THE EFFICACY OF USING INVENTORY DATA TO DEVELOP OPTIMAL DIAMETER INCREMENT MODELS

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ABSTRACT.—Most optimal tree diameter growth models have arisen through either the conceptualization of physiological processes or the adaptation of empirical increment models. However, surprisingly little effort has been invested in the melding of these approaches even though it is possible to develop theoretically sound, computationally efficient optimal tree growth models using inventory data. The Potential Relative Increment (PRI) methodology is a good example of a flexible potential growth modeling system developed under these auspices. I present a series of suggestions for ecological consistency, variable and parameter assumptions, statistical properties, data quality, and model flexibility that should be considered when developing optimal increment models, exemplified with white oak (*Quercus alba* L.) equations from the Midsouth region.

The increased utilization of ecological simulators has led to a proliferation of models designed to forecast tree growth. Several general types can be distinguished, ranging from empirical regression models to process-based theoretical constructs or other mathematical designs. The increment model of the Forest Vegetation Simulator (FVS) (Wykoff and others 1982) is a classic example of an empirical design:

$$\Delta D = \sqrt{dib^2 + dds} - dib \qquad (1)$$

Following this formulation, periodic diameter increment (ΔD, in this case, inside bark growth over a 10-yr period) is a function of inside bark diameter at breast height (d.i.b.) and squared inside bark diameter (dds):

$$\ln(dds) = HAB + LOC + b_1 \cos(ASP)SL + b_2 \sin(ASP)SL + b_3 SL + b_4 SL^2 + b_5 EL + b_6 EL^2 + b_7 (CCF/100) + b_8 \ln(DBH) + b_9 CR + b_{10} CR^2 + b_{11}(BAL/100) + b_{12} DBH^2 \qquad (2)$$

where HAB is a habitat type constant, LOC is a location constant, ASP is stand aspect, SL is stand slope ratio, EL is stand elevation, CCF is a crown competition factor, CR is crown ratio, BAL is the total basal area of trees larger than the subject tree, DBH is diameter at breast height (d.b.h.), and b_1 to b_{12} are species-specific regression coefficients. Not every empirical model has this number of variables and coefficients, but all are usually designed to maximize statistical fit to inventory data (often using transformed independent variables). Process-based growth models assume a specific, meaningful, and mechanistic relationship between increment and the independent variable(s). This approach is increasingly popular in ecological models (e.g., Botkin and others 1972, Chertov 1990, Pacala and others 1993) because they attempt to emphasize biological processes rather than just enumerating tree growth. For example, some approximate photosynthesis, like Pacala and others (1993):

$$\Delta D = \frac{g_1 GLI}{g_1/g_2 + GLI}\left(DBH/2\right) \qquad (3)$$

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where g_1 is the asymptotic growth rate of the species under high light conditions, g_2 is the slope at zero light, and GLI is the global light index (a proxy for photosynthetic potential). Most mathematical growth models (e.g., Chapman-Richards, Bertalanffy, Gompertz) associate age and tree size to realized increment. Zeide (1993) decomposed these models into two general forms, the power decline (PD) and exponential decline (ED):

$$PD: \qquad \Delta D = kDBH^{p} a^{q} \tag{4}$$

$$ED: \qquad \Delta D = kDBH^{p} e^{qa} \tag{5}$$

where a is current tree age and k, p, and q are species-specific regression coefficients (k > 0, p > 0, and q < 0). Key features of these models involve the expansion (e.g., DBH^p) and contraction (e.g., a^q or e^{qa}) components, which permit rapid growth when the trees are small and constrain growth as trees age (Zeide 1993).

Two philosophies on growth prediction can also be identified. One concentrates on fitting a response curve through a cloud of data points and parameterizes the resulting model to be sensitive to measured variables. Predicted increment is either adjusted upwards or downwards from the fitted curve to reflect good or poor growing conditions. The second approach defines an upper increment response curve for "ideal" growing conditions and then rescales growth downward to reflect suboptimal performance. Most empirical and mathematical models (e.g., Lessard and others 2001, Wykoff and others 1982, Zeide 1993) typify the first approach, while process models are more characteristic of the second (e.g., Botkin and others 1972, Bragg 2001a, Hahn and Leary 1979). Either approach can yield realized increment, yet there are instances when potential growth is more interesting. While not specifically designed for this task, most empirical or mathematical designs have (under the correct circumstances) a set of conditions that would produce an increment prediction analogous to optimal growth. However, their adaptation for potential increment modeling is an inefficient solution to a problem better addressed through a hybridized, inventory-based system.

The Potential Relative Increment (PRI) methodology shares many of the attributes of mathematical and process-based growth models while retaining strong empirical roots (Bragg 2001*a*). The PRI equation can be generalized as:

$$\Delta D_{OPT} = DBH \left(b_1 DBH^{b_2} b_3^{DBH} \right) \tag{6}$$

where b_1, b_2, and b_3 are species-specific non-linear regression coefficients. Note that only d.b.h. is needed to predict the potential increment (ΔD_{OPT}) of the species, which differs from other strategies that embed environmental factor(s) directly in the model (e.g., Botkin and others 1972, Pacala and others 1993, Wykoff and others 1982) or assume that tree age is known (Zeide 1993). Using the PRI methodology as an example, this paper considers the efficacy of adapting large-scale inventories to create optimal increment models by providing suggestions to assist in model development and describing the challenges and potential of inventory-based approaches.

RULES OF THUMB FOR OPTIMAL INCREMENT MODELS

As with any statistical model, a number of rules of thumb should be followed to ensure theoretical robustness and statistical reliability (and, concurrently, user confidence). The order of this list is not meant to impart any ranking of importance, but rather is intended to track the logical flow of events when developing an optimal diameter increment model.

1. An optimal increment model must be consistent with ecological theory, including reasonable assumptions about the variables and parameters involved.

In general, an ecological growth model should fit basic expectations of biological behavior. For instance, an optimal growth model should predict neither infinite nor negative growth. The first optimal growth models often included compromises between computational efficiency and ecological relevance. The resulting efforts may have seemed practical at the time, but they have imposed some unfortunate requirements. As an example, the optimal increment equation of the gap models (e.g., Botkin and others 1972, Mielke and others 1978, Shugart and West 1977) distributed potential growth along a sigmoidal function:

$$\Delta D_{OPT} = \frac{G \cdot DBH \left(1 - \dfrac{DBH \cdot HT}{DBH_{\max} \cdot HT_{\max}} \right)}{274 + 3b_2 DBH - 4b_3 DBH^2} \tag{7}$$

where maximum diameter (DBH_{max}, in centimeters) and maximum height (HT_{max}, in centimeters) are estimated from the literature, and the growth parameter (G) equals:

$$G = \frac{4HT_{max}}{AGE_{max}}\left\{\ln\left[2\left(2DBH_{max}-1\right)\right]+\frac{\alpha}{2}\ln\left[\frac{9/4+\alpha/2}{4DBH_{max}^{2}+2\alpha\,DBH_{max}-\alpha}\right] - \frac{\alpha+\alpha^{2}/2}{\sqrt{\alpha^{2}+4\alpha}}\ln\left[\frac{3+\left(\alpha-\sqrt{\alpha^{2}+4\alpha}\right)\left(4DBH_{max}+\alpha+\sqrt{\alpha^{2}+4\alpha}\right)}{3+\left(\alpha+\sqrt{\alpha^{2}+4\alpha}\right)\left(4DBH+\alpha-\sqrt{\alpha^{2}+4\alpha}\right)}\right]\right\} \tag{8}$$

when AGE_{max} is an estimate of species longevity (in years) and $\alpha = 1 - 137/HT_{max}$. Because of some inconsistencies in the calculation of equation 8, Botkin and others (1972) approximated values of G and greatly simplified the derivation of optimal increment. Equation 7 terminates (reaches zero growth and mandatory senescence) when DBH × HT = $DBH_{max} \times HT_{max}$, placing considerable importance on DBH_{max} and HT_{max}. Neither of these parameters is definitively known and may not be maximized under the most favorable growing conditions. As an example, it is not unusual for some species to reach their maximum age in less than ideal environments (e.g., bristlecone pine (*Pinus longaeva* D.K. Bailey) on alpine sites or northern white-cedar (*Thuja occidentalis* L.) on cliff faces (Kelly and others 1992)).

Furthermore, the sigmoid nature of equation 7 produces two-thirds of the tree's growth under optimal conditions by half of the tree's lifespan, thus noticeably underestimating tree growth potential (Bragg 2001a), especially when AGE_{max} is large. Figure 1 contrasts the diameter growth potential for white oak (*Quercus alba* L.), using the gap model FORAR (Mielke and others 1978, assuming G = 100.7, AGE_{max} = 400 years, DBH_{max} = 122 cm, and HT_{max} = 3,470 cm) and the PRI coefficients from Bragg (in preparation a). FORAR appreciably underestimated potential increment, especially for small to moderate diameter trees (fig. 1). The PRI approach eliminates most of the unnecessary ecological limitations of the gap models by providing more realistic curve shapes and not enforcing a maximum tree size.

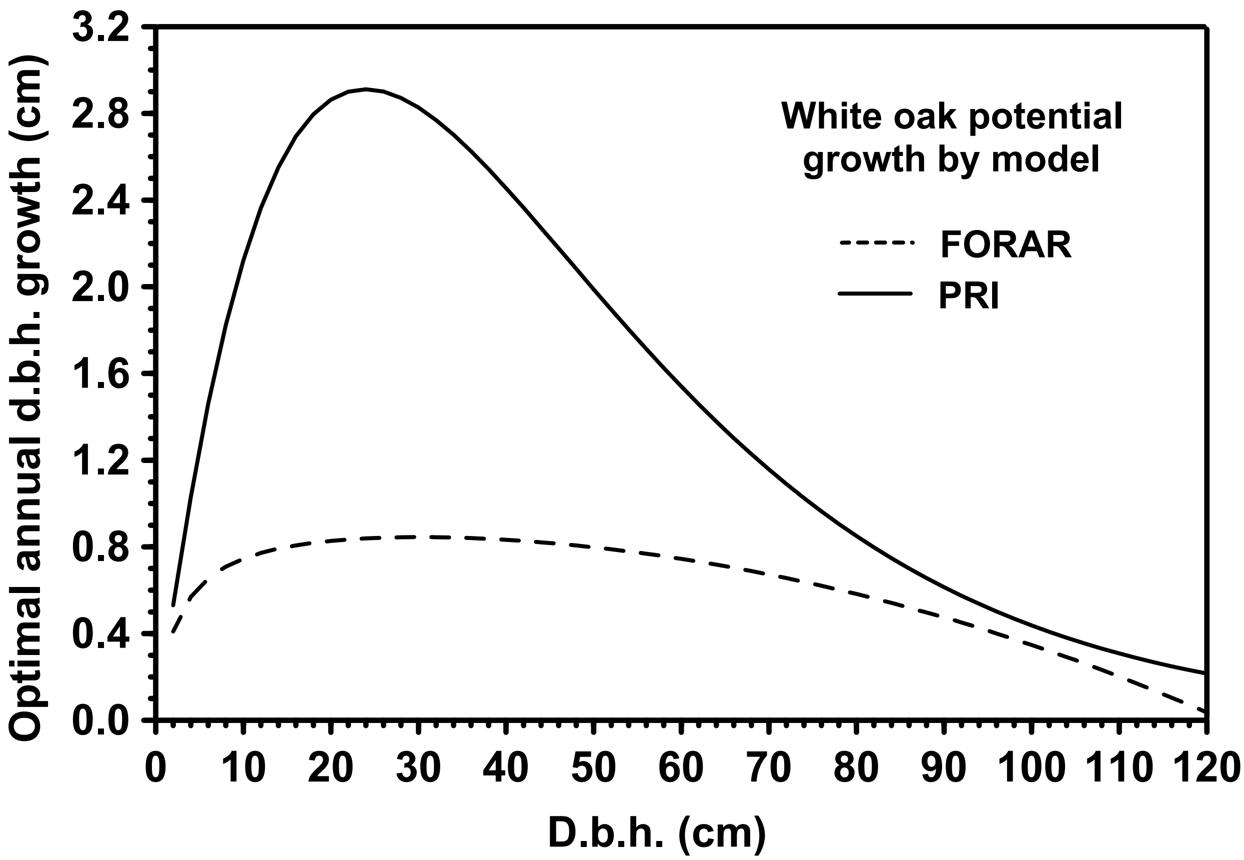

Figure 1.—*Differences in white oak optimal diameter growth predicted by the PRI methodology (solid line) and the gap model equation (dashed line).*

2. An optimal increment model should be statistically robust yet efficient and parsimonious.

Advances in computational ability make the development of large-scale empirical models technically possible, often giving these efforts an advantage over other designs in that hundreds or perhaps thousands of individual observations have been used to fit the equations. This large sample, collected over an extensive geographic distribution, conveys statistical power (even when many parameters are used). Inferring ecological meaning, however, from many coefficients and variables becomes challenging as more and more factors with increasingly obscure relevance are added. More parameters also do not equate to a better model design. Zeide (1991) pointed out that a high-degree polynomial with the same number of parameters as data points would provide an exact fit (fig. 2) yet would prove less biologically relevant than a simpler but noisier model (e.g., a power function) that does not attempt to explain all data variation but can be related to a known process (e.g., mitosis).

Unfortunately, many process-based optimal growth models also suffer from inadequate designs. One major problem is that their developers used very few samples of the relevant populations, thereby limiting their utility. The gap models, for example, use coefficients fit to a handful of trees (i.e., the one(s) contributing AGE_{max}, DBH_{max}, and HT_{max}), while other approaches have been developed from limited field sampling (e.g., Pacala and others 1993), thus constraining extrapolation. Other authors are quick to point out that process-based increment equations rarely (if ever) outperform empirical growth predictions (Fleming 1996, Vanclay 1994). Hence, statistical efficiency and biological relevance can sometimes fail at a critical juncture of increment model: realized performance.

The PRI methodology's compact equation form produces a parsimonious model while still considering large samples of known quality and distribution. PRI equations are directly linked to actual trees in real environments and follow reasonable rules of behavior. For example, tree diameter growth peaks at a relatively early age and then declines over

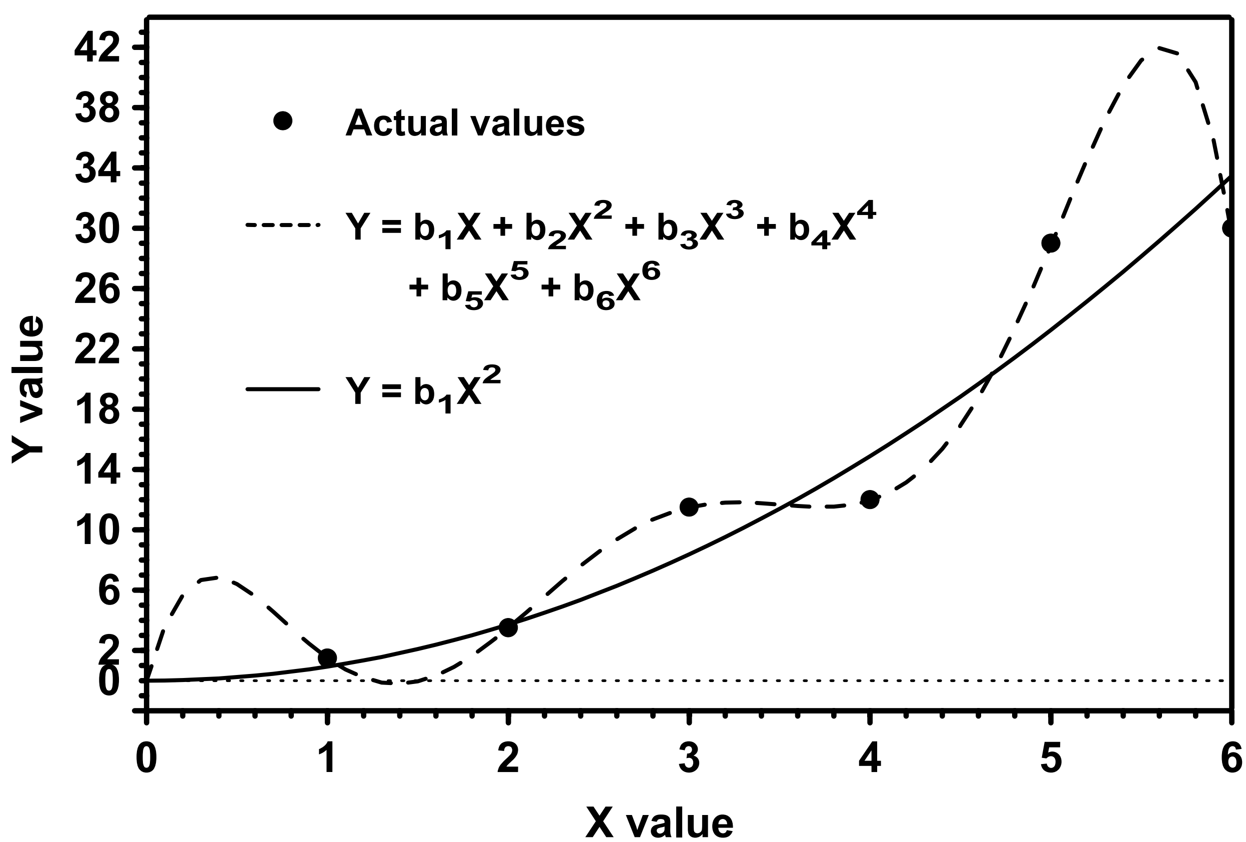

Figure 2.—*Example of Zeide's (1991) assertion that simple models, while usually noisier, can be more appropriate approximations of natural systems than high-order polynomials designed to maximize fit.*

time (Zeide 1993). The information used to develop PRI models, while not as formidable as in some empirical designs, is still considerable (Bragg 2001a). PRI's realized performance also does not hinge solely upon optimal increment model behavior, but rather upon the quality of the modifying functions (see next section).

3. An optimal increment model should be flexible and general in its application.

Many empirical and mathematical growth models incorporate environmental parameters directly into their equations. To illustrate this point, FVS uses up to 9 specific parameters (12 if the squared variables are included) and 12 coefficients to forecast growth. The behavior of each must be simultaneously considered to determine the appropriate combination, so interpreting parameter behavior (especially in the context of optimal growth) can be virtually impossible. As an example, beyond a small improvement in statistical fit, what is the biological significance of squared elevation or slope length? How valid is this model if the user prefers not to apply one or more of these parameters?

Even process-based designs sometimes require users to adopt assumptions they may be uncomfortable with. For instance, unless one is willing to use the global light index imbedded in the growth model of SORTIE, applying equation 3 to predict growth is not possible. Any model that explicitly includes such variable force that factor to become an integral part of the factors affecting growth performance (diameter is directly related to future increment, so it should always be included). Greater flexibility is possible in model designs that separate potential from limitations (e.g., Botkin and others 1972, Bragg 2001a, Hahn and Leary 1979). The PRI methodology considers optimal performance solely from species and current diameter, so any reasonable set of constraining functions could be used to inhibit growth.

4. The data used to develop an optimal increment model should be reliably sampled and broad in scope.

Rarely are inventories so poorly implemented that the quality of the data threatens the interpretation of the results. More likely, data limitations arise from the inadequacy of sampling conditions. Large-scale databases like the Eastwide Forest Inventory Data Base (EFIDB— Hansen and others 1992) are well suited for developing optimal increment models because of their considerable sample extent. For example, table 1 lists the 25 most abundant species in the Midsouth optimal growth models developed by Bragg (in preparation a). The EFIDB inventory systems also have rigorous training, data quality, and accuracy standards that contribute to their reliability. While the EFIDB has supported the PRI effort to date, it is anticipated that any data set (e.g., Miles and others 2001) that includes species, original tree diameter, current tree diameter, and the remeasurement period could be similarly processed.

What makes the PRI approach unique in empirical growth modeling is that the regression coefficients are fit using a restricted subset of the increment data, rather than the entire collection. Only the trees growing the fastest within their respective diameter classes are selected for PRI analysis (fig. 3), and even these are pared down (resulting in some PRI equations being fit with 10 or fewer observations). However, depending on the species and spatial extent of the project,

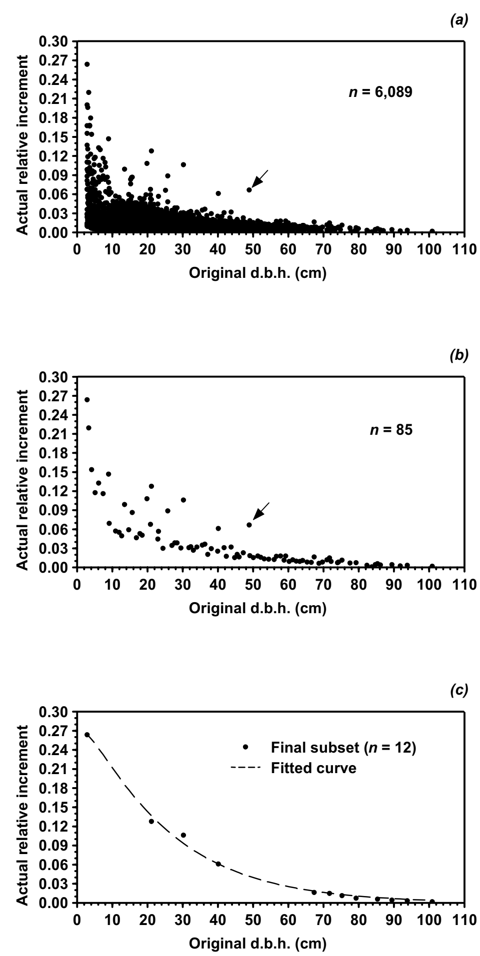

Figure 3.—*PRI model derivation process for white oak in the Midsouth showing the initial data (a), a tentative selection of highest actual relative increment (b), and the final subset of points chosen to fit the equation (c). Note the outlier identified by an arrow in (a) and (b).*

Table 1.—*Twenty-five most abundant species used to derive Midsouth optimal diameter growth models (Bragg 2002a).*

Common name	Scientific name	FIA code	n	Min. d.b.h.	Ave. d.b.h.	Max. d.b.h.	Std. dev.
				centimeters			
Loblolly pine	*Pinus taeda* L.	131	37,672	2.8	24.8	103.4	14.42
Shortleaf pine	*Pinus echinata* Mill.	110	17,114	2.8	25.3	89.7	10.96
Post oak	*Quercus stellata* Wangenh.	835	13,308	2.8	25.2	104.4	14.02
White oak	*Quercus alba* L.	802	13,085	2.8	25.6	101.9	14.60
Sweetgum	*Liquidambar styraciflua* L.	611	12,250	2.8	24.3	111.3	14.59
Black oak	*Quercus velutina* Lam.	837	7,145	2.8	26.3	133.1	15.04
Water oak	*Quercus nigra* L.	827	5,464	2.8	34.8	140.7	20.91
Southern red oak	*Quercus falcata* Michx.	812	5,128	2.8	30.5	138.2	16.22
Blackgum	*Nyssa sylvatica* Marsh.	693	3,942	2.8	22.5	93.0	16.01
Black hickory	*Carya texana* Buckl.	408	3,937	2.8	17.8	70.9	11.01
Baldcypress	*Taxodium distichum* (L.) Rich.	221	3,695	3.3	42.8	250.2	28.44
Northern red oak	*Quercus rubra* L.	833	3,319	2.8	29.2	94.5	14.36
Winged elm	*Ulmus alata* Michx.	971	3,170	2.8	13.6	67.8	10.32
Red maple	*Acer rubrum* L.	316	3,113	2.8	16.1	89.4	12.81
Green ash	*Fraxinus pennsylvancia* Marsh.	544	3,058	2.8	28.1	96.5	17.21
Willow oak	*Quercus phellos* L.	831	2,867	2.8	37.6	149.1	20.30
Mockernut hickory	*Carya tomentosa* Poir. Nutt.	409	2,572	2.8	19.0	85.3	12.84
Sugarberry	*Celtis laevigata* Willd.	461	2,486	2.8	28.5	114.3	14.92
Eastern redcedar	*Juniperus virginiana* L.	68	2,435	2.8	15.2	72.9	9.57
Blackjack oak	*Quercus marilandica* Muenchh.	824	2,304	2.8	20.4	72.1	11.35
Cherrybark oak	*Quercus pagoda* Raf.	813	2,112	2.8	37.7	117.3	20.22
Water tupelo	*Nyssa aquatica* L.	691	2,061	2.8	31.5	95.5	13.09
Overcup oak	*Quercus lyrata* Walt.	822	2,010	2.8	40.4	129.8	19.92
American elm	*Ulmus americana* L.	972	1,734	2.8	24.3	148.6	18.28
American hornbeam	*Carpinus caroliniana* Walt.	391	1,271	2.8	11.8	44.5	7.53

hundreds to thousands of individual growth measurements are first evaluated to produce the final set of high-performing trees. In fact, PRI models must be derived from extensive inventories if the methodology is to remain ecologically and statistically robust (Bragg 2001a, in press a).

CHALLENGES TO THE APPLICATION OF INVENTORY INFORMATION

As can be seen in the preceding paragraphs, every growth model comes with potential and challenges, not the least of which is user confidence. Ecological modelers are often leery of purely empirical designs because of their less than satisfactory mechanisms, yet many also find mathematical or process approaches troubling because of shortcomings in their structure and implementation. New techniques like PRI that blend empirical, mathematical, and process-based principles have considerable promise in the development of optimal growth equations because of their flexibility and robustness. However, as with any system derived from inventory data, the PRI methodology has two areas of concern (missing data and outlier handling) that need to be addressed further.

Missing Data

Regardless of how accurately measurements are taken, errors of omission can be problematic for optimal increment models (Bragg 2001a). Fortunately, while the PRI methodology is sensitive to limited data, the process is designed to conservatively estimate potential increment (Bragg 2001a). Regression models can also suffer from inadequate interpolation or inappropriate extrapolation (Vanclay 1994) due to missing or limited data. Problems may also arise if certain environmental conditions are absent from the inventory (e.g., no wet bottomlands or steep, rocky slopes), so as much of the expected range of site conditions as possible should be included to promote model applicability. Narrow sampling conditions can still be projected, so long as it is recognized that the resulting models should not be extrapolated to other environments.

Much of this challenge can be addressed by broadening the sampling extent. For instance, Bragg and Guldin (in preparation) considered the role of regional distribution and sample size on PRI models for white oak in the Interior Highlands of Arkansas, Missouri, and Oklahoma. Pronounced differences in white oak performance appeared in regional PRI models, primarily due to limited representation in critical size classes rather than insufficient total sample size (fig. 4). Fortunately, we found that pooling the regional models ameliorated differences and improved model outcome. Bragg (2001a) recommends at least 100 trees per species before developing PRI equations, but even this sample size may be inadequate, especially for widely distributed taxa. Missing species, diameters, or site conditions could be addressed by strategically supplementing the inventory with additional field sampling or the incorporation of other data sets.

Outlier Handling

Every inventory-based optimal diameter increment model needs to address outlier handling because even the best implemented system will have some records that dramatically differ from the rest of the data. In most cases, either a measurement or a transcription error slipped undetected through quality control. However, it is possible that some presumed outliers may actually represent growth at a heretofore unobserved rate and thus reflect the true potential. To date, the PRI outlier strategy has been to simply delete points that noticeably depart from other data (figs. 3a and 3b). This has worked well, because few species have more than one or two obviously errant points. For example, Bragg (in press a) was able to easily identify an outlier in an Arkansas loblolly pine EFIDB data set because the large loblolly averaged 4.2 cm of growth annually for 7.2 years (i.e., during this period, it increased from 61 to 90 cm d.b.h.).

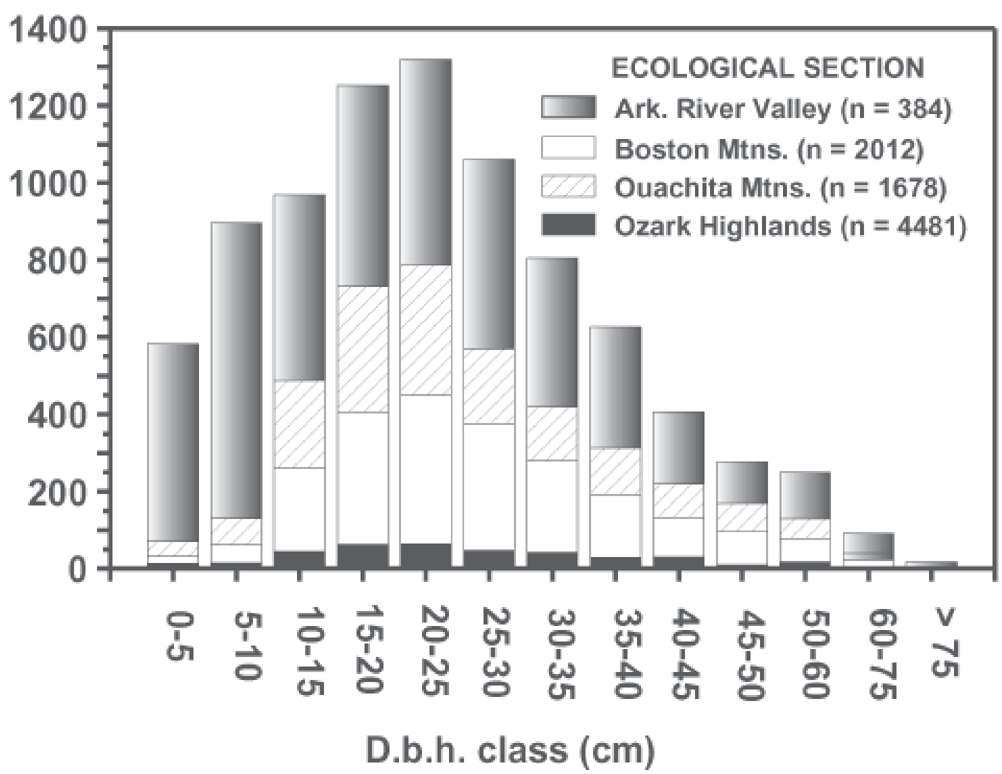

Figure 4.—*White oak sample distribution from the Interior Highlands ecoregion of Arkansas, Missouri, and Oklahoma (adapted from Bragg and Guldin (in preparation)). Ecological section PRI sample sizes were unevenly distributed, both geographically and statistically.*

But what happens with data points that do not deviate appreciably from the "good" data? Is there a better way to fit equations to incorporate more of the information otherwise excluded via the PRI methodology? These questions invariably arise when hundreds or thousands of data points are discarded when the final subset is chosen. Bragg (in preparation b) suggested the possibility of using a top-averaging design in which a fixed number (or percent) of the fast growing records per size class are averaged and then fitted. While a top-averaging approach increases the information content of the equations, it results in optimal growth equations that are even more conservative than the PRI models. Other statistical approaches like quantile regression have been used to identify curves corresponding to various quantiles (e.g., Koenker and Bassett 1978, Yu and Jones 1998). While these have fit splines rather than specific functions like PRI, they could identify different thresholds of acceptance based on predetermined criteria (e.g., optimal increment curves developed using the 95th quantile).

EFFICACY OF INVENTORY DATA

Inventories provide the opportunity to "model from the extremes" (Zeide 1991) and to develop optimal tree increment models based strongly in ecological theory and statistical robustness. Unfortunately, many ecologists have avoided using inventories to develop increment models because they are not aware of their availability or do not appreciate the usefulness of these data. Other growth and yield researchers have not grasped the opportunity to craft alternative approaches to forecasting increment potential. For example, Lessard and others (2001, p. 302) dismissed the value of potential growth models for projecting FIA plots into the future because "…potential growth cannot be observed and historical procedures for estimating it are complex and time consuming." While it is essentially true that potential growth cannot be measured directly, it can be estimated with relatively simple and quick techniques (Bragg 2001a, in press b).

CONCLUSIONS

Inventories have considerable potential to contribute to the development of empirically based, ecologically valid, and statistically robust optimal tree diameter increment models. To ensure the optimal increment models developed from inventory data reasonably approximate species potential, steps can be taken to increase model reliability. The most salient points include the application of accepted ecological theory, robust yet economic statistical derivation, flexibility and generality in model design, and use of reliable inventory information. Providentially, blending the important biological and statistical properties markedly improves the development and application of increment models. While some notable challenges remain, the PRI methodology is a good example of how these desirable aspects can be incorporated with existing inventories to develop models of potential growth.

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