte Carlo simulation, which involves a large number of iterations of the basic deterministic model (Morgan and Henrion 1990, Vose 1996, Cullen and Frey 1999). Random selections of values are made for each probabilistically defined variable for each iteration of the model. The outcome of each iteration will differ slightly depending on random selections among the probabilistically defined variables. In this way, a frequency distribution will form as the resulting model prediction. Latin hypercube sampling was used. It is a stratified sampling procedure within a Monte Carlo simulation that draws samples from equal-probable intervals, without replacement, from each PDF (Iman and Shortencarier 1984). Advantages of Latin hypercube sampling include the ability to more precisely specify joint probability distributions among variables, where such covariability exists, and a reduction in computational effort, since simulations require fewer repetitions of sampling before achieving a stable output distribution. The Monte Carlo simulation produces results that reflect the joint uncertainties of all parameters by simultaneously sampling from all specified PDFs.

Monte Carlo simulations are relatively straightforward and are flexible means of incorporating probabilistic values in a numerical simulation model such as FORCARB. Modification and reanalysis can be accomplished simply and quickly, and this readily facilitates comparison of alternate forms of a model. Other means of determining results in the form of distributions that reflect effects of input distributions also can be em-

ployed, most notably methods that are termed first-order approximations (Beck 1987, Iman and Helton 1988, Bobba and others 1996). Monte Carlo simulations, however, are less reliant on assumptions about distributions, such as a need to know central moments. Bayesian methods also have been applied to the problem of improving estimates of input or model parameter uncertainty (Green and Strawderman 1985, Lexer and Hoenninger 1998). We have not pursued these methods with FORCARB because of the scarcity of information to establish a likelihood given a prior estimate of uncertainty, and the ease and rapidity of each Monte Carlo simulation. The merits of alternate approaches to simulating model uncertainty have been compared by a number of researchers (Morgan and Henrion 1990, Guan and others 1997, Cullen and Frey 1999). Tractable use of “what if” scenarios makes Monte Carlo simulation the choice for our application.

This application of Monte Carlo uncertainty analysis is useful at an early stage of model implementation. Because the parameter PDFs are only preliminary values, we extend the analysis to individual components of the PDFs in addition to whole distributions, which is more commonly the case. We define these components as the range of values, the likelihood of values along that range, and the covariability among PDFs. The range and likelihood can be considered the marginal uncertainty, and the covariability describes joint uncertainty among FORCARB parameters. Our analysis assumes that methods of identifying influences of whole PDFs can also identify influences of the components of PDFs.

Measuring Influence of Parameter Uncertainty

Not all uncertain values affect results equally, and not all uncertain values are equally well defined. A goal of this analysis is to identify influences of whole PDFs as uncertain input values and extend the same analyses to components of PDFs. Uncertainty in some model parameters may have a large effect on uncertainty in results, yet even large uncertainties in other parameters may have negligible effects on uncertainty in results. Similarly, everything that goes into defining a PDF may not be equally important in affecting model results. To this end, we employ measures of parameter influence that are sensitive to how uncertainty is defined and are amenable to iteratively developing those definitions of uncertainty. These measures include two simple and commonly used indexes to express influence of model PDFs on result PDFs; we label these the “importance index” and “contribution index” for this manuscript. These methods are not dependent on assumptions about the respective distributions.

The importance index is the coefficient of rank correlation (Vose 1996, Cullen and Frey 1999), which reflects relative influence of model parameters on total uncertainty in results. Random samples taken from parameter distributions during the Monte Carlo simulation will differ in their degree of influence on model results. The cumulative effect of all such samples is reflected in the output distribution of the simulation. The relative influences of each parameter distribution on model output can be identified by means of partial nonparametric correlations (Morgan and Henrion 1990). The nonparametric Spearman coefficient of correlation (Conover 1971, p. 244) between two distributions is based on the rank order of samples drawn from a parameter distribution and those resulting in the output distribution. We determine an estimate of the partial correlation between a parameter and model output in cases where the distribution of values calculated from one parameter are dependent on another parameter (Conover 1971, p. 254).

Importance index values range from -1 to $+1$. Greater absolute values indicate a greater degree of influence and values approaching zero indicate decreasing influence. These values incorporate the effect of both the median value and the dispersion of each parameter by allowing simultaneous changes in all such sampled parameters. That is, model results reflect effects of joint probability among parameter distributions, yet the importance index identifies only the added effect of individual parameters during simultaneous sampling among all parameters.

The second index, the contribution index, allocates total uncertainty among the parameters as a percentage contribution to the total (Vose 1996). This is based on selecting a common measure of uncertainty for parameter and inventory distributions; we use the difference between the 95th and 5th percentiles of each distribution. The effect of uncertainty in a given parameter is determined by two separate simulations: one with the parameter defined precisely (single value) and one with the parameter defined as a PDF. The difference in model uncertainty between the simulations represents the effect of that parameter on total uncertainty. This is repeated, in turn, for all parameters, and the ratio of the individual contribution to the sum of contributions is expressed as a percentage. These can be a positive or negative value for each parameter, yet the total for all parameters sums to 100%.

Extending the uncertainty analysis to components of parameter PDFs is accomplished by systematically altering the shape of PDFs, covariability among PDFs, and range of PDF values. The sensitivity of projected inventory uncertainty to such changes is reflected in index

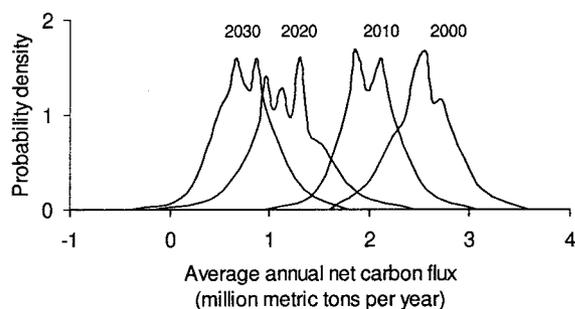


Figure 1. Model estimates of average annual carbon flux for projection years 2000, 2010, 2020, and 2030 described as probability densities (PDFs) obtained from Monte Carlo simulation of the FORCARB model. Note that the area under a PDF sums to 1.

values. Uncertainty in estimates of average annual net flux are subject to influence by similar components of carbon inventory PDFs. Sensitivity analyses are employed to examine effects of these simulated intermediate PDFs on uncertainty of projected flux.

Results and Discussion

Basic Simulation Results

Probability distributions describing likely forest carbon inventory and average annual net carbon flux were the initial products of Monte Carlo simulations. These PDFs approximated normal distributions (Figure 1), with the 5th, 50th, and 95th percentiles given in Table 2. A positive value for flux reflects a net gain in forest carbon inventory over the period simulated. The median values presented here approximately equal the analogous deterministic estimates of FORCARB simulations performed to produce regional carbon budgets in conjunction with the base scenario for the 1993 RPA timber assessment update (Birdsey and Heath 1995, Haynes and others 1995). We use the range of the central 90% of the result PDF simply as a convenient summary of uncertainty. This also can be expressed as plus or minus a percentage of the median for the symmetrical distributions produced here.

The proportion of total carbon inventory contained in soils and trees (softwood plus hardwood) remained at about 66% and 26%, respectively, for all simulations. Uncertainty in soil carbon inventory for 2010 was about $\pm 10\%$ of the median (5th–95th percentile), and that for hardwood carbon was about $\pm 13\%$ of its median (data not shown). These contributed to an uncertainty for total carbon inventory in 2010 of just over $\pm 7\%$ of the median (Table 2). Uncertainty for projected flux at 2010 was about

$\pm 20\%$ of the median. Note that relative uncertainty did not increase as separate soil and tree (and other) inventories were summed. Summing independent distributions in a Monte Carlo simulation will tend to decrease relative dispersion of values in the resulting distribution. In fact, this characteristic of the central limit theorem contributes to the same effect in forest inventory sampling statistics, which are considered increasingly precise as larger areas of inventory are considered (Hansen and others 1992). The same is true for pooling separate forest groups. Aggregate values are relatively more precise, to the extent that they are independently estimated. For this reason, preliminary estimates of uncertainty for maple–beech–birch forests should not be considered representative of uncertainty for the region, which is likely to be relatively smaller.

The choice of number of iterations to include in the simulation depends on the purposes of the model. Variance and fractile values of the output distribution will change with each successive iteration. Stability of distributions increases with number of iterations, with a greater number required to stabilize extremes (or tails). We are concerned with the range of most likely values. Five hundred iterations produced carbon inventory distributions that differed by less than 5% (that is, 95th minus 5th percentiles, as in Table 2) as the entire simulation was repeated with different random sampling sequences. This level of precision resulted in stability in the indexes of parameter influence we chose to employ for our uncertainty analysis. Models with a focus on precision of estimated probability in the tail regions of output distributions (that is, small probabilities of extreme events) may require hundreds to thousands of iterations, depending on individual standards set for confidence (Iman and Helton 1988, Morgan and Henrion 1990, Cullen and Frey 1999). Simply put, the sensitivity of the result should dictate number of iterations.

A number of approaches exist for propagating uncertainty through simulation models (Morgan and Henrion 1990, Cullen and Frey 1999). Error propagation methods can produce results that reflect input uncertainties. Without a high degree of confidence in the input uncertainty, however, error propagation alone has limited value. Thus, we focus on the relation between how these input uncertainties are initially defined and the overall model results. We next examine alternate indexes of parameter influence on result uncertainty, and we then focus on how individual components of PDFs might contribute to overall uncertainty.

Table 2 Median values (50th percentile) of simulated carbon inventory (million metric tons carbon) and average annual net carbon flux (million metric tons carbon per year) with 5th and 95th percentiles of distributions indicating range of modeled results, which encompass 90% of the distribution

Year of simulation	Carbon inventory			Average annual net carbon flux		
	5th	50th	95th	5th	50th	95th
1990	430	465	496			
2000	454	490	522	2.0	2.5	3.0
2010	471	510	545	1.6	2.0	2.4
2020	479	522	561	0.7	1.2	1.7
2030	484	530	573	0.4	0.8	1.2

Indexes of Parameter Influence Identify Sources of Carbon Budget Uncertainty

Monte Carlo simulation is ideally suited for examining the consequences of all or a selected part of the PDF defining uncertainty about a FORCARB parameter (Morgan and Henrion 1990, Vose 1996). The influence of uncertainty in a particular parameter on overall model results will be modified by uncertainties in other parameters. The two indexes of parameter influence we employ for analysis reflect the strength of the parameter-to-result relation under conditions of simultaneous uncertainty in other variables.

Importance index values for the effect of ten FORCARB parameters on carbon inventory projected for 2010 are shown in Table 3. Pools of tree carbon for both softwood and hardwood species are each based on two parameters applied in sequence. The indexes presented for the expansion to total aboveground carbon therefore represent partial correlations after having removed the effect of uncertainty of carbon in merchantable volume. Clearly, uncertainty in parameterized estimates of soil carbon, with an importance index of 0.833, had the greatest importance in determining uncertainty in total inventory. Estimates leading to projections of hardwood tree carbon had the second greatest importance in these maple-beech-birch forests. Uncertainty in understory carbon had effectively no influence.

The importance indexes (such as presented in Table 3) are produced by pairwise rank correlation of the 500 samples drawn from a parameter's distribution with the 500 samples forming the model output distribution. As such, these indexes are random variables subject to the sampling of the 500 values from parameter distributions. To assess variability in this index, ten independent Monte Carlo simulations were performed. Values in Table 3 are means and standard deviations of the resulting set of importance indexes. Variability in index values was inversely proportional to absolute index values. Standard deviations are provided simply to illus-

Table 3 Importance index (rank correlation coefficients) between total carbon inventory at 2010 and ten parameters with uncertainty as defined in Table 1^a

Parameter	Mean importance index (standard deviation)
Soil carbon in older stands	0.833 (0.012)
Total aboveground carbon in hardwood trees	0.360 (0.031)
Soil carbon in younger stands	0.281 (0.039)
Hardwood carbon in merchantable volume	0.240 (0.031)
Forest floor carbon in older stands	0.104 (0.033)
Total aboveground carbon in softwood trees	0.089 (0.041)
Understory carbon in older stands	0.046 (0.042)
Softwood carbon in merchantable volume	0.041 (0.034)
Understory carbon in younger stands	0.031 (0.046)
Forest floor carbon in younger stands	-0.011 (0.058)

^aValues are means and standard deviations (in parentheses) of 10 Monte Carlo simulations.

trate variability of importance indexes, for we have no reason to test for differences among the rank correlations in the analyses we present here. Some test statistics (Conover 1971, p. 248, Kendall and Gibbons 1990, p. 74) have been developed for Spearman's rho, which is only slightly different from the calculation for our importance index (see related discussion concerning ties in rank ordering in Conover 1971). We do not present estimates of variability with importance indexes subsequent to Table 3 because the values are not means and our purpose in presenting them is only to indicate rank or relative degree of influence.

These indexes of parameter influence reflect the effects of simultaneous uncertainties in all parameters

Table 4 Importance index of major carbon pools estimated by FORCARB model parameters^a

Parameter distribution	1990	2000	2010	2020	2030
Carbon inventory					
Total tree	0.38	0.43	0.44	0.42	0.40
Understory/floor	0.16	0.17	0.17	0.18	0.18
Soil	0.91	0.88	0.88	0.89	0.90
Average annual net carbon flux					
Total tree		0.37	0.40	0.22	0.21
Understory/floor		0.07	0.15	0.16	0.12
Soil		-0.09	0.75	0.87	0.90

^aIndex values range from -1 to +1, with larger values expressing greater parameter influences on model estimates of uncertainty in carbon inventory and average annual net carbon flux.

Table 5 Contribution index of major carbon pools estimated by FORCARB model parameters^a

Parameter distribution	1990	2000	2010	2020	2030
Carbon inventory					
Total tree	10.1	15.1	12.4	13.2	11.7
Understory/floor	-1.6	2.9	2.0	2.3	2.5
Soil	91.5	82.0	85.6	84.5	85.9
Average annual net carbon flux					
Total tree		11.4	9.2	3.9	3.9
Understory/floor		1.2	-0.4	-1.7	-0.3
Soil		87.4	91.2	97.8	96.4

^aIndex expresses percentage of total uncertainty due to parameter influence on model estimates of uncertainty in carbon inventory and average annual net carbon flux.

during simulation. This interdependence represents a difference from information provided by a more conventional sensitivity analysis (Iman and Helton 1988) and implies that model changes to one variable may also produce a measurable effect on the degree of influence exerted by other variables. For example, removing uncertainty from the parameter for soil carbon for older stands would increase the importance for total above-ground carbon in hardwood trees from 0.36 to 0.77 (Table 3). Thus, as uncertainties of one variable are reduced, others can become very influential, even without revising the level of uncertainty of the other variables.

Total carbon inventory is the sum of five intermediate carbon pools that are, in turn, each predicted from two FORCARB parameters. To simplify presentation of the remainder of the results, we consider three pools of carbon: tree, soil, and understory/floor (Table 4). Clearly, soil carbon was most influential in contributing to overall uncertainty for most years. This was due to the relative size of the soil carbon pool. Understory and forest floor carbon pools were consistently not influential, and reducing uncertainties associated with these values will have little effect on estimated uncertainty for total carbon budgets. Importance indexes in Table 4 were based on the aggregate effect of parameters in

producing the three intermediate carbon pools. That is, rank correlations of model results were with the three intermediate pools rather than the ten parameters. Similarly, correlations to determine the importance index for the 2010 flux were based on correlations of estimated flux with the three inventory pools for 2010.

The index showing the contribution of PDFs to total uncertainty is given in Table 5. These results correspond to the importance index of Table 4; that is, the index reflects the contributions of PDFs for the three intermediate carbon pools. The two indexes are generally very consistent (Vose 1996). Notable exceptions were values of flux for the interval between 1990 and 2000. Clearly the two methods differed in identifying the contribution of soil carbon to the total flux. The difference was based on the use of the three intermediate pools and the fact that underlying parameter influences changed between 1990 and 2000. Relative strength of volume and age contributions to soil carbon changed considerably during that interval; this produced a change in the sampling sequence of the combined pool. The importance index was sensitive to that change, and the percentage contribution index was not. Differences between indexes do not represent a

failure of one index or another, but rather, point out slight differences in assumptions and sensitivities.

Both measures of uncertainty indicate the influence of input parameter uncertainty on uncertainty in model results. They are not qualitatively identical, however, as indicated by the occasional negative values for percentage contribution of understory/floor (Table 5) and the differences in soil flux for 2000 (Table 4). Small pools of carbon can have a negative percentage contribution on a sum. The negative value is not incorrect but is potentially confusing. The complete dependence of the importance index on Monte Carlo sampling (as rank correlation) of a single distribution makes it dependent on only one of the inventories contributing to the flux estimate and sensitive to changes in sampling between the two. These measures are both easy to apply to numerical simulations, flexible, and relatively free of assumptions. The importance index is perhaps a simpler application in most cases because the contribution index always requires two simulations and the choice of a base model can become difficult when making multiple comparisons.

Individual Elements in Definition of PDFs Can Affect Parameter Influence

The same analysis to identify influential model parameters can be applied to improve parameter PDF definitions. Low confidence in starting definitions for uncertainty limits the value of simply propagating uncertainty through a model. Because Monte Carlo simulation can be easily adapted to look at the effect of selected parts of input distributions, we next examine what aspects of defining uncertainty do and do not make a parameter important in determining model results. This can be viewed as part of an iterative process of increasing precision in the PDFs used as inputs. We systematically examined the response of model uncertainty to changes in shape of the probability distributions, covariability among distributions, and range of uncertainty surrounding the expected value. Results in this section are limited to pooled effects of tree and soil carbon parameters for the year 2010 to focus discussion on potentially important influences.

The shape of a probability distribution used to define uncertainty about a parameter value implies an expectation of the most probable values from among the set of possible values (Seiler and Alvarez 1996, Haas 1997). Symmetrical distributions were assumed for parameter definition. Triangular and uniform distributions were defined for the ten model parameters such that the 5th, 50th, and 95th percentiles corresponded to the same values as for the normal distributions defined in Table 1. The net effects of altering shape were

differences in the concentration of possible values around the median versus the tails; that is, differences in level of information about the expected value, or entropy in the sense of Shannon and Weaver (1949). Results analogous to those in Tables 4 and 5 (but not shown here) suggest that this level of varying shape of parameter distribution had no real effect on the most influential parameters.

Covariability, or joint probability, among PDFs can influence simulation results (Ferson and Long 1995, Vose 1996, Cullen and Frey 1999). Such specifications are possible with Monte Carlo simulations irrespective of the shape of the distribution (Iman and Conover 1982). We explored this effect with FORCARB parameters related to tree and soil carbon. Tree carbon is determined by two successive parameterized conversions from merchantable volume to total tree carbon, for each hardwood and softwood. Specifying perfect correlation in sampling among tree carbon probability distributions increased the importance index of tree carbon inventory and flux (Table 6, row 2). The correlation effect on tree carbon inventory also resulted in a slight decrease in influence of soil carbon inventory. Similarly, perfect correlation in sampling the age- and volume-based estimators for soil carbon increased soil importance relative to tree.

The range of values included in a PDF (or in other words a measure of its variance) is probably the most obvious and direct influence determining importance of a model variable. However, simply increasing the dispersion of values alone does not automatically translate into immediate increased importance of the parameter; it is still subject to the effects of other such parameter specifications. For example, a doubling of the relative range of tree carbon parameters was required before overall tree uncertainty had an importance index similar to soil carbon (Table 6, row 5). Conversely, a reduction by one half in the range of soil carbon parameters was required to achieve approximately the same effect. The effects of covariability and relative size of distribution appeared to be additive in affecting the importance index and relative uncertainty (Table 6, row 7). The range and relatedness of parameter uncertainties can together affect uncertainty of estimated carbon inventory.

Because we analyzed uncertainty in FORCARB parameters while excluding other exogenous uncertainties, the importance index appeared to provide a simple quantitative restatement of the obvious: larger pools of carbon with larger uncertainties had greater influence. As future model complexity increases by inclusion of additional system uncertainties, however, all newly considered effects can be represented as effects

Table 6 Comparison of effects of covariability among distributions and level of uncertainty (range of values) in PDFs on influence of parameters on model results for 2010^a

	Carbon inventory			Average annual flux		
	Importance index		Model estimate (%)	Importance index		Model estimate (%)
	Tree carbon	Soil carbon		Tree carbon	Soil carbon	
Base model ^b	0.44	0.88	510 ± 7	0.40	0.75	2.0 ± 20
Covariability						
Two steps to calculate tree carbon, perfect correlation among samples	0.56	0.78	510 ± 8	0.52	0.67	2.0 ± 20
Age and volume bases for soil carbon, perfect correlation among samples	0.38	0.92	509 ± 9	0.31	0.95	2.0 ± 26
Both tree and soil estimates of carbon, perfect correlation among samples	0.46	0.86	510 ± 9	0.37	0.91	2.0 ± 26
Level of uncertainty						
2× uncertainty of tree carbon	0.70	0.67	510 ± 9	0.66	0.61	2.0 ± 24
0.5× uncertainty of soil carbon	0.69	0.66	509 ± 5	0.65	0.61	2.0 ± 12
2× uncertainty of tree carbon with perfect correlation among tree samples	0.79	0.54	512 ± 11	0.75	0.51	2.0 ± 28

^aImportance indexes are for influences of tree and soil carbon pools. Median values of simulated inventory (million metric tons carbon) and average annual net flux (million metric tons carbon per year) include the range of values (\pm percent of median) that encompasses approximately 90 percent of the distribution.

^bValues in this row are repeated from Tables 2 and 4 to facilitate comparison of effects.

on size and dispersion of values for the major carbon pools. That is, an additional contribution to total uncertainty may affect a “balance” of magnitude and relative uncertainty between the tree and soil carbon pools. This relation is shown in Figure 2. Inventory estimates for 2010 were modified to produce the given range in the ratios of expected values (x axis) and relative uncertainty (y axis, defined here as coefficients of variation). Any point on the graph can be defined by the paired values: (relative parameter value, relative parameter uncertainty). The area above the two lines represents conditions where soil carbon uncertainty is most important in affecting overall inventory, and the area below the two lines represents conditions where tree carbon uncertainty is most important. The area between the two lines represents conditions where importance indexes were not significantly different based on paired t tests of results from five sets of ten independent simulations ($\alpha = 0.05$). The \times 's represent the position of the five base model projections of carbon inventory for 1990 through 2030 from Table 2. The \circ 's represent the position of the carbon inventory estimates made for Table 6.

The point of Figure 2 is to provide a chart of “what if” analyses illustrating conditions where soil or tree carbon uncertainty most contribute to total uncer-

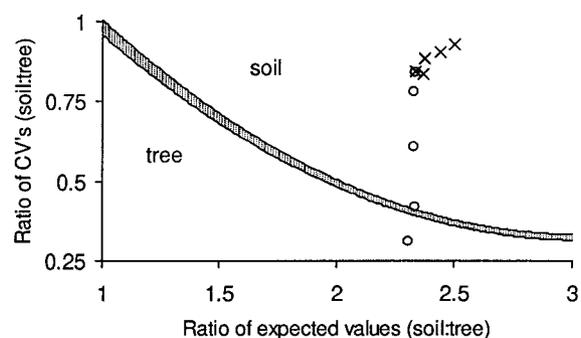


Figure 2. Plot identifying conditions in which soil carbon uncertainty is more important in determining model uncertainty versus conditions in which tree carbon uncertainty is more important in determining model uncertainty. Here, importance in determining total uncertainty is expressed as a function of magnitude of the carbon pools (x axis) and relative uncertainty (y axis). Importance of soil carbon uncertainty increases as ratios indicate greater absolute amounts of soil carbon or greater relative uncertainty in soil carbon—the area above the two lines. Conditions for tree carbon control of model uncertainty are below the lines. The area between the lines represents no significant difference between the two. The \times 's represent the position of the five base model projections for 1990 through 2030. The \circ 's represent the position of the inventory estimates made for Table 6.

tainty. Uncertainty in soil carbon inventory clearly has the greater influence on most model projections. The base model assumes that the four tree-carbon parameters are independent values, yet if they covaried while retaining the same values as in Table 1, uncertainty of tree carbon would increase considerably. This would move the ratio of coefficients of variation for the 2010 inventory from 0.867 to 0.532 (that is, change the position of an \times along the y axis in Figure 2). Even this significant change in assumptions about FORCARB parameters still would produce results more influenced by uncertainty about soil carbon than uncertainty about tree carbon.

Monte Carlo simulations may produce PDFs as intermediate values. Obviously, the same whole-PDF and components-of-PDF analyses are possible with these values as well. A useful distinction here is that intermediate PDFs are partly determined by model structure and assumptions in addition to influences of input PDFs. We illustrate how components of intermediate PDFs can reflect model assumptions by extending the above analysis to estimates of average annual net carbon flux based on differences between two projections of inventory.

Flux uncertainty is dependent on factors not apparent in simple expressions of inventory uncertainty. For example, the same two inventory distributions for 2000 and 2010 in Figure 3A are used to produce the distributions of estimated flux in Figure 3B. The differences are based on assumed covariability, or correlation in Monte Carlo sampling, of consecutive soil or tree carbon inventories. The narrowest distribution results from assuming high correlation between the 2000 and 2010 inventories for both soil and tree carbon (that is, the base model). The two wider distributions result from reducing the covariability of the tree carbon or soil carbon pools to a correlation coefficient of $r = 0.6$. Further decreases in covariability will tend to further disperse projections of flux. Thus, flux uncertainties are strongly influenced by the relatedness of inventory uncertainties, a relation highly dependent on modeling assumptions.

Total flux, in the context of the FORCARB model proper, is simply the sum of the component fluxes. Thus, whatever influences the expected value and uncertainty of these component distributions (for example, tree or soil carbon flux) will be important in determining overall flux uncertainty. Expected value of flux depends on the difference between consecutive inventories. Flux uncertainty depends on both relative uncertainty and covariability between two consecutive inventories. We compared response of flux uncertainty

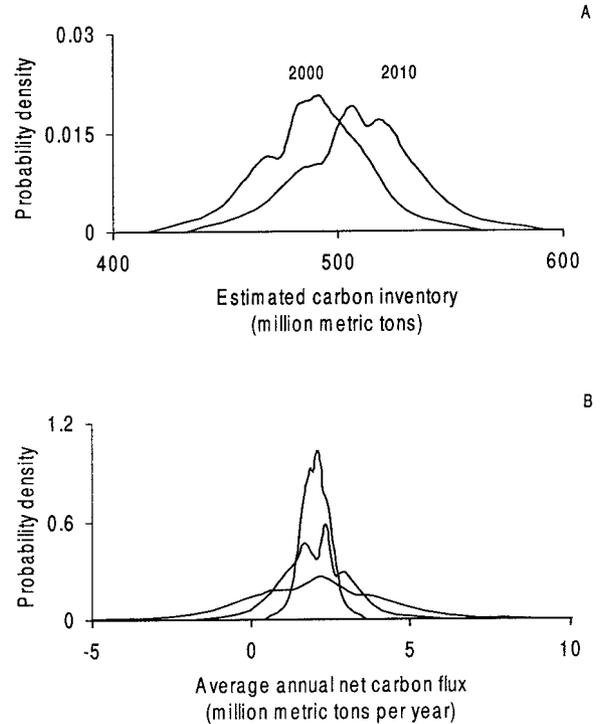


Figure 3. Base model projections of (A) carbon inventory for projection years 2000 and 2010 and (B) estimates of average annual flux derived from the two inventory distributions under different assumptions of covariance between constituent pools and inventories. The central curve represents the base model projections that assumes very high correlation between two consecutive tree and soil estimates ($r \geq 0.99$). The two wider distributions represent flux uncertainty with between-inventory tree or soil carbon correlation coefficients set at $r = 0.6$. Note that the wide disparity in the range of uncertainty for flux (B) resulted from the same two distributions of inventory (A), but with different assumptions of covariance.

to these three effects by manipulating the distributions contributing to the 2010 flux (Figure 4).

Inventory for 2010 was increased and decreased to systematically alter expected difference between inventories and thus expected value of flux. This effect had little apparent influence on range from the 5th to the 95th percentiles of the distribution of flux values produced by the model (Figure 4A). Relative uncertainty was manipulated by modifying the dispersion (but not the shape or sampling sequence of the distribution) of the 2010 inventories to modify the relative coefficients of variation of the two inventories (Figure 4B). The effect was apparently linear, with a greater influence by increased uncertainty in soil carbon inventory. Manipulating the Monte Carlo sampling sequence to set the sampling correlations between distributions also had a greater effect through soil carbon estimates (Figure

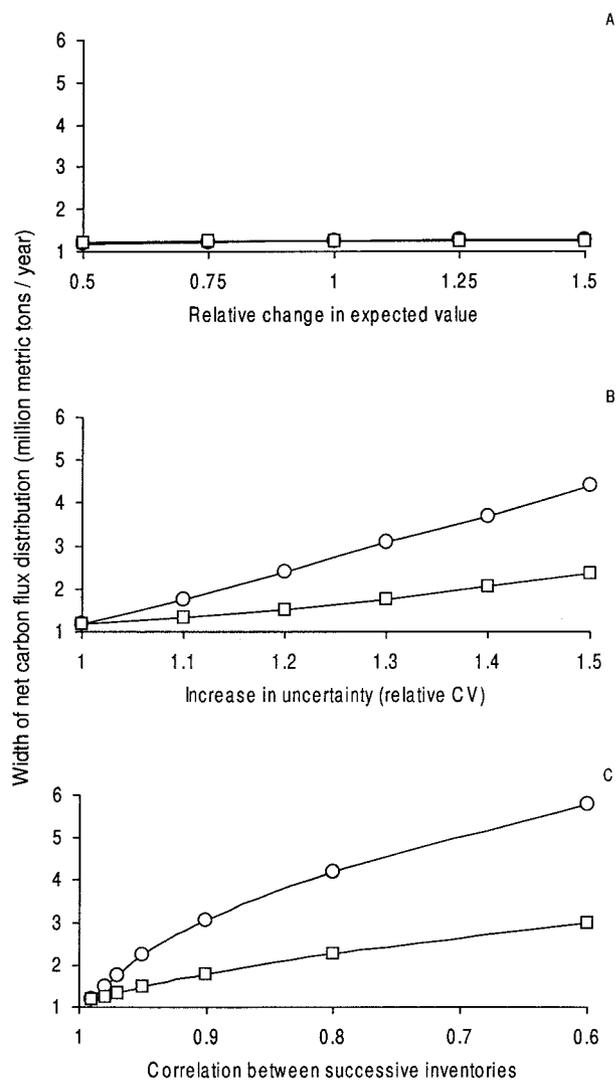


Figure 4. Sensitivity of model uncertainty to elements defining probability densities of two consecutive inventories. Sensitivities to changes in (A) mean of estimates, (B) increase in uncertainty, and (C) covariation of distributed values were determined for both tree (\square) and soil (\circ) carbon pools. Model response was represented by the width of the central 90% of the probability density. Values were obtained from manipulation of inventory distributions used to project average annual flux for 2000 to 2010.

4C). An interaction of the effects shown in Figures 4B and 4C was such that as relative uncertainty about carbon inventory decreased, the influence of between-year covariability increased (note the greater slope for high correlation in Figure 4C). This illustrates the possible value in simply knowing what inputs influence results, such as when explicitly accounting for covariability may matter. Because joint distributions can be

key to successful links among models (Morgan and Henrion 1990), the importance of recognizing this simple relation increases as additional parts of the larger forest sector modeling system are considered.

The 5th to 95th percentile interval was more appropriate as a dependent variable in this analysis than were the importance and contribution indexes. The importance index identifies the effect of a single distribution on the resulting model distribution, while flux uncertainty was influenced by the relative definitions of two inventories. Determining percentage contribution would require many additional simulations because each is based on a treatment effect relative to a base level. This analysis underscores the point that a single measure of model results is usually not best for all analyses.

Value of This Approach

Not everything always matters. Projections of uncertainty about forest carbon budgets are dependent on assumptions and information that go into the models. When such information is very limited, a pragmatic approach to quantifying this uncertainty is to develop preliminary estimates and then sort out conditions where specific uncertainties may or may not matter. This can be followed by iteratively revising estimates of uncertainty and improving empirical bases for models. A single approach is seldom best for all applications. Our use of Monte Carlo simulation is simple and flexible and allows for alternate approaches to uncertainty analysis.

Some simulation models make the distinction between uncertainty associated with variability, or natural heterogeneity in the system modeled, and uncertainty due to lack-of-knowledge (Morgan and Henrion 1990, Hattis and Burmaster 1994, Hoffman and Hammonds 1994, Ferson and Ginzburg 1996, Cullen and Frey 1999). Lack-of-knowledge uncertainty assumes the existence of a single best value that we do not know, while variability expresses changing values among occurrences. Such a distinction is not absolute and may depend on the understanding of appropriate mechanisms or even the scale of the model. Variability may contribute to overall uncertainty in parameterizing relations at moderately large temporal (year) and spatial (10^5 – 10^7 ha) scales as applied in this model. We are not, however, addressing annual fluctuations in carbon inventory or growth, and we do not simulate more than one forest industry maple–beech–birch forest in the northeastern United States for a given year. Thus, estimates of uncertainty applied to this analysis do not contain a distinct component of variability, yet the contrast was made because the presence or absence of

variability affects the structure of the Monte Carlo simulation. A two-way Monte Carlo model can separately identify the effects of each on overall model results (Cullen and Frey 1999); however, a one-way Monte Carlo model is most appropriate in our case, where variability is not a distinct component of uncertainty.

Quantitative expressions of uncertainty produced through Monte Carlo simulation are unambiguous but highly dependent on the adopted definition of uncertainty and the context in which they are used. As we present it, uncertainty is essentially a model imposed on the modeled projection of carbon, and like any model, results are simply functions of inputs and assumptions. However, acceptance that such PDFs are appropriate to the questions asked of the simulation requires an additional level of confidence beyond correctly translating input numbers to output numbers. Such "confidences" on the part of the decision-makers are not readily quantified and can differ with experience and background. Some insight into a model is usually required for acceptance by users. For this reason, simple and tractable models that allow for user insights into behavior and sensitivities are commonly recommended for integrated assessments (Morgan and Dowlatabadi 1996).

Simply knowing inventory uncertainty is insufficient for estimating flux uncertainty. The distinction between the uncertainty necessary to characterize inventory and that required to estimate flux points out the need for accounting for covariability among uncertainties defined as PDFs. Marginal probabilities alone are insufficient for these analyses. Joint distributions, where necessary, can be either empirically determined or developed through the process modeled. Just as viewing inventory as an end will limit uncertainty analysis of flux, so viewing discrete parts of a system as entirely separate also will limit uncertainty analysis of the aggregate system.

The analysis presented here is based on assumptions that FORCARB is a subset of a larger system of influences that determine forest carbon budgets and the need for these values extends beyond single "best" estimates. Adding a probabilistic simulation did not change the expected values for the model. As system size and complexity increase, however, expected values for the probabilistic and single-point forms of a model can easily diverge. Thus, the value of this application is not in a specific application of FORCARB, but in identifying important links to the larger system. Ongoing uncertainty analyses of forest carbon budgets address a system of models (Birdsey and Heath 1995) and presume that questions asked about uncertainty, definitions assigned to uncertainty, and means of analyzing uncertainty are interrelated.

Conclusion

The uncertainty analysis techniques we employed were clearly applicable to developing quantitative estimates of input uncertainty through examining components of PDFs. These results underscored two useful considerations when applying this approach to analysis. First, no single measurement of influence, or index, applied equally well to all analyses: for example, the importance index had limited usefulness in identifying influences of flux estimates. Second, results are subject to reanalysis as uncertainties are revised and influences shift. The iterative nature of this modeling is based on identifying where additional information most affects results: obviously, needs change as information is obtained. The consequent updating and reanalysis will affect the quality of results and confidence in the simulation. The simple and tractable modeling structure we employ contributes to the flexibility necessary for these analyses.

Using these techniques with limited initial information, we were able to identify relative influences of both model parameter PDFs and components of parameter PDFs, reducing and ordering the list of unknown uncertainties. Distributional forms of the PDFs were not as important as covariability or ranges of values. Covariability among FORCARB model parameters emerged as a very influential component of uncertainty; however, the implications of this result were different for estimates of carbon inventory versus carbon flux. Uncertainty about model parameters had a simple, direct effect on estimates of carbon inventory, and this extended to covariability between PDFs. Overall uncertainty increased in proportion to influence and covariability between parameters. The effect was essentially a linear function of *a priori* assumptions input as parameter uncertainty. In contrast, input parameter definitions had relatively less influence on flux uncertainty. Flux uncertainty was largely a function of simulated covariability between annual inventories and was dependent on structure and assumptions of the modeling process. Thus, our results suggest that explicit accounting of covariability may be necessary for adequately defining flux uncertainty in a forest carbon budget model.

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