Application of the Urban Mixing-depth Concept to Air-pollution Problems

by PETER W. SUMMERS, *chief, Atmospheric Dispersion Division, Atmospheric Environment Service, Environment Canada, Toronto, Canada.*

ABSTRACT.—A simple urban mixing-depth model is used to develop an indicator of downtown pollution concentrations based on emission strength, rural temperature lapse rate, wind speed, city heat input, and city size. It is shown that the mean annual downtown suspended particulate levels in Canadian cities are proportional to the fifth root of the population. The implications of this and other results of the model are discussed.

A LARGE AND INCREASING proportion of the population of North America lives in large cities. Thus by far the largest time-integrated exposure of people to air pollution occurs in urban areas. In spite of improved pollution-control technology, the continued growth of cities and especially of vehicular traffic suggests that there will be no near-term decrease in this exposure. To make rational choices between various land-use and pollution-control strategies, decision-makers require informaton about how the dstribution of emissions and meteorological factors combine to affect urban air-pollution concentrations.

This information can be generated in many ways and can be provided in a wide array of formats. For example, numerical modelling techniques can simulate the ground-level pollution concentrations over the whole city for any given set of source configurations and several typical sets of meteorological conditions. Whilst such results have many valuable uses, they require very detailed source-emission inventories and meteorological data on space and time scales that are usually not available. On the other hand, a simple approach, which gives the pollution concentration in the centre of a city in terms of the major causative factors, also is valuable for urban planners.

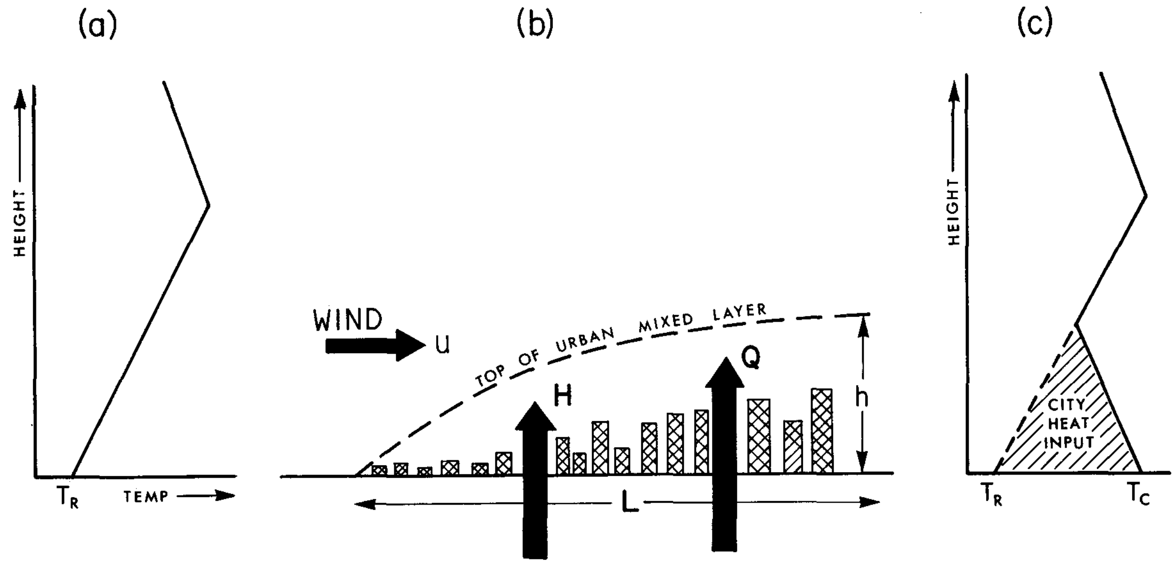
In this paper the simple urban heat-island model developed by Summers *(1965)*, using the mixing-depth concept, will be extended to develop an urban pollution indicator in terms of several easily understood and measurable parameters. Some of the implications of this indicator will be discussed—especially in relation to city size and energy use.

THE HEAT-ISLAND MODEL

The phenomenon of the "urban heat island," where night-time surface temperatures on clear nights with light winds are considerably higher in urban areas than in surrounding countryside, has been observed for over 100 years and has been documented extensively in the last 50 years. The most recent comprehensive review on the subject is by T. R. Oke *(1974)*. However, a physical model that could satisfactorily explain the general features of the heat island was not developed until relatively recently *(Summers 1965)*.

The basic concept of this model is shown schematically in figure 1. On clear nights with light winds (<5 m sec^{-1}), a surface-based temperature inversion usually develops in rural areas (that is, the temperature increases with height above the ground). The difference between this vertical temperature profile and the dry adiabatic lapse rate

Figure 1.—Schematic representation of the development of the urban mixed layer. (a) The vertical temperature profile upwind of the city with a surface-based inversion. (b) The development of the mixed layer due to urban heat input as air moves from left to right. (c) The vertical temperature profile modified by the city to produce a ground-based unstable layer.



(a temperature decrease of 1°C per 100 m of height, which normally occurs in the lower atmosphere) is defined as α. The stronger the inversion in the rural air, the greater the value of α. As rural air moves in over the city, it is heated from below in two ways.

1. The release of sensible heat stored in the concrete and asphalt fabric of the city from the solar heating of the previous day.
2. The release of waste heat to the atmosphere as a result of the space heating of homes and commercial buildings, industrial activities, and to a lesser extent automobiles.

Source 1 is stronger in the summer, but in the high latitudes of Canada is almost negligible in winter. Source 2 is small in summer, but is the stronger in mid-winter when its strength is proportional to heating degree-days.

Heating of air, combined with increased mechanical turbulence induced by the buildings, produces a layer of well-mixed air over the city, with a temperature lapse rate between the dry adiabatic and isothermal. As the air moves across the city, more and more heat is added, thus causing a steady increase in the height of the mixed layer. As shown by Summers *(1965)* the following relationships can be developed.

$$T_{CITY} - T_{RURAL} = \left[\frac{2\,HL\alpha}{\rho C_P u}\right]^{1/2} \qquad [1]$$

$$\text{mixing depth } (h) = \left[\frac{2\,HL}{\rho C_P u_a}\right]^{1/2} \qquad [2]$$

in which:

T_{RURAL} = Temperature in the upwind rural area (°C)

T_{CITY} = Temperature in the downtown core (°C)

L = Distance from the edge of the city to the centre along the wind direction (m)

H = Heat input per unit area (cal $m^{-2}\,sec^{-1}$)

u = Wind speed (m sec^{-1})

ρ = Density of air (gm m^{-3})

C_P = Specific heat of air at constant pressure (cal gm^{-1} per °C)

α = Difference between the upwind temperature profile and the lapse rate over the city (°C m^{-1})

THE POLLUTION MODEL

If the urban area acts as a uniform area source of pollutants that are uniformly mixed in the mixing layer, then the concentration of pollution in the city centre is given by

$$\chi_{CITY} = \frac{QL}{uH} = Q\left[\frac{\rho C_P \alpha L}{2uH}\right]^{1/2} \quad [3]$$

in which:

χ_{CITY} = Pollution concentration in the city (gm m^{-3})

Q = Pollutant input per unit area (gm m^{-2} sec^{-1})

RELATION BETWEEN POLLUTION AND CITY SIZE

For most urban areas that are nearly circular in shape, the population (P) is given by

$$P = \text{const. x } L^a$$

If the population density is uniform, a = 2. However, in most cities the population density is higher close to the centre due to the high density of older buildings, and especially high-rise apartments. Thus a $\geqslant$ 2, hence

$$L = \text{const. } P^{1/a} \text{ where } a \geqslant 2$$

Substituting in equation 3

$$\chi_{CITY} = \text{const. x } Q \text{ x} \left[\frac{\rho C_P \alpha}{2uH}\right]^{1/2} \text{x } P^{1/2a} \quad a \geqslant 2 \quad [4]$$

The mean annual average of a pollutant is heavily influenced by the large values, which in turn occur under those meteorological conditions favouring poor ventilation (simultaneous low values of u and h; or, alternatively, simultaneous low values of u and high values of α). On a cross-Canada basis,

Table 1.—National air-pollution surveillance network: suspended particulates in commercial downtown core areas vs population for 1971

City	Population	Suspended particulates
	Thousands	*μg m*$^{-3}$
Fredericton, N.B.	38	36
St. John, N.B.	107	54
Moncton, N.B.	71	54
Montreal	2,743	132
Ottawa—Hull NCR	602	92
Toronto	2,628	99
Hamilton	499	142
Peterborough	64	61
London	286	125
Sarnia	78	105
Windsor	259	122
Sault Ste Marie	81	55
Thunder Bay	112	69
Winnipeg	540	73
Regina	141	57
Saskatoon	126	72
Edmonton	496	60
Calgary	403	105
Red Deer	28	64
Medicine Hat	29	57
Lethbridge	41	41

Source: Environment Canada (1974).

u is almost constant for Canadian cities, and H tends to be highest where winter inversions are strongest. Thus, a/H does not vary much from city to city; and hence as a first approximation

$$\chi \begin{pmatrix} \text{annual} \\ \text{mean} \end{pmatrix} = \text{const. x Q x } P^{1/2a} \quad [5]$$

Apart from a few large industrial sources, particulate emissions in a city are due to a large number of low-level point sources such as home heating units, cars, etc. Assuming that Q is nearly constant from city, then equation 5 reduces to

$$\chi \begin{pmatrix} \text{annual mean} \\ \text{suspended} \\ \text{particulates} \end{pmatrix} = \text{const. } P^{1/2a} \; a \geqslant 2 \quad [6]$$

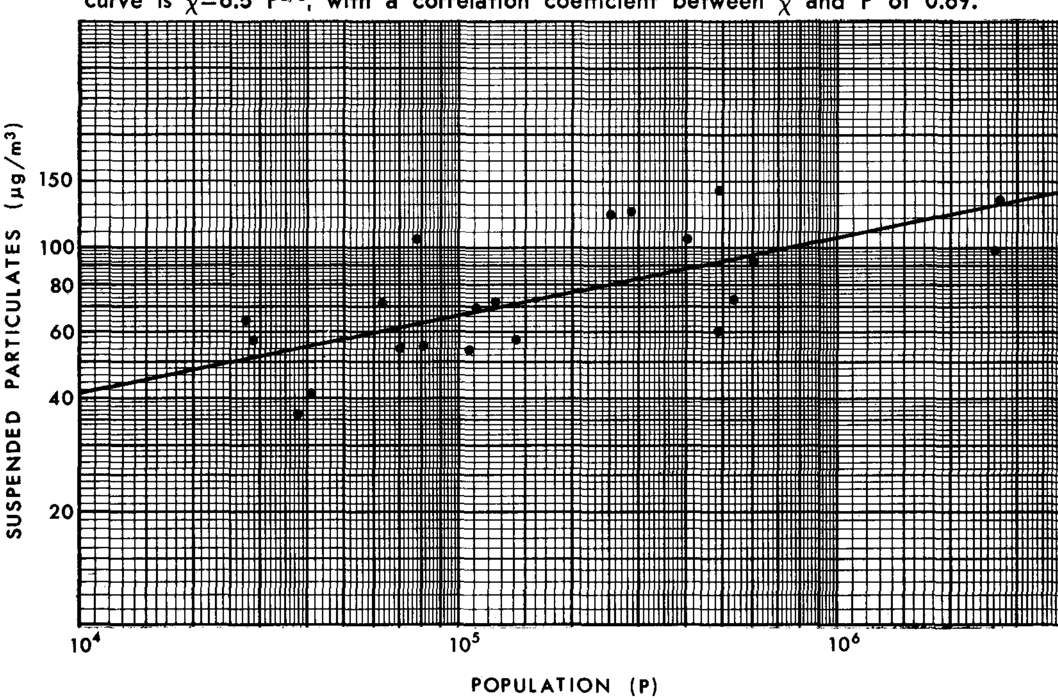
To test equation 6, data from the Canadian Air Pollution Surveillance Network were used (table 1). The population of 21 cities at the time of the last national census in 1971 are listed together with the mean annual suspended particulate levels in the commercial downtown core for the same year *(Environment Canada 1974)*. Note that an effort has been made to locate all these stations in as representative a location as possible away from heavy industry and other large point sources. These data are plotted on log-log paper (fig. 2) and fitted to a power-law curve as shown. The best fit curve is given by

$$\chi = 6.5 \; P^{1/5} \quad [7]$$

with a correlation coefficient between χ and P of 0.69.

Note that the data points above the best-fit line correspond mainly to the heavily industralized cities in southern Ontario. Thus there are probably several lines corresponding to different values of Q (see equation 5), all having a slope of approximately 1/5. It also appears that for Canadian cities a = 2.5.

Figure 2.—Relationship between population of Canadian cities and the mean annual downtown suspended particulate levels in 1971. The best-fit power-law curve is χ=6.5 $P^{1/5}$, with a correlation coefficient between χ and P of 0.69.



SOME IMPLICATIONS OF THE MODEL

$$\chi_{CITY} = \text{const. x Q x} \left[\frac{a}{uH}\right]^{1/2} P^{1/5} \quad [8]$$

This simple indicator shows how various factors combine to produce a measure of the downtown air pollution. Some of these (a and u) are meteorological only. One (P) is dependent on city size only. Others (Q and H) are dependent on both city parameters (mainly population density) and meteorological parameters. For example, one of the components of H, and hence Q, is dependent on heating degree-days.

Several important conclusions can be drawn from this indicator of downtown pollution levels:

1. The main controlling factor is the source strength Q (dependent on population density).
2. Meteorological factors, whilst still important, have less controlling effect due to the square-root relationship.
3. Total population is the least important controlling factor because of the fifth-root relationship.
4. A small town with an average value for Q can have a significant value for χ, but once a city is large (population $>$ 1,000,000), then decreasing the rate of growth will have only a minor impact on χ compared to reducing Q.
5. Increasing the heat input H or decreasing the value of Q, with other factors unchanged, reduces χ. Thus the value of switching to cleaner fuels for home heating: coal → oil → natural gas.
6. Energy conservation could possibly increase χ. If a substantial part of Q were not related to heat production and H were cut back, (due to improved home insulation or other conservation measures) at a greater rate than Q^2, then the value of $Q/H^{1/2}$ could increase, resulting in higher values of χ.

REFERENCES

Environment Canada.
1974. THE CLEAN AIR ACT ANNUAL REPORT 1973-1974. Environ. Can. Inf. Can. Cat. En 41-1974. 34 p.

Oke, T. R.
1974. REVIEW OF URBAN CLIMATOLOGY 1968-1973. World Meteorol. Organ. Note 134. 132 p.

Summers, P. W.
1965. AN URBAN HEAT ISLAND MODEL ITS ROLE IN AIR POLLUTION PROBLEMS, WITH APPLICATIONS TO MONTREAL. Can. Conf. Micrometeorology Proc. 1: 435-436. Toronto.