Northeastern FIA Tree Taper Study: Current Status and Future Work

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Abstract.—The northeastern unit of the Forest Inventory and Analysis program (NE-FIA) is engaged in an ongoing project to develop regionwide tree taper equations. Sampling intensity is based on NE-FIA plot data and is stratified by species, diameter class, and height class. To date, modeling research has been aimed largely at evaluating existing model forms (and hybrids thereof) and incorporating mixed-effects parameters to account for correlations among measurements. In conjunction with the taper study, bark thickness estimates are being developed from wood utilization studies. When fully implemented, the bark thickness/taper equation system will provide a wide range of analytical flexibility for tree species in northeastern forests, and may reduce or eliminate the costs of collecting data on merchantable heights.

Introduction

To compute merchantable volume of standing trees, the northeastern unit of the U.S. Department of Agriculture (USDA) Forest Service Forest Inventory and Analysis program (NE-FIA) has traditionally taken field measurements of tree height at certain diameter limits. Often, these measurements lack repeatability due to difficulties in observing the bole in upper portions of the tree and determining the point at which the diameter limit occurs. To improve data quality and increase fieldwork efficiency, NE-FIA is developing regionwide taper models. A parallel effort is underway for estimating the bark thickness of northeastern tree species. Prediction of bark thickness will increase analytical flexibility by enabling computation of inside-bark diameters. These models will be applicable across the 13 States in which NE-FIA collects resource inventory data.

Tree Taper Sample Development

The sampling list was developed from inventory data obtained on NE-FIA sample plots to represent the range of geography and tree size. The primary goal was to sample 150 trees in each of 18 species groups for a total sample of 2,700 trees. The species groups arise from species assignments used by NE-FIA for tree volume estimates.

Information on frequency of occurrence was tabulated and stratified by species group, tree species, diameter class, and height class (table 1). This stratification indicated that the sampling intensity for a specific species/diameter class/height class combination (S/D/H) could not be based on frequency alone due to the dominance of certain S/D/H combinations within some species groups. To spread the sample more evenly among species and tree sizes, a limit of six sample trees was imposed for any S/D/H arrangement. Conversely, it would be undesirable to devote the necessary resources to sample relatively rare S/D/H combinations. Thus, to be included in the sample, S/D/H combinations must have at least five observed trees across all NE-FIA sample plots. S/D/H combinations with at least five observations but comprising less than 0.1 percent of a species group are limited to a sample size of one.

Table 1.—*Tree diameter (inches) and height (feet) classes used to stratify tree sample.*

Tree d.b.h.	d.b.h. class	Tree height	Height class
3.0”–4.9”	1	0.0’–29.9’	1
5.0”–8.9”	2	30.0’–49.9’	2
9.0”–12.9”	3	50.0’–69.9’	3
13.0”–16.9”	4	70.0’–89.9’	4
17.0”–23.9”	5	90.0’ +	5
24.0” +	6		

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An example of the development of the sample list for red pine is given in table 2. The $Sample_0$ column indicates the S/D/H combinations found in the data and the original sample size based on frequency information alone. The numbers in this column are determined from the values in the percent of Group column, which includes frequency percentages for each S/D/H combination. Using the first row as an example, this S/D/H combination comprised 4.82 percent of the trees in the species group (i.e., 4.82 of every 100 trees in the group were in this category). However, we wanted to determine the number of trees in this category that should be sampled among the 150 sample trees for the entire species group. This is indicated in the percent of 150 column, which shows that the sample size should be 7.23 percent of the 150 sample trees. The result is 10.8 (11 in the $Sample_0$ column). The remaining values in $Sample_0$ column also were computed in this manner.

The limit of six sample trees per S/D/H combination is imposed in $Sample_1$ column (note that the total number of trees is reduced from 41 to 30). The $Sample_2$ column shows the reallocation of trees into cells with fewer than six sample trees and comprising more than 0.1 percent of the species group (i.e., an increase of two sample trees for each eligible S/D/H combination). The remaining sample tree to be accounted for is placed into the most common S/D/H category with fewer than six sample trees (column $Sample_3$), and the overall total is reconciled at 41 sample trees. This approach to reallocation of sample trees maintains the original sample size while permitting sampling of most S/D/H combinations found on NE-FIA inventory plots. Sample lists for each State were created from the overall sample list.

Data Collection

During the 2002-03 leaf-off season, tree taper data were collected in Ohio, Maryland, Pennsylvania, and West Virginia. Tree form was measured with a Barr & Stroud dendrometer. Paired height/diameter data were obtained at 1, 2, 3, 4.5, 6 feet, and at taper intervals of about 1 inch thereafter. A measurement also was taken at the base of the live crown. Additional data were collected for each sample tree (d.b.h., crown ratio, crown class, etc.) and plot-level characteristics (slope, aspect, etc.) were noted. In all, 267 sample trees were measured; yellow poplar was the most common species.

Efforts to collect taper data have been greatly expanded during the 2003-04 leaf-off season, with collection occurring in all 13 States under NE-FIA auspices. Cooperators in this effort include Ohio State University, State University of New York College of Environmental Science and Forestry, and Maine Forest Service. Additional data were obtained from studies by the USDA Forest Service Eastern Region (Region 9).

Data on bark thickness were obtained primarily from wood-utilization studies conducted by NE-FIA; Region 9 also contributed information. All of these data are from studies of felled trees. Protocols have differed over time and between studies, but there are measures of bark thickness for most trees from 1-foot stump height to a 4-inch top diameter limit.

Table 2.—*Development of sample list for red pine from frequency information and abundance limitations.*

D class	H class	Percent of group	Percent of 150	$Sample_0$	$Sample_1$	$Sample_2$	$Sample_3$
3	4	4.82	7.23	11	6	6	6
3	3	4.35	6.52	10	6	6	6
2	2	3.31	4.97	8	6	6	6
2	3	1.21	1.81	3	3	5	6
4	4	0.72	1.08	2	2	4	4
4	3	0.61	0.92	2	2	4	4
1	1	0.19	0.29	1	1	3	3
5	4	0.13	0.19	1	1	3	3
1	2	0.07	0.11	1	1	1	1
3	2	0.06	0.09	1	1	1	1
2	1	0.04	0.06	1	1	1	1
			Count =	**41**	**30**	**40**	**41**

Taper Modeling

Data on yellow poplar collected during 2002-03 were used to initiate the taper modeling process. Initial analyses consisted of comparing existing taper equations. Max and Burkhart (1976) presented a segmented polynomial model with estimated join points. Each segment was specified as representing the neiloid, parabolic, or conic sections of a tree. This is consistent with the approach taken by many other researchers, i.e., the lower portion of the bole is similar to a neiloid; the middle section is parabolic in shape; and the top section generally is conic:

$$d^2/DBH^2 = \beta_1(h/H - 1) + \beta_2(h^2/H^2 - 1) + \beta_3(\alpha_1 - h/H)^2 I_1 + \beta_4(\alpha_2 - h/H)^2 I_2 + \varepsilon \qquad (1)$$

where: d = diameter outside bark (in)
DBH = diameter at breast height (in)
h = height (ft) at diameter d
H = total tree height
I_1 = indicator (= 1 if $_1 \geq h/H$; = 0 if $_1 < h/H$)
I_2 = indicator (= 1 if $_2 \geq h/H$; = 0 if $_2 < h/H$)
α_1, α_2 = segment join points (estimated from data)
$\beta_{1\text{-}4}$ = parameters to be estimated from data
ε = random deviation

Kozak (1988) eliminated the necessity for specifying different functions for various parts of the stem by developing a variable-exponent taper equation. This approach allows the exponent to change with relative tree height, which allows a single function to describe neiloid, paraboloid, and conic forms:

$$d = \beta_5 DBH^{\beta_6} \times \beta_7^{DBH} \times X^{\beta_8 Z^2 + \beta_9 \ln(Z+0.001) + \beta_{10}\sqrt{Z} + \beta_{11} e^{Z} + \beta_{12}(DBH/H)} + \varepsilon \qquad (2)$$

where: $X = \left(1 - \sqrt{h/H}\right) \times \left(1 - \sqrt{p}\right)$
p = percentage of total height where change from neiloid to paraboloid occurs
Z = h/H
ln = natural logarithm
e = base of natural logarithm
$\beta_{5\text{-}12}$ = parameters to be estimated from data
other variables as previously defined

Valentine and Gregoire (2001) described a taper model in which numerical switching functions are used to smooth the transition between the neilod, parabolic, and conic forms. The model is similar to that Max and Burkhart (1976) in that the three classic shape descriptors provide the basis for the model. Rather than being estimated from the data, their join points were fixed at 4.5 feet and height to live crown. To account for repeated measures on individual trees, 2 random-effects parameters were specified in one of the switching functions:

$$A_d = A_{DBH}\left(\frac{H-h}{H-4.5}\right)^{\alpha_1+S_1} \times \left(\frac{H-h}{H-C}\right)^{\alpha_2 \times S_2} + \varepsilon \qquad (3)$$

where: A_d = cross-sectional area (ft^2) at diameter d
A_{DBH} = cross-sectional area (ft^2) at diameter at breast height
C = height to base of live crown (ft)
α_1 = estimated shape parameter of the middle segment
α_2 = estimated shape parameter of the top segment
S_1 = numerical switch exhibiting switch-off behavior
S_2 = numerical switch exhibiting switch-on behavior
other variables as previously defined

When the Max and Burkhart (hereafter MB) and Kozak models were developed, there was no practical means by which correlations among measurements on individual trees could be accounted for. However, advances in statistical theory and computing capabilities now allow researchers to account for this lack of independence when fitting equations. With respect to correlated observations, one approach is specifying a mixed-effects model, as was done by Valentine and Gregoire (hereafter VG). To make valid comparisons among models, the MB and Kozak equations were modified by incorporating random-effects parameters. Random components were added to the estimated join points in the MB equation, allowing the join points to vary among trees. The Kozak model also was altered to incorporate random effects into parameters associated with tree size.

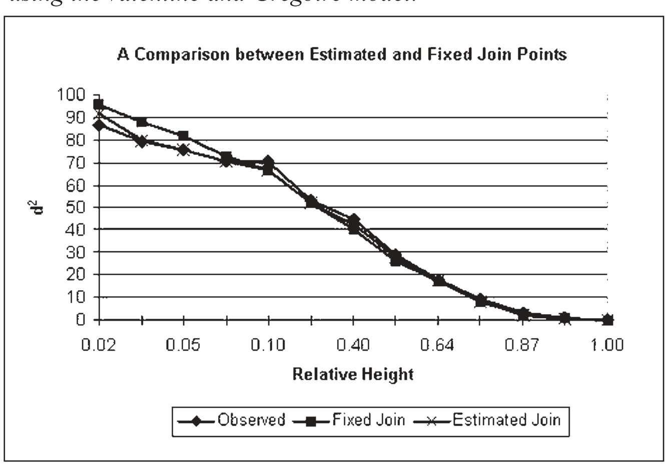
The ability of these three models to describe bole shape was evaluated by fitting each of the equations to taper data from 34 yellow poplar trees. For comparisons among models, each was modified to produce diameter outside bark squared (d^2) as the dependent variable. The models were fitted using the SAS NLMIXED (Version 8.01) procedure. The efficacy of each model was measured by Akaike's Information Criteria (AIC) (see Gregoire *et al.* 1995). A smaller AIC value indicates a better model fit:

Model	AIC
Max and Burkhart	4,153.0
Valentine and Gregoire	4,254.5
Kozak	4,324.3

The results indicate that the MB model outperforms the other models in predicting tree taper for yellow poplar. On the basis of this limited analysis, two observations can be made. First, both the MB and VG models have smaller AIC values than the Kozak model. This implies that specifying the neiloid, paraboloid, and conic terms in the model provides better predictions of tree taper than the variable-exponent approach. The primary difference between the MB and VG models is that the former utilizes estimated join points while the latter has fixed values. The smaller AIC for the MB specification suggests that estimating join points provides a better fit to the data.

To test this assumption, the VG model was generalized to have estimated join points. This was accomplished by recasting the model to use relative rather than actual tree height and replacing fixed join points at 4.5 feet and crown height with parameters. The fitted regression produced an AIC of 4,041.2, which was a notable improvement over 4,254.5 obtained from the original model. This specification also surpassed the MB model in minimizing AIC. The primary gain in predictive accuracy is found in the lower section of the bole (fig. 1). This is particularly important if the taper equation is used to derive tree volume because a relatively large percentage of the volume occurs in this area. Both the estimated and fixed join point models performed similarly above 0.10 relative tree height.

Figure 1.—*Comparison between estimated and fixed join points using the Valentine and Gregoire model.*



An additional investigation was undertaken to determine whether moving the random-effects parameters to another location in the VG model would improve the fit. The original specification by VG placed the random effects in the S_1 switch. The improvement in fit statistics for the VG model obtained by estimating join points led to the supposition that moving the random effects into the estimated join points could result in further improvements in AIC. Fitting of this specification produced an AIC statistic of 3,962.2, a reduction of 2.0 percent from the previous formulation and 6.9 percent from the original model.

The work thus far provides evidence that a segmented model with estimated join points provides the best description of the shape of the bole. Also, it appears that specification of random effects in the join points produces better fit to the data than other formulations, and it is thought that the use of switching functions improves model fit, though additional evaluation is needed. These findings are based on limited analyses of a single tree species. Research on the applicability of these results to other tree species and species groups is warranted.

Bark Thickness Estimation

Estimates of bark thickness are needed to obtain diameter inside bark (dib) and diameter outside bark (dob) for volume estimation. In most previous work on bark thickness, an average dib/dob ratio has been applied (Martin 1981) or the ratio was predicted as a function of tree size (Hilt 1985).

To date, data on bark thickness indicate that for many species, the dib/dob ratio depends on d.b.h. and height along the bole. Further, the dependence of dib/dob ratio on d.b.h. and height along bole can be described adequately by a linear model. The following model was fitted to tree species (as opposed to groups) for which there were a minimum of 30 observations of bark thickness:

$$\mathrm{dib}/\mathrm{dob} = \beta_{13} + \beta_{14}\mathrm{DBH} + \beta_{15}h + \varepsilon \qquad (4)$$

where: dib = inside-bark diameter (in.) at height *h*
dob = outside-bark diameter (in.) at height *h*
$\beta_{13\text{-}15}$ = parameters to be estimated from data
other variables as previously defined

However, because the ratios for certain species show considerable variability (e.g., white ash), it is difficult to justify a modeling approach (table 3). For these species, application of an average ratio may be sufficient. Predicted dib/dob ratios for 3 d.b.h. sizes of slippery elm are shown in figure 2. As expected, the ratio increases as tree size increases due to bark thickness occupying a relatively smaller portion of the overall diameter.

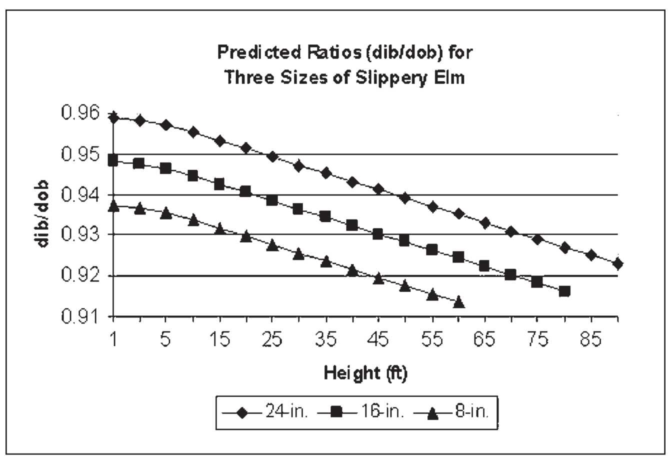
Future Work

Taper and bark thickness data still are being collected. As additional data become available, expanded analyses will be possible. The application of taper modeling results presented in this article to other species needs to be addressed, and the need to model bark thickness vs. applying an average value requires additional study. If bark thickness models are developed, mixed-effects parameters should be used to account for correlations among observations. These analyses should allow for determination of the best approaches for modeling tree taper and estimating bark thickness by comparisons of fit statistics and validation using independent data. When data collection is completed, the estimates of model parameters and other necessary statistics (e.g., average dib/dob ratios) can be finalized. When fully implemented, the bark thickness/taper equation system will provide a wide range of analytical flexibility for tree species in northeastern forests, and may reduce or eliminate the costs of collecting data on merchantable heights.

Table 3.—*Nonsignificant variables and adjusted R^2 for equation [4] fit to various species (minimum of 30 observations).*

Species	Nonsignificant	Adj. R^2
American basswood		0.399
Bigtooth aspen	h	0.086
Bitternut hickory	d.b.h.	0.179
Black cherry		0.192
Chestnut oak		0.544
Cucumbertree	d.b.h.	0.331
Eastern white pine	d.b.h.	0.111
Northern red oak	d.b.h.	0.119
Pignut hickory		0.474
Red maple		0.315
Scarlet oak		0.442
Slippery elm		0.315
Sugar maple		0.237
Sweet birch		0.200
White ash	d.b.h., h	0.024
White oak	d.b.h.	0.097
Yellow-poplar		0.199

Figure 2.—*Predicted dib/dob ratios for three different sizes of slippery elm.*



Literature Cited

Gregoire, T.G.; Schabenberger, O.; Barrett, J.P. 1995. Linear modeling of irregularly spaced, unbalanced, longitudinal data from permanent-plot measurements. Canadian Journal of Forest Research. 25: 137–156.

Hilt, D.E. 1985. OAKSIM: an individual-tree growth and yield simulator for managed, even-aged, upland oak stands. U.S. Department of Agriculture Forest Service Research Paper NE-562. 21 p.

Kozak, A. 1988. A variable-exponent taper equation. Canadian Journal of Forest Research. 18: 1363–1368.

Martin, A.J. 1981. Taper and volume equations for selected Appalachian hardwood species. U.S. Department of Agriculture Forest Service Research Paper NE-490. 22 p.

Max, T.A.; Burkhart, H.E. 1976. Segmented polynomial regression applied to taper equations. Forest Science. 22(3): 283–289.

Valentine, H.T.; Gregoire, T.G. 2001. A switching model of bole taper. Canadian Journal of Forest Research. 31: 1400–1409.