DEVELOPMENT OF A STAND DENSITY INDEX EQUATION FOR SLASH PINE STANDS

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ABSTRACT.—Stand density index (SDI) is commonly used as the basis for density management guides for even-aged forest stands. Many tree species follow the same self-thinning trajectory, allowing for the use of stand density index in such guides. Slash pine (*Pinus elliottii* Englem.) has been shown to depart from the self-thinning trajectory exhibited by other tree species. However, slash pine stands do follow a self-thinning trajectory. A stand density index equation for slash pine is herein developed. Although different from the form generally accepted for other species, the new equation can still be used to estimate relative densities of varying slash pine stands, and it may potentially serve as the basis for a stand density index-based management guide for slash pine plantations.

L.H. Reineke first introduced his concept of stand density index in the early 1930s. Reineke (1933) reported that even-aged stands of a variety of species followed the same size-density relationship or self-thinning pattern. Once this pattern was quantified, a relative measure of stand density, called stand density index (SDI), was created. Stands with the same SDI are of the same relative density regardless of the individual ages or sizes (quadratic mean diameter) of the stands (see also Avery and Burkhart 2001).

SDI has been used for a variety of purposes for a multitude of species since the original work of Reineke (1933). Most of this work has centered on the creation and subsequent use of density management diagrams. Stand managers can use density management diagrams to determine the relative density of a stand, compare relative densities between stands, and schedule harvesting (partial or final) activities based on the SDI. The goal of SDI-based management is to maintain a given stand at a target density or within the levels (thresholds) of two target densities. For example, SDI can be used to identify the self-thinning threshold and the minimum site occupancy threshold for loblolly pine (see Dean and Baldwin 1993).

Drew and Flewelling (1979) introduced the concept of density management diagrams with their work on Douglas-fir (*Pseudotsuga menziesii* [Mirb.] Franco). McCarter and Long (1986) extended the concept of density management diagrams to lodgepole pine (*Pinus contorta* Dougl. ex Loud.) and incorporated height of dominants and codominants into the guide, allowing for the approximate age of the harvesting activities to be determined as long as the stand's site index is known. SDI-based management research on lodgepole pine continues as evidenced by Whitehead and others (2001) and their work on using SDI-based management to control beetle outbreaks. Newton and Weetman (1994) developed an SDI-based density management tool for black spruce (*Picea mariana* Mill.), and Newton (1998) reported a computerized version for black spruce. Dean and Baldwin (1993) developed a density management diagram for loblolly pine, as did Williams (1994, 1996), who added yield information to the diagrams. Doruska and Nolen (1999) introduced a spreadsheet version of SDI-based density management for loblolly pine plantations; their use of a spreadsheet allowed for thinned and nonthinned stands to grow and develop differently within an SDI-based management scenario.

Reineke (1933) noted that slash pine (*Pinus elliottii* Englem.) and longleaf pine (*Pinus palustris* Mill.) departed from the self-thinning pattern of most other species he examined. Therefore, the slope of the size-density relationship is different for these two species. SDI-based research involving these species has been sparse. Dean and Jokela (1992) developed a management guide for slash pine based on the concept of annual growth potential as opposed to SDI.

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This paper traces the development of an SDI equation for slash pine. The equation developed herein can then be used to compare relative densities of slash pine stands and potentially serve as the basis for density management diagrams or spreadsheets for slash pine plantation management.

THE DATA

A large, anonymous landowner in the Southeastern U.S. granted access to its slash pine inventory data. The data obtained and used were from the southwest Gulf Coastal Plain. A total of 1,638 stand inventories were used, with each stand inventory providing the following information: trees per acre, quadratic mean diameter (in.) and thinning status. Table 1 contains summary statistics of these data taken as a whole (combined data) and separately (nonthinned and thinned data).

METHODS

The following equational form was fit to the nonthinned, thinned, and combined slash pine data sets, respectively:

$$Log_{10}(N_i) = \beta_0 + \beta_1 Log_{10}(D_{qi}) + \varepsilon_i \qquad (1)$$

where: N_i = trees per acre of stand i,
D_{qi}= quadratic mean diameter (in.) of stand i,
β_0 , β_1 are parameters to be estimated, and
å$_i$ is the random error associated with stand i, assumed NORM(0,σ^2).

Following Avery and Burkhart (2001), a stand density index equation can then be created by requiring the equation to yield SDI=N when D_q = 10 in. This is accomplished via the intercept term

$$\beta_0 = Log_{10}(SDI) + \beta_1 \qquad (2)$$

Equation (2) can be substituted into equation (1), to yield the SDI equation of the form

$$Log_{10}(SDI) = Log_{10}(N_i) + \hat{\beta}_1 Log_{10}(D_{qi}) - \hat{\beta}_1 \qquad (3)$$

The fitted slope from equation (1) is used to parameterize equation (3), which subsequently can be used to calculate SDI for a particular slash pine stand, as long as the stand's trees per acre and quadratic mean diameter are known.

Equation (1) can also be used to determine if the slope (self-thinning trajectory) is the same for both thinned and nonthinned slash pine stands by incorporating an indicator variable (I_i)

$$Log_{10}(N_i) = \beta_0 + (\beta_1 + \beta_2 I_i) Log_{10}(D_{qi}) + \varepsilon_i \qquad (4)$$

where: I_i = 1 if stand i has been thinned, 0 otherwise,
β_2 is the parameter to be estimated, and
all other terms as previously described.

If β_2 statistically differs from 0 (α=0.05) then the self-thinning trajectory varies between nonthinned and thinned stand status.

RESULTS AND DISCUSSION

The parameter estimates from fitting equation (1) to the nonthinned, thinned, and combined slash pine data sets, respectively, are shown in table 2. The slopes of all three fits, as evidenced by the p-values in table 2, were significantly different than 0 at α=0.05. The approximate R^2's reported in table 2 were obtained by transforming the predicted values of each fit into the original units of the Y-variable (trees per acre), thus expressing the proportion of variation in trees per acre explained by the independent variables (as opposed to the proportion of variation of Log_{10} [Trees per Acre] explained by the independent variables).

Table 1.—*Summary statistics of the slash pine data*

		Quadratic mean diameter (in.)		Trees per acre	
	n	Mean	Std. dev.	Mean	Std. dev.
Nonthinned stands	95	4.78	0.83	733.78	230.17
Thinned stands	1,543	9.31	2.23	186.90	95.51
Nonthinned and thinned stands combined	1,638	9.05	2.42	218.62	167.28

Table 2.—*Parameter estimates from the fit of equation 1 to the nonthinned, thinned, and combined slash pine data sets*

	Parameter	Estimate	Std. error	p-value	Approx. R^2	MAR[a]
Nonthinned data	β_0	3.71536	0.08836	<0.0001	0.5967	116.65
	β_1	-1.29634	0.13057	<0.0001		
Thinned data	β_0	4.13437	0.01704	<0.0001	0.8704	23.29
	β_1	-2.00530	0.01771	<0.0001		
Combined data	β_0	4.17800	0.01462	<0.0001	0.8913	29.91
	β_1	-2.04817	0.01542	<0.0001		

[a]MAR is the mean absolute residual, the average of absolute values of the errors, expressed in trees per acre.

Figures 1 to 3 depict the fitted regression lines (eq. 1) to the nonthinned, thinned, and combined slash pine data sets, respectively. The slopes of the regression lines depicted in figures 1 to 3 and reported in table 2 are used in the SDI equation (eq. 3). The slope of the self-thinning trajectories of most species as reported by Reineke (1933) and Avery and Burkhart (2001) is –1.605. The null hypotheses of t-tests of the form

$$H_0: \beta_1 = -1.605$$
$$H_1: \beta_1 \neq -1.605$$

were rejected at α=0.05 for the slopes of all three model fits (p-value <0.03 in all cases) as expected. Recall, slash pine had been reported to depart from the self-thinning trajectory of other species.

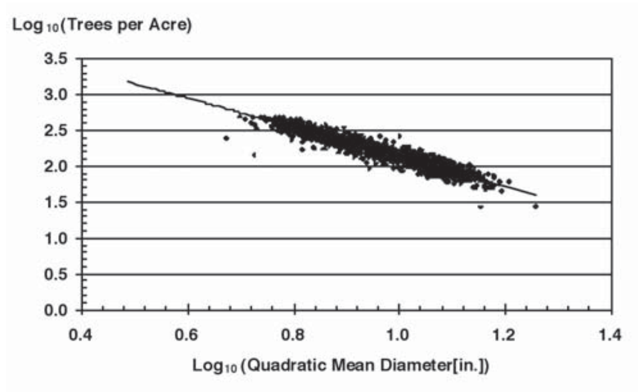

Figure 2.—*Regression fit (line) obtained when fitting equation (1) to the thinned slash pine data (points).*

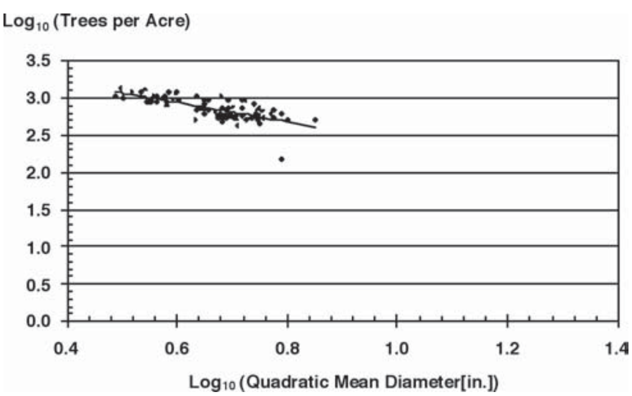

Figure 1.—*Regression fit (line) obtained when fitting equation (1) to the nonthinned slash pine data (points).*

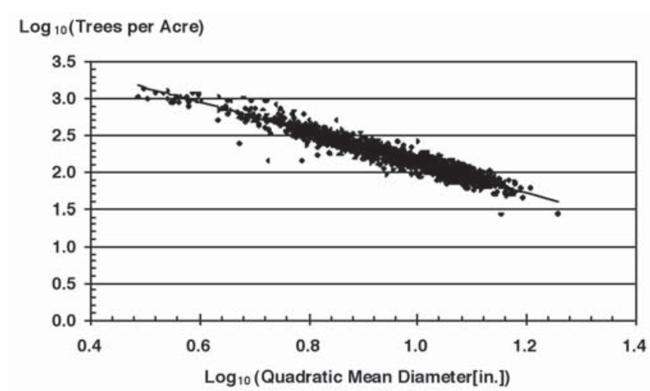

Figure 3.—*Regression fit (line) obtained when fitting equation (1) to the combined slash pine data (points).*

Table 3.—*Parameter estimates from the fit of equation 4 to the combined slash pine data*

	Parameter	Estimate	Std. error	p-value	Approx. R^2	MAR[a]
Combined data	β_0	4.10698	0.01707	<0.0001	0.8970	29.28
	β_1	-1.87117	0.01373	<0.0001		
	β_2	-0.10584	0.02752	<0.0001		

[a] MAR is the mean absolute residual, the average of absolute values of the errors, expressed in trees per acre.

What was unexpected was the apparent difference in the self-thinning trajectories of the nonthinned and thinned slash pine stands. As a result, an indicator variable was introduced into equation (1) to test if the slopes were significantly different between the nonthinned and the thinned data (eq. 4). Parameter estimates from fitting equation (4) to the combined slash pine data set are shown in table 3.

The p-value for the test

$$H_0: \beta_2 = 0$$
$$H_1: \beta_2 \neq 0$$

was <0.0001; therefore, the slope is indeed statistically different between the nonthinned and the thinned slash pine stands. Perhaps this is the result of the management style of this particular landowner.

The landowner that provided the data employs thinning regimes; thus, older, nonthinned stands were not included in the data set. This may mean the nonthinned stands never reached the self-thinning threshold, possibly influencing the slope estimate from that portion of the data. More nonthinned slash pine data will need to be examined to address this issue.

The following SDI equation obtained by fitting equation (1) to the combined slash pine data set is thus cautiously recommended for use until the nonthinned slope can be further evaluated

$$\text{Log}_{10}(\text{SDI}) = \text{Log}_{10}(N_i) - 2.04817\text{Log}_{10}(D_{qi}) + 2.04817 \quad (5)$$

It is hoped that equation (5) will serve as the basis for continued research into SDI for slash pine and eventually lead to the creation of an SDI-based spreadsheet management tool for slash pine similar to the loblolly pine spreadsheet presented in Doruska and Nolen (1999).

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