Base-Age Invariance and Inventory Projections

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Abstract.—One of the most important functions of forest inventory is to facilitate management decisions towards forest sustainability based on inventory projections into the future. Therefore, most forest inventories are used for predicting future states of the forests. In modern forestry the most common methods used in inventory projections are based on implicit functions describing time and site dependent relationships derived from panel data. The essence of the implicit functions used for inventory projections is that each function is defined by its own value at one point in time—usually at the inventory time—called the initial conditions or reference values. For this reason, these functions are also called self-referencing, and initial conditions are obtained from sampling, measurements, re-measurements, or other type of inventories. Classic examples of such functions, although not exclusive, are the site index models. They can have different algebraic forms using fixed or variable base ages and be base-age invariant or base-age variant. We explain the implications of different algebraic forms of the self-referencing models that can be used for inventory projections and discuss the forestry literature on base-age variant models under the base-age invariance agenda.

BACKGROUND

The models most frequently used in forestry to describe panel data, i.e., pooled cross-sectional and time-series or longitudinal or repeated measurements use self-referencing (Northway 1985) forms of site equations. In forestry almost all dynamic processes are necessarily dependent on the cross-sectional aspect of forest dynamics relating to different ecological productivity sites, hence: the site models. The site models are in principle the same as mixed-effects models (Lindstrom and Bates 1988, 1990), random-effects models for longitudinal data (Chi and Reinsel 1989, Laird and Ware 1982, Racine-Poon 1985, Stiratelli *et al.* 1984), and panel data models (Furnival *et al.* 1990). Yet, in forestry literature, they are most frequently referred to as site index models or site-dependent height over age models.

Different types of site models contribute immeasurably to efficient forest management by facilitating inventory updates and projections, growth and yield forecasting, and site productivity identification and stratification. These models are also the best illustrations of the evolution of site models in forestry and historical changes in site model forms and expectations.

The earliest efforts in height modeling concentrated on two-dimensional models (height over age). Both hand-drawn curves and the earliest equations that were capable of consistently generating more intricate shapes approximated two-dimensional relations. To enhance the applications of these two-dimensional relations, they could, at times, be developed separately for different sites or even individually for different stands. In a geometrical sense, the collections of curves developed separately for different sites or stands could be classified today as a discrete collection of two-dimensional polymorphic non-disjoint (Clutter *et al.* 1983) height curves.

Historically, such a collection was usually in the form of graphs or tables that were developed for a discrete collection of sites, or stands. They represented a four-dimensional height space in which the dimensions were: reference-height (discrete); age of reference-height (continuous); prediction age (continuous); and prediction height (continuous). The reference-height was reduced to discrete categories because only a discrete number of heights at any given age could be matched with existing curves.

For some applications, generic curves were anamorphically adjusted for individual stands by a simple means of manual multiplication of a guide curve using a ratio of observed to predicted height at an arbitrary age so that the newly generated curve would pass through a known height-age pair. Algebraically adjusting a single base model to specific situations or stands by scaling definitely improves the efficiency over the previous multiple-models approach. This approach also reduces the number of models involved in the prediction system and, in the analysis phase, allows data from different stands to be combined in a complementary system. It extends the discrete reference-height to a continuous reference-height through a simple but explicit multiplication and is therefore more functional. In principle, this algebraic adjustment approach is similar to some

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contemporary systems of site-tree height curves, and localization of models with the Kalman filter approach (Walters *et al.* 1991) could be considered a modern application of such a method.

Newer approaches to site-tree height modeling almost exclusively involve three and four dimensions by adding to basic height over age models additional explicit variables of site index (S, third dimension) and base age (Ab, fourth dimension). An early algebraic inclusion of S into simple anamorphic models was followed by increased model complexity necessary to describe height growth polymorphism and other desirable model characteristics. Some of these included: (1) curves through the origin, (2) variable asymptotes, and (3) equality of predicted height and S at base age.

Peschel (1938) and Prodan (1968) credit Spath in 1797, Hossfeld in 1822, and Smalian in 1937 with the first attempts to express height growth by mathematical equations. Today parameters for such equations are commonly approximated or estimated through linear, and more recently, non-linear regressions. With such equations becoming more and more complex, containing new added variables such as S and Ab, the generated curves, or rather multidimensional spaces, have brought about improvements in biological soundness such as polymorphism and variable asymptotes. With increasing sophistication in the analysis approaches used to determine model parameters, the model simulations become more exacting.

Bailey and Clutter (1974) introduced the concept of base-age invariance in which a height at any age may be predicted directly from any age-height pair without compromising consistency of the predictions. Base-age invariant models can be viewed as four-dimensional spaces that are continuous over all dimensions. The heights predicted with base-age invariant models are unaffected by arbitrary changes in base age. In their work, Bailey and Clutter (1974) applied a technique that has become know as the algebraic difference approach (ADA, see also Borders *et al.* 1984). Site models derived with this approach are mathematically sound and always compute consistent estimates.

At present, virtually all site index models are based on algebraic forms of the fixed or variable base-age formulations that can be either base-age specific, base-age invariant, or base-age variant. The different mathematical forms of these models have different implications for the model operational use in inventory projections.

MODEL FORMS AND FUNCTIONAL ASSUMPTIONS

The mathematical equations used in growth and yield modeling can be linear or non-linear. Linear models are less flexible and usually require more terms (or parameters) to satisfactorily explain the data. This may lead to model overparameterization and unreasonable predictions outside the range of the data on which the model is calibrated. Linear models with many parameters are likely to become atypical in shape and difficult to defend biologically. Non-linear models are more flexible, more likely to be biologically sound, and usually much better behaved outside of the data range (Pienaar and Turnbull 1973).

Peschel (1938), Prodan (1968), Ricker (1979), Cieszewski and Bella (1989, 1991b, 1994), Elfving and Kiviste (1997) and others give numerous examples of basic growth models and various site models used to model height in forestry. From the stand view of a model formulation, the site index models can be categorized as a fixed base age, base-age invariant, and base-age variant. These categories are described below.

Fixed Base-Age Models

Fixed base-age site index models denote models that predict height at any age as a function of this age and a site index at a fixed base age that is obtained from a direct height measurement at any age. These models can be unconstrained or constrained to define the site index as equal to height at the base age. It is important to recognize that the site index in the unconstrained S models does not define any height at any age. Assuming such a definition is one of the most prevalent misconceptions about S models of practitioners who might be using heights at base age as a known site index either in the model fitting or in computing model predictions.

There are many modifications of base models providing either anamorphic or polymorphic fixed base-age site index height models such as the modifications of the Chapman-Richards function by Hegyi (1981), Lundgren and Dolid (1970), Biging (1985), Ek (1971), and Payandeh (1974), and the modification of the logistic function by Monserud (1984).

Some of the modified models became so complicated in an attempt to obtain a better fit to the data that they became unsolvable for S as a function of a height and age. Since they require prior knowledge of S, usually calculated from an observed height at some age, separate models are developed for S as a function of the height at any age. Models for height and S that are derived separately are usually incompatible with their corresponding height models. Another problem is that some base-age specific models, e.g., all the above modifications of the Chapman-Richards and the logistic functions, generate curves that may not go through appropriate heights at their base ages. That is, they predict heights at base ages that are not equal to the site index used as a predictor variable. These and similar models can be conditioned to give site index as the predicted height when age is equal to base age, e.g., Burkhart and Tennant (1977), while analytically unsolvable models can be treated numerically by iterative search routines.

When the fixed base-age S models are simple enough to be solvable for S, they are in terms of predictions functionally equivalent to the base-age invariant models. A similar functional equivalency can be achieved with the fixed base-age S models that are not solvable for S but are applied with compatible numerical solutions for the S values.

Base-Age Invariant Models

Bailey and Clutter (1974) introduced the notion of Base-Age Invariance in the forestry literature. This concept has been defined as invariance of predictions and curves (the invariant) with respect to the selection of base age, i.e., not changing predictions for any selection of base ages within each site series. Base age is simply a common age value for all curves at which the heights on the curves represent site indices (or the reference heights) or initial conditions of the equation. If the curve is invariant, any such age-height point on a curve chosen as a reference unequivocally defines the very same curve. This is not true with the models listed above. Thus, they are not base-age invariant.

Two conditions must prevail for a model to be base-age invariant:

(1) the model must be represented in a base-age invariant algebraic form, and
(2) the model coefficients must be estimated in a way that avoids any influence of the choice of base age on the values of the model coefficients.

Using a base-age invariant algebraic form but base-age specific estimation will result in a base-age invariant equation with base-age specific coefficients. That is, it would not result in a base-age invariant model. Using a base-age specific algebraic form and base-age independent parameter estimation methods would result in a base-age specific equation with base-age invariant coefficients, e.g., Garcia (1983). A model like that of Garcia (1983) could just be solved for the site-specific coefficient (Bailey and Clutter 1974) and, with back substitution, converted to a totally base-age invariant model.

When a base-age dependent algebraic form is used, the achievement of a base-age invariant model is impossible. Clearly, if the coefficient estimation method was base-age invariant, i.e., if no base ages contributed to (or affected) the estimation of coefficients, the model could not possibly produce predictions governed by the base ages because no information about base-age influence on prediction would be acquired. In other words, the model predictions cannot be functionally changing with different selections of base ages unless such changes are defined in a multi-base-age-specific fitting of the model. Thus, the base-age variant models are neither base-age invariant in their forms nor are they base-age invariant in their coefficients. They are equivalencies of conglomerations of multiple base-age specific models with smooth and continuous changes between different base-age specific submodels. This seems to be what Goelz and Burk (1992) intended but mislabeled as base-age invariance.

The notion of base-age invariance is applicable only to models that can directly and consistently predict height at all ages from a single reference height at a single reference age, i.e., base age. Iterative models that predict only a fixed increment or height for each age cannot be base-age invariant because they do not use a single base age (selection of which could be an issue) nor do they directly predict all heights from any selection of a base age. Simply, if there are no multiple predictions of all heights from multiple individual selections of base ages, there is nothing that can be variant or invariant with respect to selection of the base ages. Thus, for example, the iterative model in Wang and Payandeh (1995) is neither base-age variant nor base-age invariant (just like it is neither tall nor short). It simply does not involve the relevance of this concept.

Furthermore, a model formulation is base-age invariant if and only if it analytically defines all heights as a function of any height with outcomes totally unaffected by any choices of base ages and without any use of other equations, numeric searches, programs, generators, guesses, or any other means. Models that require iterative numerical searches for estimation of arbitrary heights from inputs at any ages or for estimation of compatible site indexes are not base-age invariant. Clearly, if the existence of numerical approximations of a model solution was the only criterion for base-age invariance, then every base-age-specific model ever invented would be base-age invariant and the term would be meaningless.

Examples of base-age invariant models published since Bailey and Clutter (1974) include: Amaro *et al.* (1998), Borders *et al.* (1984), Borders *et al.* (1988), Begin and Schutz (1994), Cao *et al.* (1993, 1997), Cieszewski and Bella (1989, 1991a, 1993), Clutter *et al.* (1983), Clutter *et al.* (1984), DuPlat and Tran-Ha (1986), Elfving and Kiviste (1997), Lappi and Bailey (1988), McDill and Amateis (1992), Vicary *et al.* (1984), and Ramirez *et al.* (1987). Examples of different approaches to proper derivation of base-age invariant models besides Bailey and Clutter (1974) are in Amoro *et al.* (1998), Cieszewski and Bella (1989, 1994), Elfving and Kiviste (1997), and McDill and Amateis (1992).

Other forestry examples of development of true base-age invariant equations (not using this name) published before Bailey and Clutter (1974) are Bennett *et al.* (1959), Coile and Schumacher (1964), and Lenhart (1968, 1972).

Site index models are usually based on panel data (i.e., temporal and spatial attributes combined in the same measurements). Not all potentially base-age invariant models have been used for modeling panel data. In fields outside of forestry, examples of such equations and their proper derivations used for individual Y over X relationships may be found in the works of Schnute (1981) and Ratkowsky (1983, 1990).

Other equivalencies of base-age invariant equations in mathematics, physics, and other fields resulting from initial condition or boundary value solutions to differential equations or other difference equations have been in use since the early $16/17^{th}$ century. For example, Johannes Kepler (1571-1630) formulated *Kepler's second law* relating to movement of planets, which may be expressed as $A(t_2) = A(t_1) + \frac{1}{2} h(t_2 - t_1)$ -a truly base-age invariant equation, and it can be used to fit panel data in forestry as, in fact, demonstrated in Bailey and Clutter (1974) where $A = Ln\ H$.

Begin and Schutz (1994) is a kind of standout. The authors develop a truly base-invariant model according to the definition in Bailey and Clutter (1974), but they appear to be unaware of the existence of that article and cite Goelz and Burk (1992), who they, in fact, do not follow. Furthermore, treatment of coefficients and fitting in Begin and Schutz (1994) mimics that of DuPlat (1986), who admittedly also follows Bailey and Clutter (1974).

Base-Age Variant Models

The base-age variant site index models are formulations that create curves that vary under different choices of base ages during the model applications. Examples of those models include Goelz and Burk (1992), Payandeh and Wang (1994, 1995), Huang (1994a,b), Huang *et al.* (1994), and Wang and Payandeh (1996).[1]

The base-age variant models are capable of creating similar inconsistencies as the fixed base-age site index models with incompatible S solutions. However, since the base-age variant models are using as input height at any age, they do not generate inconsistencies of the imput values, only output values. This is unlike the fixed-age S models with incompatible S solutions. The degree of the inconsistencies depends on the selection of sites and base ages.

SI MEASUREMENTS VS. S ESTIMATES

SI models require the knowledge of S for their use. This knowledge comes from various height measurements and at times from S estimates from other variables. It is clear that S estimate from other variables is an estimate, but it is not clear if it should be considered an estimate or a measurement when it is defined by a direct height measurement at an age different than the base age.

Site index can be considered either a measurement or an estimate depending on the mathematical form of the applied site index model and the treatment of the site index solution for a fixed base-age S model. In a simplest case of a variable base-age model, the measured reference height enters the model directly as the S at any given base age, and it is clearly a measurement. Any S at a fixed age, e.g., 50 years, predicted by this model can be considered an estimate, but it is irrelevant to the usage of this model or to its predictions that are strictly driven by the direct height measurement at any given age.

A similar case, though not as obvious, is a fixed base-age S model with a compatible S solution. In such models the height at any age enters the mathematical formula without any deviation from its actual value, and at the age of the measured reference height, which is the age of the model initiation, the model contains no error other than the one associated with the measurement. Furthermore, any imprecision of such model estimates results from the model infidelity rather than from the fact that S is estimated from a height at age different than the base age and that this estimation in turn results in a model input different from the actual measurement.

A special comment should be made about the fixed base-age S models that have compatible S solutions but are not conditioned to predict appropriate heights at the base age, such as Hegyi (1981), Lundgren and Dolid (1970), and Biging (1985). These models are just as consistent in predicting heights as the conditioned fixed base-age S models with compatible S solutions, and for all intents and purposes, the site indexes in these models should be considered measurements, not estimates. Unfortunately, these models mis-display the site indexes due to their sloppy formulations and may mislead a careful practitioner about the misgivings of the model S inputs. The part of the formula in these models that pretends to be the site index, e.g., S or SI, is really just a mathematical constant without any necessary associated meaning other than a necessity of an intermediate calculation. This intermediate calculation must be followed indiscriminately to the

[1] *Since expressing such relationships as equalities implies contradictions such as "1=0" a proper notation for them could be defined by replacing "=" signs with "(" or for a small range of t1 values they could be expressed correctly as approximates using "(."*

age of measured height even if the input height is measured at an age equal to the base age. However, as long as S in these models is computed as the intermediate step, the fixed base-age S models will compute consistent height estimations regardless of the age at which the input height is measured, and they will always be initiated exactly at this very height. Therefore, the inherent site index in these models is measurement, not estimate, despite the superficial appearance of the seemingly erroneous formulation.

The only situation of site index applied as an estimate exists in applications of fixed base-age S models using incompatible site index prediction models. This is so because such applications result in initiating the height models at heights different from those measured and intended for the model input. Such applications disregard the basic principle of the S model being a family of height over age time-series trends, of which a unique trend is identified by a height measurement at any arbitrary age. Clearly, if a height at any age is used to identify a unique series, this height must belong to this series and no statistical considerations are relevant to this concept. Thus, in consideration of S models, treating S as an estimate that somehow needs to be statistically inferred from height and age measurements is a mistake resulting from a misunderstanding of those models and their function as merely a description of height over age trends across a range of productivity sites. Accordingly, statistical developments of the site index prediction models meant to predict the fixed base-age site indexes from heights at arbitrary ages are ill-founded and serve no other purpose than confusing the reader and discrediting the use of S models in the eyes of the general public. All S models should be used with compatible analytical or numerical S solutions. If an actual equation predicting S (from heights and ages) is practically necessary for a given S height over age model, it should be calibrated on this very model to be as compatible with it as possible. The compatibility between the two models is in this case the bottom line criteria, and for this reason it is wrong to calibrate such a model on the data. This compatibility is the only criterion because it is the height measurement that defines the series we seek—not some kind of other "mysterious" variable (like site index) that we might approximate or infer from the measurement. Consequently, in consideration of S models, the site index can be assumed an estimate only in situations of model misuse and misinterpretation. In all other uses of S models, the site index is either a measurement (variable base-age models) or a parameter of intermediate computations (fixed base-age models) serving a function equivalent to a measurement.

CONSISTENCIES OF INVENTORY PROJECTIONS

The consistencies of the inventory projections are examined for the above mentioned form of site index models, assuming the following four criteria of potential model uses:

(1) Computations of site index stability over time
(2) Predictions of heights at all ages from different base ages
(3) Predictions of heights at a harvest age using intermediate computation of S
(4) Predictions of heights at a harvest age directly from measured heights at different ages.

It is noted that the last point does not directly reflect inventory projections, but it indirectly affects them because site indexes are used in almost all growth and yield models as necessary input variables, thus affecting all computed growth and yield predictions.

Base-Age Invariant Models

Base-age invariant site index models report consistent heights regardless of the base ages used and always are consistent in correctly reporting the site indexes as equal to heights at base ages. When using these models, the site indexes calculated from generated height over age series are constant over time (fig. 1a). Regardless of what base age is used, the height over age patterns are always identical for any given site. Figure 1b shows four sets of curves for three sites. As a result of base-age invariance with respect to selection of base ages, all sets of curves on this graph are identical so it seemingly appears to represent only one set of curves.

Similarly, when computing heights at the harvest age of 150 years, the predictions are indifferent to the base age from which the harvestable height is computed. The heights at harvest age depend only on the productivity site and are indifferent to both the base age and the method of computation. When they are computed from site index that is computed from the measured height and age (fig. 1c), they are identical to the ones computed directly from the measured height and age (fig. 1d).

The base-age invariant models produce identical predictions regardless of which way they are used, and site indexes computed with these models are unaffected by age. Using any height age pair within a height over age series, these models define unequivocally the very same series.

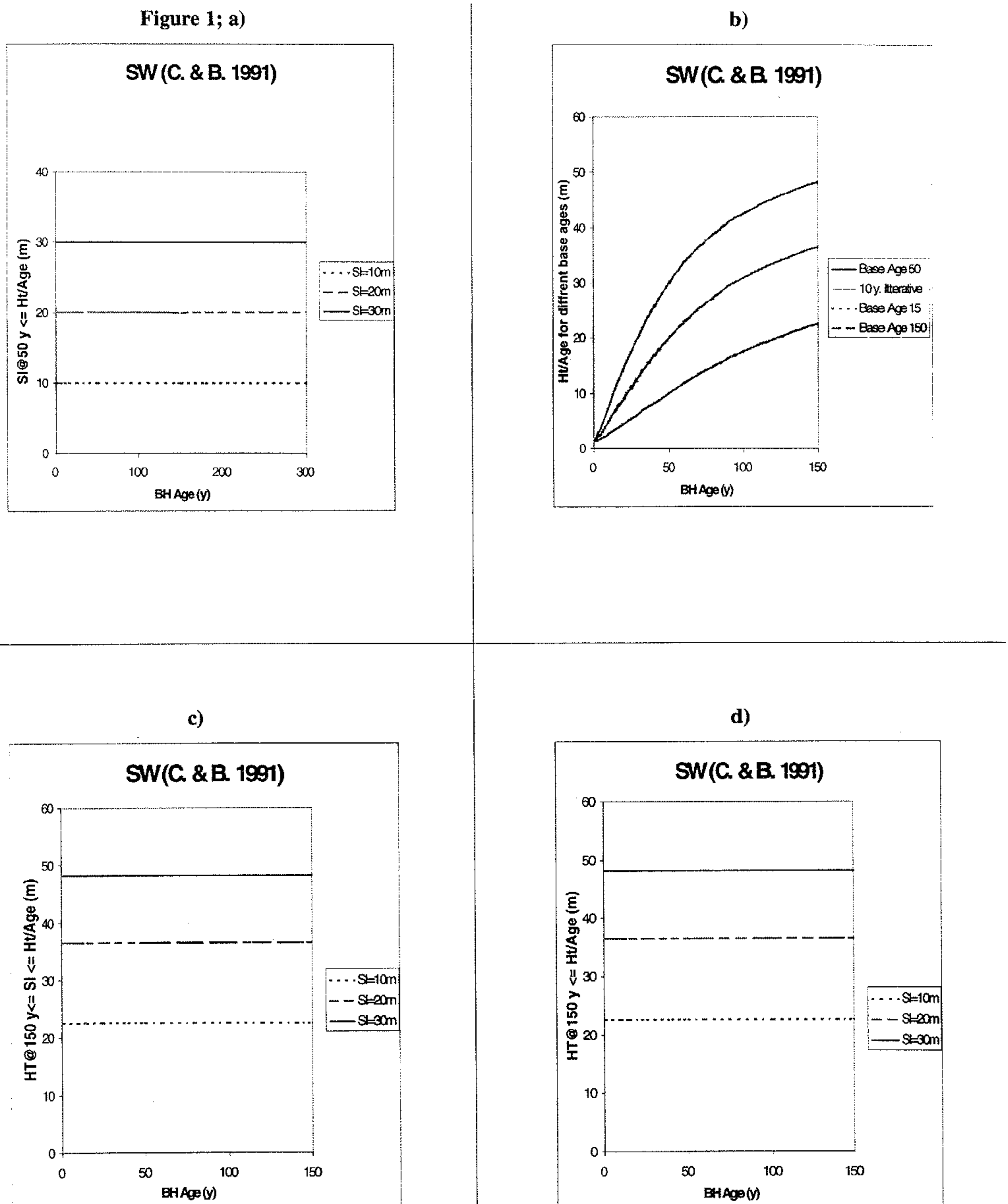


Figure 1.—*Base-age-invariant models for low, medium, and high sites: a) site index predictions unaffected by age of reference height; b) height predictions unaffected by selection of base age; c) predictions of height at harvesting age (computed with intermediate site index calculations) unaffected by age of reference height; and d) predictions of height at harvesting age (computed directly from reference height and age) unaffected by age of reference height.*

Fixed Base-Age S Models with Compatible S Solutions

When the fixed base-age S models are used consistently with intermediate S calculations, then even if they are unconstrained they should produce appropriate heights at any input age (including the S base age) equal to input heights at the input age. Of course, for the unconstrained models, S will be different than the H at base age, but this is not important because the curves are driven through appropriate input heights anyway and the site indexes are simply misreported while the height predictions are not affected by this misreporting.

Proper uses of those models with compatible S solutions will generate predictions that are similar to those from using base-age invariant models. The site indexes computed for different ages as well as heights at base age and at harvest age will be constant over the whole range of input ages. As a consequence of the stable site indexes, the height over age trends will be also consistent and invariant with respect to the selection of the input base ages in the model applications (fig. 2b). Similarly, the height at harvest age (fig. 2c) or any other age will be consistent and unaffected by the age of the input height measurement.

Unlike the case of base-age invariant models, there is a standing issue with the model ambiguity of the fixed base-age S models. The definition of site index, the method of model constraint, the methodology of model usage, the model parsimony, and the compatibility and methodology of obtaining site index solutions are just some of the problems of fixed base-age S models. It is probably these and similar associated dilemmas and misuses that contribute to a broad criticism of site index models and a broad public mistrust and disapproval of site index models in general.

Fixed Base-Age S Models with Incompatible S Solutions using S Estimates

Fixed base-age site index models used with incompatible S solutions compute predictions affected by different choices of base ages. The site indexes in these models are estimates because they are not a mere consequence of the model form transforming the height measurement into a point of reference; instead, they are statistical inferences based on isolated from the S model data interpretations. As a consequence, the site index estimates in these systems are inconsistent over different ages (fig. 3a). This means that the selections of growth series for inventory projections are ambiguous (fig. 1b,c) even though the heights over age series are consistent (fig. 1c).

Due to the change in site index with input age, any predictions of heights will be base-age dependent and will yield inconsistent values varying with different ages of height measurements (fig. 3c). This in turn may result in erroneous height predictions. In the example illustrated in figure 3d, the measured height at age 100 yields 41.4 m, which implies the site index of 25.1, which in turn predicts a height of 41.2 m at a harvest age of 150 years.

Base-Age Variant S Models with Variable Base-Age S Measurements

The base-age variant S models, e.g., Goelz and Burk (1992), Payandeh and Wang (1994, 1995), Huang (1994a, b), Huang *et al.* (1994), and Wang and Payandeh (1996), are the most inconsistent and ambiguous of all the models discussed, and their algebraic formulations are malformed and ill conditioned. According to these models:

(1) site indexes vary over time (fig. 4a);
(2) heights over age trends change with base ages (fig. 4b);
(3) heights at harvest age (fig. 4c and d) or any other age can be anything; and
(4) predictions are biased by different types of model usage (fig. 4b and c vs. d);

A more complete discussion of these models and their use go beyond the scope of this paper.

CONCLUSIONS

The consistency of model estimations in using self-referencing models depends on the type of model used. The highest consistency of model predictions and interpretations is expected from base-age invariant models with direct use of height age measurements for their input and lack of ambiguity about the model input vs. output interpretation.

Fixed base-age S models with adequate compatible S solutions in analytical or numerical forms alike can provide the equivalent of base-age invariant S model consistency in model predictions despite their ambiguity about usage and model interpretation. The misleading forms of these models suggesting a special role of S in the model interpretation are only superficial and can be ignored if a base-age invariant methodology for parameter estimation is used to calibrate these models (Bailey and Clutter 1974, Garcia 1983, DuPlat 1992).

Fixed base-age S models with incompatible S solutions developed separately from the height over age models produce inconsistent results that are varying predictions with different choices of base ages. The use of those models is ambiguous and unreliable in terms of inventory projections with no conclusive numbers to relay on. The ambiguity is in the future forecasting determination of the input heights (the input height is different from intended)

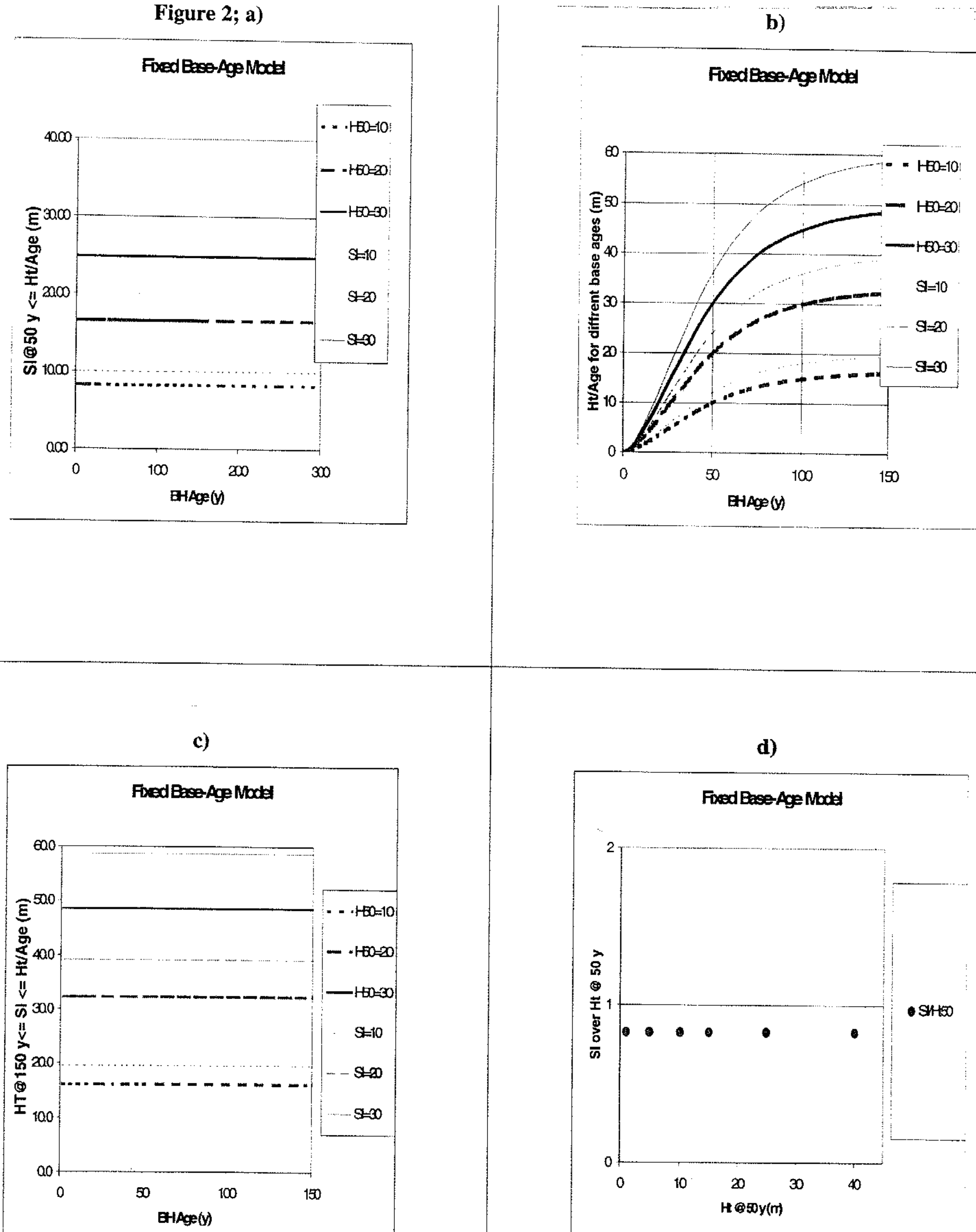
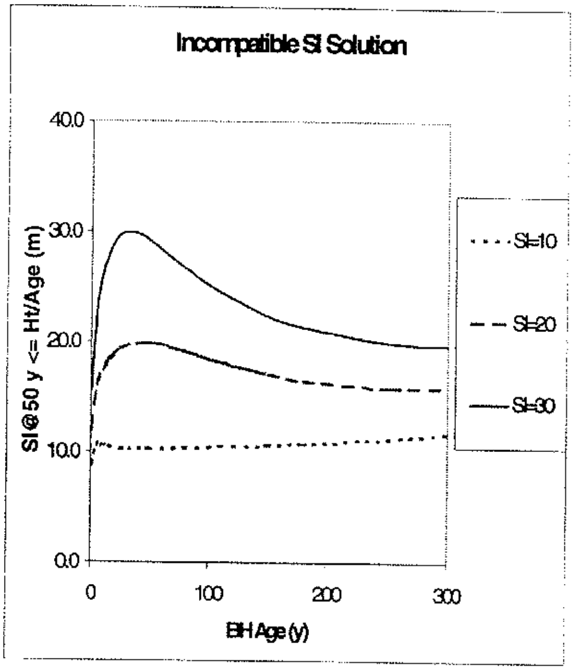
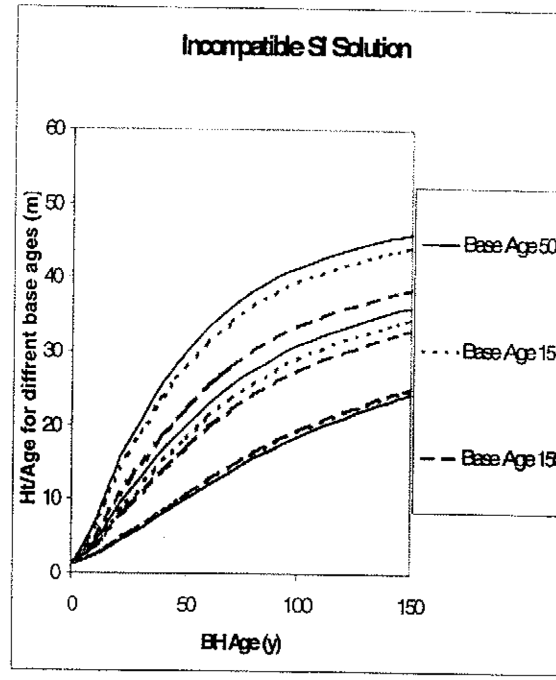


Figure 2.—*Unconstrained (hence the lack of consistency between the site indices and the predicted heights at base age) fixed-base-age site index models with compatible site index solutions for low, medium, and high sites: a) site index predictions unaffected by age of reference height; b) height predictions unaffected by selection of base age; c) predictions of height at harvesting age (computed with intermediate site index calculations) unaffected by age of reference height; and d) relative error in height predictions at harvesting age (computed directly from reference heights and ages) unaffected by age of reference height.*

Figure 3; a)



b)



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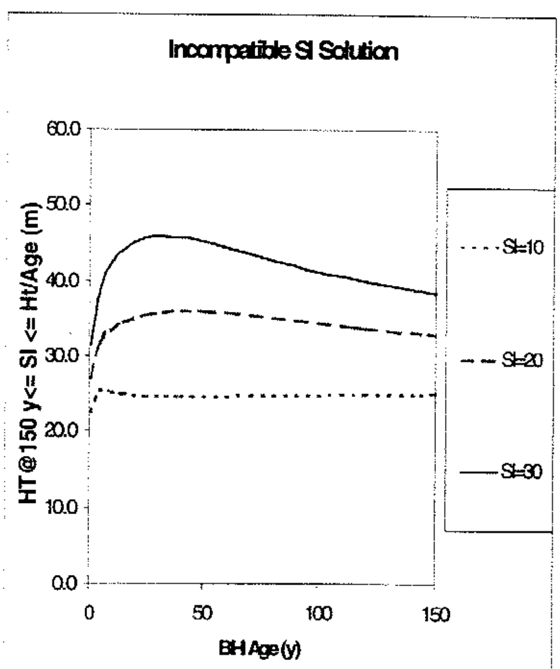

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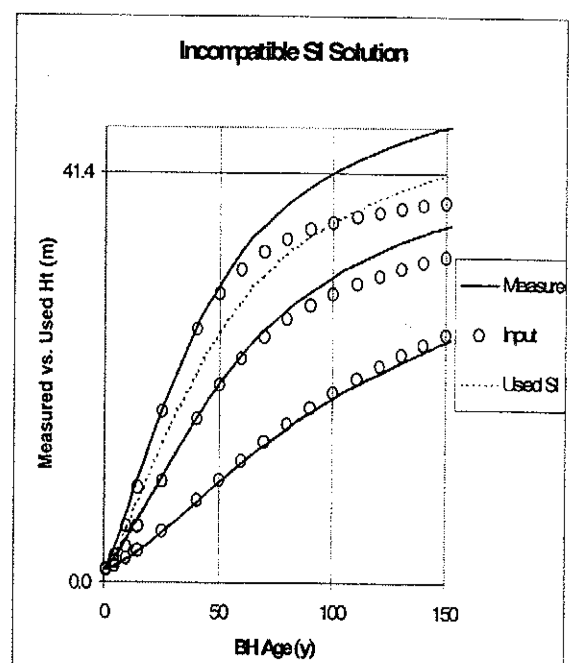
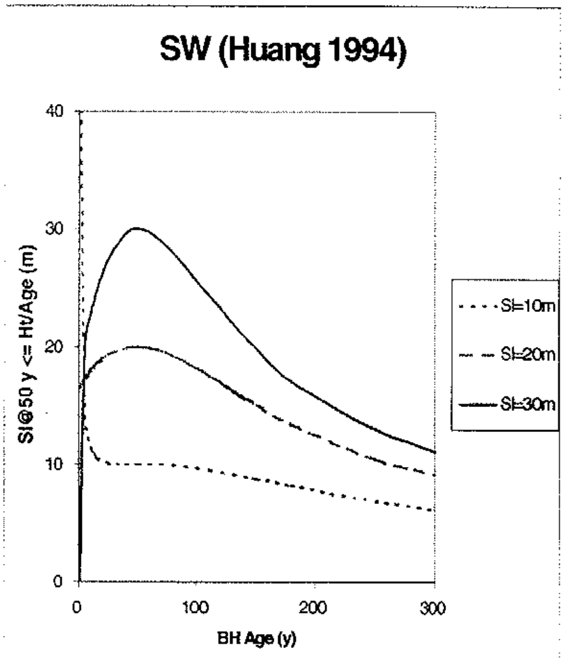


Figure 3.—*Constrained fixed-base-age site index models with incompatible site index solutions for low, medium, and high sites: a) site index predictions change with age of reference height; b) height predictions change with selection of base age; c) predictions of height at harvesting age (computed with intermediate site index calculations) change with age of reference height; and d) 25.1 m site index (predicted from 41.4 m height at age 100 yr) predicts 41.2 m height at age 150 yr.*

Figure 4; a)



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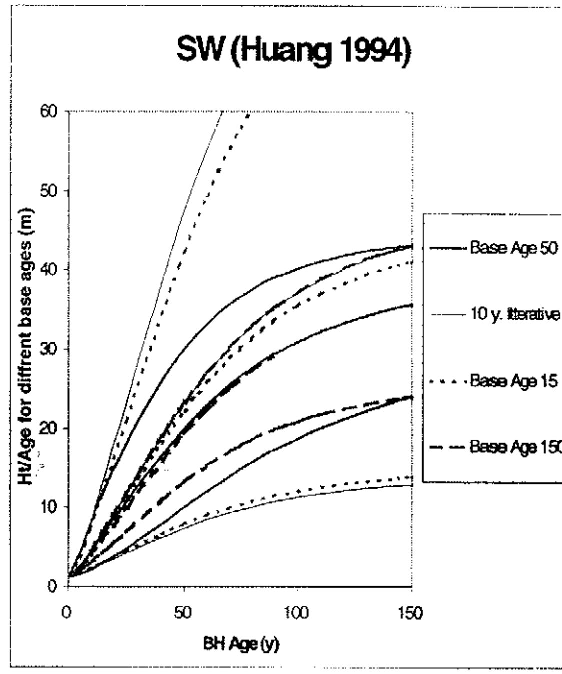

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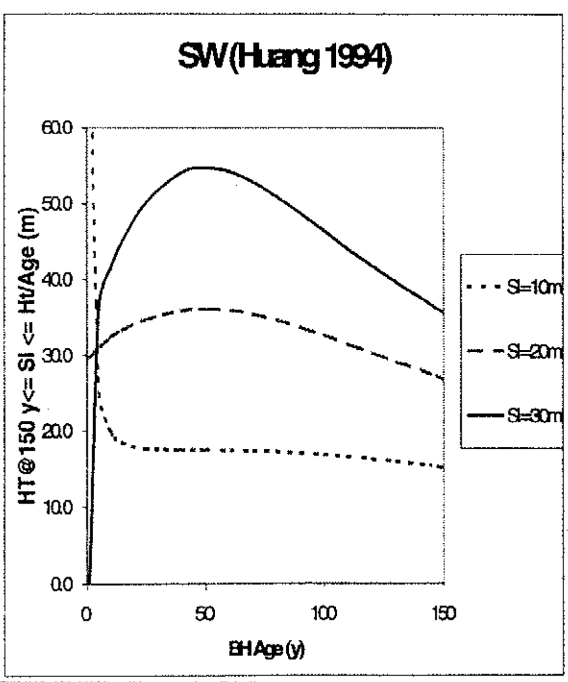

d)

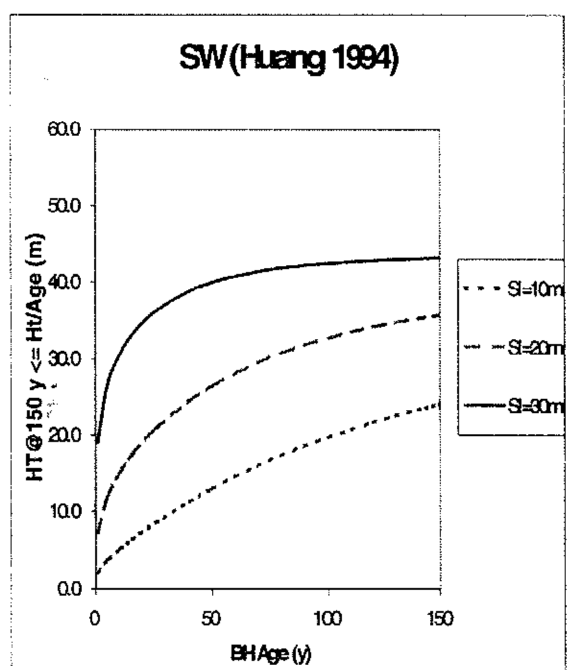

Figure 4.—*Base-age-variant models for low, medium, and high sites: a) site index predictions change with age of reference height; b) height predictions change with selection of base age; c) predictions of height at harvesting age (computed with intermediate site index calculations) change with age of reference height; and d) predictions of height at harvesting age (computed directly from reference height and age) change with the reference age.*

and in consistency of the S estimation as an attribute of growth (SI changing with age).

The base-age variant models (Goelz and Burk 1992; Payandeh and Wang 1994, 1995; Huang 1994a,b; Huang *et al.* 1994; and Wang and Payandeh 1996) are the most ambiguous in terms of their predictions and usage. Since these models maximize effects of the stochastic predictive variables, they are incompatible with their own predictions and cannot be used in the usual variety of common scenarios such as iterative simulations, and back and forth computations. They are inadequate for computation of S inputs into other models or even their own. The degree of this ambiguity varies with site and age selections. Typically it is most dramatic at the extremes of their values.

Inconsistency in site index estimations can result in major economical implications. For example, in forestry administration systems similar to those in British Columbia, a substantial portion of the land base might be taken out of sustainable constant yield calculations due to underestimation of S values in old-growth stands. This in turn would result in a substantial reduction of the AAC on false premises and would hurt the industry and provincial economy with implications for employment and revenues.

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