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Model for multi-stand management based on structural attributes of individual stands

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Abstract

A growing interest in managing forest ecosystems calls for decision models that take into account attribute goals for large forest areas while continuing to recognize the individual stand as a basic unit of forest management. A dynamic, nonlinear forest management model is described that schedules silvicultural treatments for individual stands that are linked by multi-stand management constraints. A growth model useful for many eastern forest types accounts for stand dynamics resulting from cutting decisions. This modeling approach provides a framework for coordinating management goals over many stands while meeting the practical need for stand-level cutting prescriptions. An example problem demonstrates how to measure the tradeoff between economic efficiency and tree species diversity. Dimension limits and solver efficiency are discussed. © 1997 Published by Elsevier Science B.V.

Keywords: Optimization; Nonlinear programming; Ecosystem management; Harvest scheduling

1. Introduction

A growing interest in managing forest ecosystems calls for the development of decision models that take into account management goals for larger forest areas while continuing to recognize the individual stand as a basic unit of forest management. In the context of an ecosystem management model, each stand exhibits certain attributes over time as a result of timing and intensity of silvicultural activities. For example, cutting treatments lead to specific residual stand attributes (i.e., basal area, species, diameter and height distribution) that determine the characteristics of wildlife habitat, landscape aesthetics, or light conditions for reproduction of woody and herbaceous species. Attributes of individual stands contribute to aggregated conditions over much larger areas of interest to the forest manager.

This paper describes a dynamic forest management model that schedules optimal harvest treatments for individual stands subject to constraints on a larger, multi-stand management scale. In general, a problem solution is defined in terms of the harvest sequence of timber species over time for all stands included in the problem. Management goals are quantified in terms of stand structure required for outputs such as wildlife habitat or aesthetics, and included as constraints on the problem so that prescriptions provide for such attributes in individual

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stands or in aggregate for all stands in the management unit. This multi-stand modeling approach provides a framework for coordinating management goals over many stands while continuing to recognize the practical need for cutting prescriptions for the individual stand.

In this model, feasibility is defined by two types of mathematical constraints: (1) management goals and (2) stand growth resulting from harvest decisions. Constraints that represent management goals simply place limits on harvest decisions so that the optimization procedure only considers cutting strategies that achieve and maintain desired attributes. Stand structures or aggregated forest attributes that meet management goals must be defined by the user. Constraints that represent growth dynamics can be derived from an appropriate stand growth model. Several such models are available for eastern hardwood cover types: SILVAH for Allegheny hardwoods (Marquis and Ernst, 1992), OAKSIM for even-aged upland oaks (Hilt, 1985), NE-TWIGS for mixed eastern hardwoods (Teck, 1990), and FIBER for spruce-fir and northern hardwoods (Solomon et al., 1987). For this study, the FIBER model, which provides reliable projections of stand growth for central Appalachian hardwoods (Schuler et al., 1993), was formulated to represent stand dynamics in a nonlinear programming format (Miller and Sullivan, 1993).

In modeling the single-stand management problem, a common approach is to choose a silvicultural system based on biological conditions needed for regenerating desired species, harvesting economics, and associated attributes required for other management goals such as habitat or aesthetics. Management problems are then structured to define the optimal harvest sequence for a particular stand over time, subject to constraints that define the chosen silvicultural system (Adams and Ek, 1974; Buongiorno and Michie, 1980; Haight, 1987).

Adams and Ek (1974) constructed a nonlinear programming (NLP) problem to determine optimal structure, stocking, and transition harvesting for uneven-aged northern hardwood stands. Tree growth and mortality were expressed as nonlinear functions of initial tree size and total stand basal area at the beginning of each growth period. Problem solutions defined optimal number of trees in each DBH class at the beginning of a growth period such that the value of periodic growth was maximized. The prescribed periodic cut was derived as the number of surplus trees in each size class.

Buongiorno and Michie (1980) later developed a stand management model that used the same plot data (Adams and Ek, 1974) to estimate survival and growth probabilities for each diameter class, and to express growth capabilities in a fixed coefficient matrix. The matrix simply transformed an initial tree list to a new list 5 years later, accounting for tree growth and mortality. By using a matrix of fixed transition probabilities, rather than nonlinear functions, the management model could be constructed as a linear programming (LP) problem. Although LP formulations usually are easier to solve, growth probabilities do not change in response to cutting decisions or subsequent residual stand conditions, thus the usefulness of such models is limited to problems involving narrow fluctuations in residual stocking.

Haight (1987) later described a more general NLP for stand management. Solutions prescribed a sequence of harvests over time to maximize the present value of an existing stand. Example problems again used the Adams and Ek (1974) stand growth model. This dynamic model determined superior solutions for problems that were solved earlier by static methods.While the choice of a silvicultural system for the single stand may coincidentally reflect goals associated with neighboring stands, the single-stand optimization approach does not explicitly recognize important relationships with other stands in a multi-stand or landscape management problem. The model described in this paper extends the scope of the model of Haight (1987) to address problems for which attributes from individual stands contribute to achieving management goals at a multi-stand scale.

FORPLAN, the USDA Forest Service's linear programming model (Johnson, 1986; Johnson et al., 1986), does provide a decision tool that allocates homogeneous land strata among predefined management prescriptions. However, the predefinition of prescriptions is performed exogenously and may be suboptimal under a given set of objectives and stand or forest-level constraints. Further, FORPLAN prescriptions do not track stand-level attributes such as basal area, diameter distribution, and species compo-

sition which have a direct bearing on many concerns related to ecosystem management. In the model reported here, stand-level parameters are tracked explicitly and their optimal levels are determined endogenously.

Model performance was illustrated using a series of selected forest management problems, each designed to demonstrate an important function of the model. Examples were formulated as NLP problems and solved using a projected Lagrangian algorithm (Robinson, 1972). Problems were coded using the general algebraic modeling system (GAMS), a Fortran-based equation generator capable of representing complex models in compact form (Brooke et al., 1992).

2. General management model

A general model is described that can be used to optimize harvest schedules for multi-stand management problems involving a variety of management goals. Problem solutions define the optimal sequence of cutting prescriptions for individual stands over a designated time period. Earlier reports (Haight, 1987; Getz and Haight, 1989) described a model for single-stand management that defines the optimal sequence of harvests for a given planning period. The general model presented here is an extension of their single-stand model for multi-stand, multi-resource management problems.

The objective function maximizes net present value (NPV) of the forest according to

$$\text{Max NPV} = \sum_{i=1}^{\text{MD}} \sum_{j=1}^{J} \sum_{k=1}^{K} \sum_{t=1}^{T} \delta^t p_{i,j,k}(t) h_{i,j,k}(t) - \sum_{t=1}^{T} \sum_{k=1}^{K} \delta^t fc_k(t) \quad (1)$$

where $h_{i,j,k}(t)$ defines the number of trees to harvest per unit area in size class i, species group j, stand k, time period t. The first term in Eq. (1) sums the product of price per tree $p_{i,j,k}(t)$ times number of trees harvested $h_{i,j,k}(t)$, and discounts each harvest revenue to time $t = 0$, where δ is the discount factor $1/(1 + r)$ and r is a positive annual discount rate. Variables MD, J, K and T represent the maximum diameter class, number of species groups, number of stands, and number of time periods, respectively. The second term in Eq. (1) $fc_k(t)$ represents costs associated with each stand discounted to $t = 0$. This general formulation can be expanded to include value of ending inventory, planting costs, and so on. In formulating growth constraints, $x_{i,j,k}(t)$ defines the initial number of trees per unit area in time period t. Stand growth constraints take the general form

$$\mathbf{x}(0) = \mathbf{x}_0 \quad t = 0 \quad (2)$$

$$\mathbf{x}(t+1) = G(\mathbf{x}(t), \mathbf{h}(t)) + F(\mathbf{x}(t), \mathbf{h}(t))$$
$$t = 0, 1, 2, \cdots, T, \quad (3)$$

where the current stand structure $\mathbf{x}(0)$ is given as $\mathbf{x}_0$ in Eq. (2). In subsequent time periods, the number of trees $\mathbf{x}(t+1)$ is defined by stand growth given in Eq. (3). The first term in Eq. (3) $G(\mathbf{x}(t), \mathbf{h}(t))$ is a function that estimates growth of the residual stand from the previous period. The residual stand is simply the difference between the initial stand $\mathbf{x}(t)$ and the harvest $\mathbf{h}(t)$. The second term in Eq. (3) $F(\mathbf{x}(t), \mathbf{h}(t))$ is a function that estimates ingrowth into the smallest tree diameter class from the previous period. Note that growth is a function of the initial stand structure and the manner in which it is altered by a harvest decision.

Stand growth models applicable to central hardwood forests can be formulated for use in a NLP of this type designed to optimize stand management (Miller and Sullivan, 1993). Transformed functions from the growth model take the form of Eqs. (2) and (3) and serve to define the dynamics of stand development brought about by harvesting decisions. For example, the general form of the constraint for $x_{46,j,k}(t+5)$, the number of trees in the 46 cm DBH class in species j, stand k, at time $(t+5)$, is given by (4):

$$x_{46,j,k}(t+5) = a_{46,j,k}(t)\left(x_{46,j,k}(t) - h_{46,j,k}(t)\right) + u_{41,j,k}(t)\left(x_{41,j,k}(t) - h_{41,j,k}(t)\right), \quad (4)$$

where $a_{46,j,k}(t)$ and $u_{41,j,k}(t)$ are transition probabilities that residual trees at time t will remain in the 46 cm class or grow into the 46 cm class from the 41 cm class, respectively, during the growth period from t to $t+5$. The transition probabilities also are

functions of the initial and residual stand structure at time t, thus equations defined by Eq. (4) are nonlinear functions of the decision variable $\mathbf{h}(t)$. While FIBER derives $\mathbf{a}(t)$ and $\mathbf{u}(t)$ to form a matrix of transition probabilities in one linear operation and updates $\mathbf{x}(t)$ to $\mathbf{x}(t+5)$ in a second linear operation, the NLP constraints Eq. (4) represent growth as a single nonlinear operation. In large models representing forests composed of varying cover types, growth functions from a variety of simulators can be included to account for individual stand growth.

Structural feasibility constraints are defined by

$$\mathbf{h}(t) \geq 0 \quad \mathbf{x}(t) \geq 0 \qquad t = 0, 1, 2, \cdots, T \tag{5}$$

$$\mathbf{x}_0 - \mathbf{h}(0) \geq 0 \quad t = 0 \tag{6}$$

$$\mathbf{x}(t) - \mathbf{h}(t) \geq 0 \quad t = 0, 1, 2, \cdots, T \tag{7}$$

$$M(\mathbf{x}(t), \mathbf{h}(t)) \geq 0 \quad t = 0, 1, 2, \cdots, T, \tag{8}$$

where Eq. (5) assures that all initial stands and harvests are nonnegative, Eq. (6) assures that the initial harvest does not exceed stand stocking at $t = 0$, and Eq. (7) assures that harvests do not exceed the initial stand in any time period t. Management goals Eq. (8), quantified in terms of constraints on stand structure that affect aesthetics, habitat, diversity, or other attributes, can be added to further define feasibility. Note that the general formulation can be used to model problems of varying scale in the landscape depending on the stands $k = 1$ to K that are included. Stands retain their identity and size throughout the planning period and are not subdivided as part of the problem solution. As a result, the area represented by the entire problem is determined by the sum of all individual stand areas. Similarly, the area affected by a particular constraint is determined by the stands included in that constraint.

3. Example problems

Three example problems are presented to demonstrate: (1) how the model tracks stand structure over time, (2) how to formulate single-stand problems involving constraints on residual stand structure or length of cutting cycle, and (3) how to formulate nonseparable, multi-stand problems involving constraints on aggregate attributes.

3.1. Example 1: Tracking stand structure

The first example problem projected stand growth, with harvests constrained to equal zero over a 10 year and 20 year period. Projections made by the NLP model were compared to observed stand development and to projections made directly by FIBER software. The purpose of these comparisons was to verify that regression equations from FIBER were formulated properly in the NLP model, and to assess the applicability of FIBER equations for use in central Appalachian hardwood stands.

Data were from a 5.1 ha, mixed-hardwood stand on site index 64 (average total height of codominant trees equals 20 m at base age 50) for northern red oak on the Fernow Experimental Forest near Parsons, WV. A 100% inventory was taken in 1964 when the stand was 55 years old and again in 1974 and 1984, allowing comparison of actual stands with 10 and 20 year growth projections from both FIBER and the NLP model.

The NLP for this example contained the objective function Eq. (1), stand constraints to define the initial stand structure Eq. (2), growth constraints Eq. (3), nonnegativity constraints Eqs. (4)–(6) and management constraints to allow no periodic harvests Eq. (9).

$$\mathbf{h}(t) = 0 \quad t = 0, 1, 2, \cdots, T \tag{9}$$

The solution to this simple NLP problem defines the values for $\mathbf{x}(t)$, the initial number of trees in each species and size class, for each time period from $t = 0$ to $t = 20$ that satisfy the growth dynamics derived from FIBER.

Basal area projections differed by less than 2% from direct FIBER projections (Table 1). Projected stand basal area differed from observed stand basal area by less than 6% at 10 years and by less than 8% at 20 years. Direct FIBER projections were obtained from a modified algorithm that provided a 5 cm stand structure needed for this comparison (Marquis, 1990). Discrepancies in stand structure projections are due to conversions from 2.5 cm stand structures generated by FIBER to 5 cm stand structures. Results indicated that the NLP model was adapted adequately from FIBER to allow for more meaningful analyses of alternative management strategies.

This example also demonstrates how the model

Table 1
Stand-growth projections (trees/ha) for a 55 year old, mixed-hardwood stand on the Fernow Experimental Forest

DBH (cm)	Initial	10 year			20 year		
		actual	NLP	FIBER	actual	NLP	FIBER
15	190.5	141.1	130.2	105.5	112.4	100.3	75.6
20	95.4	99.1	123.6	132.2	77.8	113.2	113.4
25	51.2	62.5	77.1	75.4	63.5	93.2	101.8
30	25.2	35.3	41.3	40.8	44.0	59.3	57.8
36	19.0	22.5	24.2	23.2	32.1	34.3	31.6
41	15.1	18.8	17.5	17.3	22.2	21.5	20.0
46	11.4	12.1	13.3	14.3	17.3	15.6	15.3
51	7.7	8.2	9.6	9.6	8.9	11.4	11.4
56	4.4	6.7	5.9	5.9	7.4	7.4	7.4
61	4.2	4.2	4.2	4.2	4.9	4.7	4.7
66	3.5	2.7	3.5	3.5	4.2	3.5	3.5
71	0.0	2.5	1.0	1.0	2.7	1.5	1.5
Basal area (m^2/ha)	22.2	24.9	26.4	26.1	28.4	30.6	30.0

tracks stand structure and related attributes over time. The solution output for this example problem defined the number of trees in each tree diameter class at 5 year intervals (Fig. 1). In practical applications in which the no-harvest constraint is relaxed, the output would define both the initial stand structure and the optimal harvest prescription at each time interval. As a result, other stand attributes such as habitat suitability index or species diversity index that are derived from stand structure also can be tracked over time for single-stand or multi-stand problems.

3.2. Example 2: Single-stand problems

This example demonstrates how to formulate separable, single-stand problems. Such problems are described as separable because constraints influencing management decisions on one stand do not influence decisions on other stands. For separable problems, optimal management of a group of stands is simply the sum of optimal management in each stand independent of other stands. Three cases of separable problems were solved.

In the first separable case, later referred to as the basic problem, NPV was maximized over a 100 year planning period subject only to growth and nonnegativity constraints, given the initial stand structure $\mathbf{x}(0) = \mathbf{x}_0$. No management constraints were imposed and harvests were feasible in any time period and at any volume level within the limits of stand growth. The basic problem does not account for many factors encountered in the real world; it is the simplest formulation from which more realistic problems can be constructed.

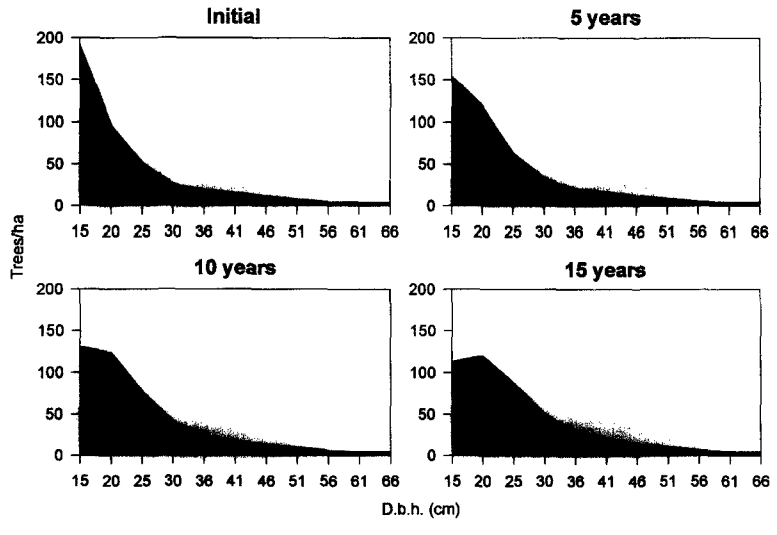

Fig. 1. Projected stand structure at 5 year intervals with no cutting treatments.

Results for the basic problem shed light on logical aspects of the model. The first harvest removed all trees 41 cm DBH and larger, an indication that such trees are financially mature according to the price function defined in the model. Product prices were derived from tree-value conversion standards (DeBald and Dale, 1991) whose values are based in part on grade. Trees 41 cm DBH and larger qualify for the most valuable grade, resulting in a high rate of value increase from 36 to 41 cm DBH, but a comparatively lower rate of value increase beyond 41 cm. The optimal cutting strategy with no management constraints was simply a 41 cm diameter limit harvest every 5 years until the final period, when all merchantable volume was removed. The objective function did not account for periods beyond 100 years, so all remaining potential revenue was taken at that time by harvesting all merchantable trees.

Results for the basic problem were not consistent with existing management guidelines which take into account operational and administrative considerations. For example, interim harvests removed volumes as low as 7.6 m^3/ha (540 bd ft/acre), well below minimum harvests required by timber buyers in most central Appalachian hardwood sawtimber markets. A more practical approach is to lessen the frequency of partial harvests to a 10 or 20 year interval, thus increasing the periodic harvest volume. A more sophisticated price function could be used in the model to account for the effects of total harvest volume, product mix and logging system efficiency.

In the second single-stand case, an operational constraint was added to the basic problem to require harvests only in certain time periods. The harvest variable $\mathbf{h}(t)$ was constrained to equal zero except in 15 year intervals from $t = 0$ through $t = 100$. Reducing the frequency of harvests lowered NPV compared to the basic problem. However, in the absence of other management restrictions the solution also was a 41 cm DBH diameter-limit harvest (Fig. 2). Harvest volume ranged from 42 to 90 m^3/ha (3,000 to 6,400 bd ft/acre) every 15 years until the final period when the remaining merchantable volume was removed.

In the third single-stand case, a constraint was added to the basic problem to improve aesthetics by leaving some large trees in the residual stand. The 'big residual tree' constraint was defined by

$$\sum_{i=36}^{MD} \sum_{j=1}^{J} b_i\left(x_{i,j,k}(t) - h_{i,j,k}(t)\right) \geq 3.2$$

$$t = 0, 1, 2, \cdots, T \qquad (10)$$

where b_i is basal area per tree in size class i. Constraint (10) requires a minimum residual basal area of 3.2 m^2/ha in trees 36 cm DBH and larger. In the solution, the residual basal area constraint was met by retaining some 41 cm trees that were cut in

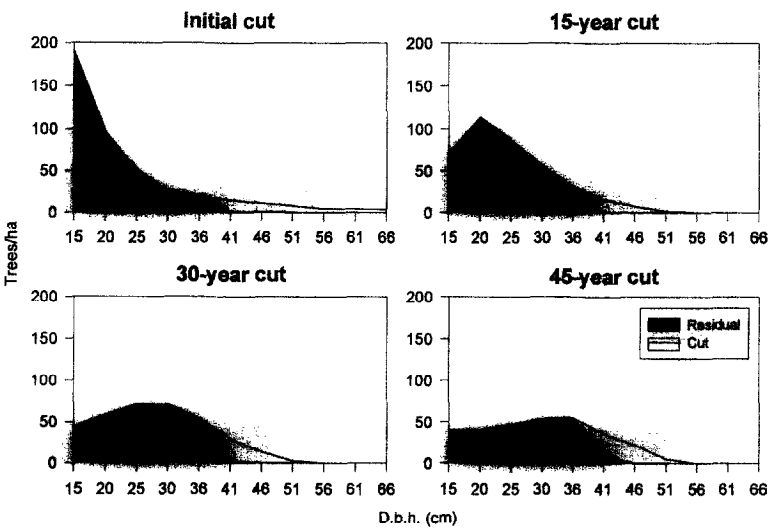

Fig. 2. Number of cut and residual trees per hectare for four 15 year periodic harvests.

the previous unconstrained case. Periodic revenues were reduced, and in turn NPV was reduced compared to the basic problem solution. Other constraints of this kind that control attributes of the residual stand can be used to develop and maintain a wide range of desired stand conditions.

The single-stand problems provided a test of mathematical and economic logic in the general management model. Growth equations derived from the FIBER growth model adequately represented stand dynamics as functions of stand density variables before and after each periodic harvest. The projected Lagrangian algorithm converged and defined the optimal harvest strategy over a 100 year planning period with only growth and nonnegativity constraints in place. Preliminary analyses further demonstrated that constraints placed on residual stand structure to represent specific management goals could be formulated within single-stand problems. Finally, arbitrary administrative constraints to control the frequency or intensity of harvests had the effect of reducing the feasible region, resulting in relatively lower NPV values compared to the unconstrained case, and further demonstrated the capability of the solution algorithm to converge for a variety of problem formulations.

For each case in example 2, a second stand with a different species composition and structure was added and the problems were resolved. In each case, including the second stand did not affect the optimal solution for the original single-stand problem. Although these test cases were simple in structure, the results indicated that management constraints can be applied to particular stands without influencing optimality in companion stands if stand management problems are separable. However, if forest-level constraints are imposed whereby optimal levels of control variables in several stands are interdependent, constraints on one stand may affect optimality on other stands included in the problem, as shown in the following example.

3.3. Example 3: A multi-stand problem

Management problems can be formulated by incorporating appropriate constraints to attain desired attributes in a single stand, a group of stands, or all stands in the management unit. As a result, the management model can be structured to solve multi-stand problems involving nonseparable constraints that link all stands together (as opposed to constraints that affect only a single stand). In this example, management goals require a certain level of diversity among commercial hardwood species to be achieved and maintained over a specified planning period. The problem is simplified to include two species groups (shade-tolerant and shade-intolerant) and two stands managed using partial harvests on a 15 year cutting cycle. Initial stocking in stand 1 was dominated by trees in the intolerant species group, while initial stocking in stand 2 was more evenly distributed between the two species groups (Fig. 3).

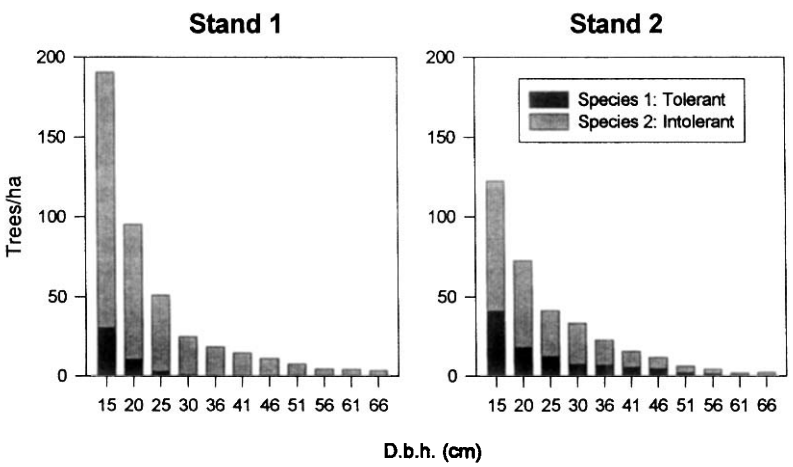

Fig. 3. Initial structure and species composition of two Appalachian hardwood stands.

This is the same management problem as the second case of example 2 described earlier, except that the stands are no longer independent due to the constraint on species diversity. The problem solution defines the optimal sequence of harvests over a 100 year planning period (as measured by NPV) subject to constraints on tree species diversity, a constraint that affects both stands in aggregate.

More practical ecosystem management problems certainly would involve additional species groups and stands, and management constraints could affect forest attributes other than species diversity. For example, specific management constraints can be written to maintain vertical structure for aesthetics for one group of stands along a visually sensitive road and/or to maintain a shade strip for another group of stands along a trout stream. The user can define the scale and pattern of stands in a problem by specifying the appropriate stand indices when constraints are written. However, this example is intended to demonstrate how multi-stand problems can be formulated and solved using the general optimization model described.

Diversity of commercial tree species among two stands was defined using the Shannon–Wiener information index value (Hunter, 1990)

$$\mathrm{DV}(t) = -\sum_{j=1}^{J} \theta_j(t) \ln(\theta_j(t))$$

$$t = 0, 1, 2, \cdots, T, \qquad (11)$$

where, $\theta_j(t)$, the proportion of trees in a species group, is defined as

$$\theta_j(t) = \sum_{i=1}^{\mathrm{MD}} \sum_{k=1}^{K} \left(x_{i,j,k}(t) - h_{i,j,k}(t)\right) \Big/ \sum_{i=1}^{\mathrm{MD}} \sum_{j=1}^{J} \sum_{k=1}^{K} \left(x_{i,j,k}(t) - h_{i,j,k}(t)\right) \qquad (12)$$

Eqs. (11) and (12) compute the aggregate diversity index based on the combination of residual stand structures in both stands. A constraint can then be imposed on DV(t) that requires a minimum level of aggregate diversity specified by the user, yet does not constrain diversity in any one stand.

The multi-stand problem was first solved without constraints on DV(t) to provide a basis for determining how such constraints affect NPV, and to provide good starting values for variables in subsequent constrained versions of the problem. Without constraints on diversity, periodic harvests result in an increase in DV(t) until time period $t = 8$ (40 years), followed by a steady decline for the remainder of the planning period (Fig. 4). Early harvests remove a greater proportion of the higher value, intolerant species to maximize NPV. As the species become distributed more evenly in the residual stands, DV(t) increases. With additional periodic harvests, the proportion of trees in the tolerant species group steadily increases and diversity of the multi-stand unit declines beyond the third periodic harvest (Fig. 4). Without constraints on diversity, the problem solution was identical to that observed in example 2.

The multi-stand problem was then solved with the constraint Eq. (13)

$$\mathrm{DV}(t) \geq d \quad t = 9, 10, 11, \cdots, T \qquad (13)$$

for values of d ranging from 75 to 100% of maximum diversity. This constraint requires a minimum level of species diversity in both stands combined for time periods beyond $t = 8$. However, no constraints were placed on diversity within the individual stands. Solutions were compared to determine how constraints on diversity affect cutting strategies and economic efficiency. In general, NPV decreased as the minimum required diversity increased (Fig. 5). Requiring maximum diversity reduced NPV by \$74/ha in this example.

Actual multi-stand problems are much more complex than the example presented. Definitions of habitat for a preferred wildlife species may involve many attributes such as species composition, age or size distribution, and spatial arrangement. Research is

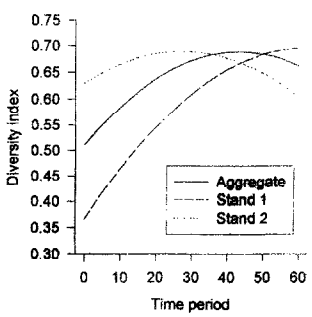

Fig. 4. Diversity index based on woody species in two stands managed by partial harvests every 15 years.

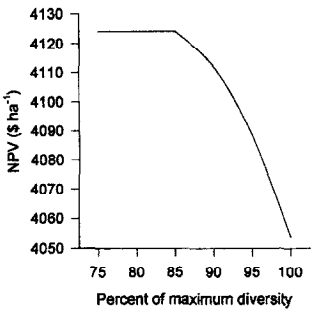

Fig. 5. Net present value of multi-stand management strategies for different levels of required diversity.

needed to integrate significant attributes of stands that represent important management goals. For example, indices of biological diversity have been used as objective functions (Hof and Raphael, 1993), yet there is considerable debate about the appropriate measure of diversity or its validity as a management objective (Hunter, 1990).

4. Discussion

The multi-stand management model described prescribes optimal stand treatments in terms of the number of trees per unit area to harvest in each stand over a specified planning horizon. The objective function maximizes net present value of all harvests, subject to user-supplied growth and management constraints defined in terms of initial and residual stand structure. Due to the structure of the stand growth model used (Solomon et al., 1987), harvest decisions may occur at intervals of 5 years within a stand included in the analysis. Problem size depends on number of tree size classes, species groups, stands, and time periods. In general the model is designed to analyze multi-stand problems, though stand-level problems can be solved by formulating the special instance in which only one stand is included.

A relatively simple example problem was used to demonstrate how attributes of individual stands can be linked, by way of specific management constraints, to form multi-stand problems. The example was based on maintaining a given level of species diversity for two species groups and two stands, though more species and stands could be added to model real-world problems. The general model allows constraints to be placed on the structure and species composition of individual stands for each time period, thus providing the user with considerable flexibility to model complex problems.

Problem solutions define a detailed harvest strategy over time, including marking guidelines for individual stands, volume estimates based on factors supplied by the user, and other outputs which can be expressed as a function of initial and harvest stand structures. The multi-stand model can be used to evaluate the impact of management constraints for a stand, a group of stands, or a forest. This is accomplished by evaluating the reduced cost or dual values associated with the binding constraints in individual stands. The user also can focus on a particular resource constraint to quantify tradeoffs associated with management goals, thus providing a basis for setting or adjusting resource priorities.

Spatial and temporal allocation of treatments among stands can be an important concern for some multi-stand problems. Techniques for scheduling activities on adjacent stands (Jones et al., 1991) and for optimizing arrangement of stands within a single time period (Hof and Joyce, 1992, 1993) have been advanced by recent research. The method for formulating multi-stand problems described in this paper does not preclude the inclusion of spatial allocation constraints to model more complex problems. Additional research is needed to integrate control of stand structure attributes with control of spatial and temporal arrangement of stands. Historically, such integration of features results in problems of unwieldy dimension, and more efficient models must be developed to reduce the number of variables and constraints required to represent such problems (Jones et al., 1991).

In multi-stand problems, optimal harvest schedules can be used to plan other management activities among various stands (for example, road construction and maintenance) to coordinate with planned harvests. This would facilitate estimating labor requirements and expenditures over time that are not formulated within the model itself. Results also can be used to analyze the effect of forest organization and stand area when establishing individual management units. By comparing solutions from various alternatives, the forest could be subdivided and har-

vest practices assigned such that overall efficiency is improved.

At the stand level, traditional management guidelines may be tested by formulating representative problems and optimizing harvests for user-supplied inputs. This aspect of the model is useful in developing expert system models and updating recommendations for existing artificial intelligence modules. Because the general formulation is based on number of trees per unit area in species groups and size classes, many resource problems (habitat, diversity, etc.) that can be quantified as a function of these units can be investigated.

Key inputs include a stand growth model that accounts for a variety of species groups and site productivity classes. It is preferable to use a growth model that estimates growth as a function of stand density as expressed by initial and residual stand structures, because density dependent models appropriately represent the dynamic nature of a managed forest, particularly when planning periods involve long rotations and/or perpetual cutting cycles. For use in nonlinear programming applications, each model must be converted from its existing format into formulations which conform to general mathematical structures required by solution algorithms. In some cases, conversion may be difficult because estimated prediction equations contained in the growth model software have not been published.

Another key input to the general management model is a reliable system for valuation of outputs so that performance of feasible alternatives can be measured and compared. The price function used for this study was based on the simplifying assumption that harvesting economies and product quality were accounted for in a stumpage price for each tree size and species. In many practical cases, this assumption is valid, particularly when harvest volumes show little fluctuation among individual stands. For instance, cost curves are relatively flat for merchantable harvests from 70 to 210 m^3/ha (5,000 to 15,000 bd ft/acre) using ground skidding equipment, and the resulting stumpage price per unit of volume is roughly constant within this range (Brock et al., 1986).

The model could be made more sophisticated by incorporating harvest cost functions to account for slight differences in average product size, terrain and skidding and hauling distance in valuing outputs. The basic management model would remain intact, but these suggested refinements would improve the reliability of price estimates $\mathbf{p}(t)$ used in the objective function.

For the example problems, the solution time was less than 10 min on a PC-compatible 586/90 MHz processor. In general, solution time increased slightly with the number of constraints included in the problem, but time increased most dramatically as the number of stands increased. In most cases, two or three species groups adequately represent eastern hardwood forest problems. Problems analyzed in this study were small compared to the size limits that affect Fortran-based solution algorithms. GAMS/MINOS can solve problems with 32,767 nonlinear nonzero elements (Brooke et al., 1992). Problems involving up to 12 DBH classes, two species groups, four stands, and 21 time periods were about one-tenth of the maximum size for the systems used. Solution time on a 586/90 MHz processor for such problems was approximately 30 min.

Options can be changed between intermediate runs to increase the speed and accuracy of major iterations. GAMS provides a means of saving work files to facilitate the evaluation of intermediate solutions. In addition, GAMS/MINOS options statements can be used to increase the speed of the optimization by adjusting feasibility, optimality, and linesearch tolerances to reduce the number of iterations.

Optimization models that are based on individual DBH classes and species groups provide great latitude in the kinds of resource problems that can be analyzed. Additional study is needed to reduce the size of such problems without losing flexibility. Bare and Opalach (1988) used a Weibull distribution function to represent stand structures, and reduced the decision space to 2 variables for each species group and stand combination.

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