Modifying Taper-Derived Merchantable Height Estimates to Account for Tree Characteristics

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Abstract.—The U.S. Department of Agriculture Forest Service Northeastern Forest Inventory and Analysis program (NE-FIA) is developing regionwide tree-taper equations. Unlike most previous work on modeling tree form, this effort necessarily includes a wide array of tree species. For some species, branching patterns can produce undesirable tree form that reduces the merchantable portion of the stem and merchantable heights to a fixed top diameter (e.g., 4 inches) are often unrealized. Thus, height estimates from a taper model tend to overestimate the actual merchantable length of the stem. This phenomenon is exacerbated as tree size increases. The extent of this problem is illustrated by comparing taper-derived merchantable height estimates to observed merchantable height measurements on trees from NE-FIA sample plots. Models were developed using individual tree attributes to adjust the taper-based estimates to account for reductions in merchantable height due to tree form.

Introduction

The volume of the merchantable portion of a tree is one of the primary characteristics of interest obtained from a forest inventory. These volumes often are derived from height measurements at specified top diameter limits (e.g., 4 inches). Sometimes to improve inventory efficiency, these heights are estimated from height prediction equations (Ek *et al.* 1984) or tree taper models (Max and Burkhart 1976). These methods are satisfactory for species with growth tendencies toward relatively straight boles from ground to tip. Bias in merchantable height (and subsequent volume) predictions, however, occur for many species because tree form can result in merchantable heights that are lower than the point where the specified diameter limit occurs.

For instance, taper equations are developed from paired height/diameter data obtained at various points along the tree bole. These data are obtained without regard to rules for determining the portion of the tree that may actually be used at a processing facility (e.g., minimum log length). Thus, assuming that the bole contains usable wood from the base to the model-predicted merchantable top height may be erroneous. For some species, the merchantable portion of the stem can often end at a point lower than where the top-diameter limit occurs. This occurrence is especially true for most hardwood species, where deliquescent form can produce relatively large decreases in diameter over a short distance. In these situations, the use of a taper-based system requires both model development and implementation strategies.

Data

The data used for this research was obtained from U.S. Department of Agriculture (USDA) Forest Service Northeastern Forest Inventory and Analysis (NE-FIA) sample plots. Although NE-FIA collects a wealth of data at sample plots, the important measurements for this research are individual tree attributes. These variables include tree species, diameter at breast height (d.b.h.), total height, and bole height for trees 5.0 inches d.b.h. and larger. Bole height is defined as the first of (1) the point beyond which no 4-foot-long section can be produced because of excessive limbs, forks, or crooks, (2) a 4-inch top diameter, or (3) the point where the central stem terminates by branching before reaching 4-inch diameter (U.S. Department of Agriculture 2004). For this study, only data from sugar maple, red pine, and eastern white pine trees 5.0 inches d.b.h. and larger were used (table 1).

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Table 1.—*Summary of tree data from NE-FIA sample plots for sugar maple, eastern white pine, and red pine.*

Species/characteristic	N	Min.	Mean (standard deviation)	Max.
Sugar maple				
d.b.h. (in)	11,590	5.1	9.4 (3.7)	25
Bole height (ft)	11,590	4	36 (14.9)	99
Total height (ft)	11,590	17	59 (15.0)	125
Eastern white pine				
d.b.h. (in)	6,985	5.1	10.1 (4.3)	25
Bole height (ft)	6,985	4	37 (18.0)	99
Total height (ft)	6,985	12	52 (17.6)	125
Red pine				
d.b.h. (in)	905	5.1	9.0 (3.0)	22.2
Bole height (ft)	905	7	36 (18.0)	82
Total height (ft)	905	14	49 (17.2)	92

Additional data were available from an NE-FIA tree taper research project. In this regionwide study, measurements include tree diameter at 1, 2, 3, 4.5, and 6 feet of height. Additional diameter measurements are taken above 6 feet at approximate 1-inch taper decrements thereafter until a final measurement is obtained at tree tip (total height). These data were also subsetted to include only the three species listed in table 2.

Analysis

One primary purpose of the NE-FIA taper research project is to eliminate the necessity for field crews to observe merchantable height attributes on sample trees. This elimination will not only improve data collection efficiency, but also result in better consistency over time and greater analytical flexibility. Merchantable height predictions from taper models, however, require a diameter limit to be specified. This necessity presents some difficulty for bole height prediction because the NE-FIA

Table 2.—*Summary of tree data from NE-FIA taper project for sugar maple, eastern white pine, and red pine.*

Species/characteristic	N	Min.	Mean (standard deviation)	Max.
Sugar maple				
d.b.h. (in)	66	3.1	10.9 (4.0)	29.9
Total height (ft)	66	25.2	58.7 (17.1)	90.2
No. of measurements	66	8	14 (4.0)	22
Eastern white pine				
d.b.h. (in)	63	3.1	12.3 (7.3)	36.3
Total height (ft)	63	18.1	59.1 (20.8)	102.2
No. of measurements	63	8	15 (5.2)	35
Red pine				
d.b.h. (in)	20	4.8	11.8 (5.5)	18.7
Total height (ft)	20	18.3	55.9 (16.1)	80.6
No. of measurements	20	9	16 (3.7)	23

data collection protocols incorporate limitations other than top diameter (e.g., minimum log length). Thus, bole height observations often are taken below the point where the 4-inch top limit occurs. This phenomenon can be illustrated by comparing observed bole height data to taper-derived heights at the 4-inch diameter limit.

For the purposes of this article, the model development strategy is simplified by adopting the segmented polynomial taper equation presented by Max and Burkhart (1976). The sugar maple, red pine, and eastern white pine data from the NE-FIA taper project were used to fit the following model for each species:

$$\frac{d^2}{d.b.h.^2} = \beta_1\left(\frac{h}{H} - 1\right) + \beta_2\left(\frac{h^2}{H^2} - 1\right) + \beta_3\left(\alpha_1 - \frac{h}{H}\right)^2 I_1 + \beta_4\left(\alpha_2 - \frac{h}{H}\right)^2 I_2 + \varepsilon \qquad (1)$$

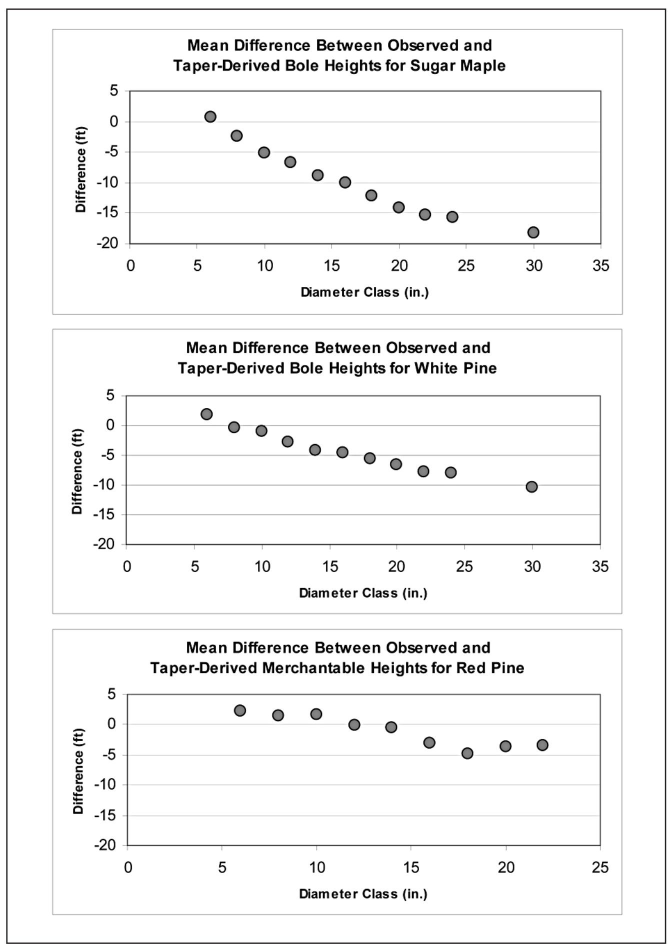
where:
d = diameter outside bark (in).
d.b.h. = diameter at breast height (in).
h = height (ft) at diameter d.
H = total tree height (ft).
I_1 = indicator (= 1 if $\alpha_1 \geq h/H$; = 0 if $\alpha_1 < h/H$).
I_2 = indicator (= 1 if $\alpha_2 \geq h/H$; = 0 if $\alpha_2 < h/H$).
α_1, α_2 = segment join points (estimated from data).
β_{1-4} = parameters to be estimated from data.
ε = random deviation.

This model was chosen because it has been found to have better overall performance than many other taper models (Cao *et al.* 1980, Martin 1981). The model fit each species well, with R^2 values of 0.92, 0.93, and 0.97 for sugar maple, eastern white pine, and red pine, respectively.

These fitted models were used to obtain predicted height to 4-inch top diameter for each tree in the NE-FIA inventory data. These three species were chosen to represent different "levels" of tree form. It was expected that the deliquescent tree form of sugar maple would result in observed bole height values occurring much lower than the 4-inch top limit. Conversely, the straight, single-stem growth tendency of red pine was expected to have bole height measures that were reasonably close to the point of 4-inches top. Eastern white pine was selected as a tree species that would have a form intermediate to the other two species.

For each species, the differences between the observed bole heights and heights predicted from the taper model were summarized by a 2-inch diameter class (fig. 1). Clearly, the differences between the observed and predicted heights become larger as tree d.b.h. increases. As surmised, the differences were greatest for sugar maple and smallest for red pine. When compared to observed data, the use of the taper-derived bole heights in conjunction with NE-FIA volume equations (Scott 1981) produce an overall increase in total volume of 10.6, 3.5, and 0.1 percent for sugar maple, eastern white pine, and red pine, respectively. These increases indicate that the development of taper models alone may not be sufficient for switching from observed field data to model-predicted values. Additional work is required to account for the effects of tree characteristics on predicted bole heights.

Figure 1.—*Mean differences between observed and taper-derived bole heights by diameter class for sugar maple, eastern white pine, and red pine.*



One approach was to model the differences between observed and predicted heights and apply these differences to the predicted values. Predicting the differences using individual tree attributes would allow for adjustments on an individual tree basis. Data were randomly split equally by species for modeling and validation purposes. To describe the differences for sugar maple trees, the following nonlinear model was fitted using least-squares regression techniques:

$$D = \hat{a}_5 DBH^{\hat{a}_6} \times \exp\left(\hat{a}_7 DBH + \hat{a}_8 HTCR\right) + \varepsilon \qquad (2)$$

where:
D = difference (ft).
HTCR = height to crown (ft).
$\beta_{5\text{-}8}$ = parameters to be estimated from data.
Other variables as previously defined.

Model (2) did not perform well for prediction of differences in eastern white pine. A linear model was found to work best for the white pine data. In this formulation, significant predictor variables were d.b.h., crown ratio (CR), and height to crown:

$$D = \hat{a}_9 + \hat{a}_{10} DBH + \hat{a}_{11} CR + \hat{a}_{12} HTCR + \varepsilon \qquad (3)$$

where:
CR = Crown ratio (%).
$\beta_{9\text{-}12}$ = parameters to be estimated from data.
Other variables as previously defined.

A linear model approach also worked well for red pine. In this case, however, different predictor variables were statistically significant. CR and HTCR were no longer important independent variables. Total tree height was not a significant variable for difference prediction for eastern white pine but was useful for prediction for red pine trees:

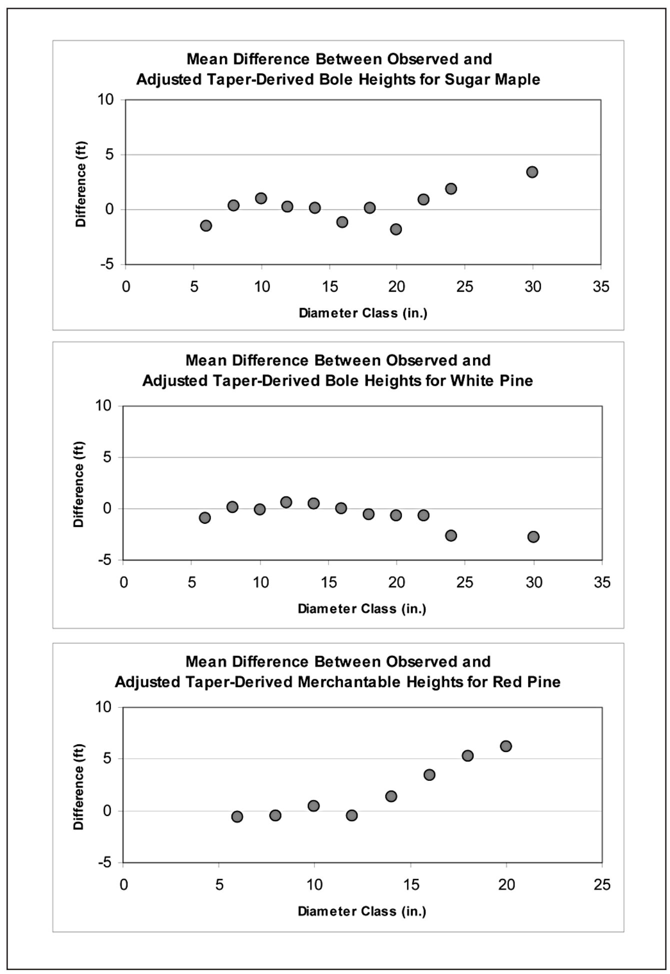
$$D = \hat{a}_{13} + \hat{a}_{14} DBH + \hat{a}_{15} H + \varepsilon \qquad (4)$$

where:
$\beta_{13\text{–}15}$ = parameters to be estimated from data.
Other variables as previously defined.

For each model (2–4), examination of plots of residuals (observed minus predicted) versus predicted values and residuals versus model predictors indicated no systematic problem with model specification or heteroscedasticity.

As anticipated, the large variability in the data was reflected in the fit statistics. R^2 values were approximately 0.30 for all three models. Model standard errors were 6.2, 4.9, and 3.9 for sugar maple, eastern white pine, and red pine, respectively. The application of the models to the validation data significantly affected the differences between observed and taper-derived bole heights (fig. 2). Improved agreement between predicted and observed bole heights was noted for both sugar maple and eastern white pine. For red pine, better agreement was obtained for the smaller

Figure 2.—*Mean differences between observed and model-adjusted taper-derived bole heights by diameter class for sugar maple, eastern white pine, and red pine.*



diameter classes, but agreement was generally poorer for the larger diameter classes. An investigation into this behavior revealed that differences between observed and taper-derived bole heights for the larger few diameter classes were greater in the fit data than in the validation data. In addition, relatively few observations exist in the larger diameter classes. These "distant" points can have a significant influence on the slope of the regression line. When applied to the validation data, the result was overprediction of the difference for larger diameter classes that produced adjusted bole heights that were too small.

The differences between the volumes computed from observed data and those from adjusted taper-derived heights were reduced significantly for sugar maple and white pine. Due to the issue mentioned above for red pine, the differences increased. The volume differences were 0.3, – 0.4, and 1.0 percent for sugar maple, eastern white pine, and red pine, respectively. For sugar maple, the volume difference between observed data and taper-derived bole heights was statistically significant ($p < 0.0001$) when evaluated using a paired t-test, but the difference using adjusted heights was not significantly different ($p = 0.3758$). Results of the eastern white pine analysis show that the use of adjusted heights reduced the difference by nearly 90 percent, although the difference from observed data was still significant from a statistical standpoint ($p = 0.0009$). As expected, the bole characteristics of red pine resulted in nonsignificant volume differences computed from observed and taper-derived bole heights ($p = 0.7899$). The use of adjusted heights had the undesirable effect of producing a volume estimate that was different from the observed data ($p = 0.0117$). Table 3 summarizes results for all three species.

Discussion/Conclusion

For two of the three species studied, a modeling approach to adjusting the taper-derived heights was effective in terms of obtaining close agreement between volume estimates. The model forms and predictor variables that provided the best predictive ability, however, were not consistent across the three species studied. Species-specific models that differ in form and content are difficult to implement in inventories in which many species are encountered. Limiting the number of models may be possible, however, by creating groups of species with similar form characteristics. This approach requires further investigation.

Another issue raised by the analysis is whether an adjustment equation is needed for all species. For red pine, only a 0.1-percent difference between volumes from observed data and volumes based on heights predicted from the taper model (without adjustment) existed, although an offsetting trend occurred between positive and negative values. One could argue that this discrepancy is not large enough to warrant application of an adjustment model. In our analysis, the use of the adjustment model did not have the anticipated effect, and results were poorer. At this point, how to determine the necessity for application of an adjustment model to taper-derived merchantable heights is unclear.

Taper-based merchantable height estimates for certain species should account for tree form characteristics if merchantability is not defined solely by a top diameter limit. Taper-derived merchantable height estimates can be modified to account for individual tree characteristics. In this study, variables assumed to be well-correlated with tree form characteristics (e.g., branching)

Table 3.—*Evaluation of differences in volume between observed data and taper-derived bole heights, and between observed data and adjusted taper-derived bole heights for sugar maple, eastern white pine, and red pine.*

		Taper-derived bole height				Adjusted taper-derived bole height			
Species	**N**	**Mean difference (ft^3)**	**Standard error**	**t**	**p-value**	**Mean difference (ft^3)**	**Standard error**	**t**	**p-value**
Sugar maple	5,795	682	19.8	34.44	< 0.0001	11	12.3	0.89	0.3758
Eastern white pine	3,491	244	11.4	21.37	< 0.0001	– 29	8.6	– 3.32	0.0009
Red pine	452	– 4	16.6	– 0.27	0.7899	– 52	20.5	– 2.53	0.0117

were used as predictors to model differences between observed and taper-derived bole heights. Appropriate model forms and/or significant predictor variables, however, appeared to be species specific. In addition, the need to apply an adjustment model may be questionable for some species. Other approaches to modifying taper-derived merchantable heights include average reductions by diameter class (or some other tree characteristic), indirect manipulation through increases in estimates of cull, and incorporation of modifiers directly within the taper equation. More research is needed to determine a modification method that is both accurate and easy to implement.

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