Growth and Yield Model Application in Tropical Rain Forest Management

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Abstract.—Analytical tools are needed to evaluate the impact of management policies on the sustainable use of rain forest. Optimal decisions concerning the level of management inputs require accurate predictions of output at all relevant input levels. Using growth data from 40 1-hectare permanent plots obtained from the semi-deciduous forest of Ghana, a system of 77 differential equations was developed to quantify the growth dynamics of the mixed-species rain forest. The models underlying the equations were developed for seven species groups, each with similar growth characteristics. This minimized the parameter estimation error. Simultaneous solution of the differential equations with initial stand conditions produced estimates of yield at some future time that was independent of stand age. The solution provides the means to project the status of the timber stand at any future time given well-specified initial stand conditions. The models are useful for inventory updating, allowable annual cut calculations, and management planning for natural or managed stands.

The absence of analytical skills and tools to determine optimal exploitation levels and evaluate the impact of alternative policies has hampered the sustainable use of the rain forest to provide the greatest benefit to all generations. Optimal decisions concerning the level of management inputs, felling cycles, cultural operations, and financial investments require accurate predictions of output from growth and yield models at all relevant input levels.

Therefore, the purpose of this study was to extend the existing analytical techniques to develop a planning tool that can help resource managers objectively evaluate the impact of management policies on the forest resource and implement procedures to maintain forest health. The accomplishment of this objective necessitated the development of a compatible *Growth and Yield Prediction System* that incorporated mechanisms for evaluating land-use policy scenarios based on cutting cycle, volume control, and economic harvests.

Using growth data from 40 1-ha permanent plots obtained from the semi-deciduous forest of Ghana, a system of 77 differential equations was developed to quantify the growth dynamics of the mixed species rain forest.

The models underlying the equations were developed for seven species groups, each with similar growth characteristics. This minimized the parameter estimation error (Atta-Boateng and Moser 1998). Simultaneous solution of the differential equations with initial stand conditions produced estimates of yield at some future time that was independent of stand age. In addition, stand structure data, expressed as stand and stock tables, were obtained from 110 1-ha temporal plots (Ghana Forestry Department 1989). These data were used to specify initial stand conditions.

METHODS

In modeling mixed-tropical rain-forest stand dynamics, this study identified individual components of growth, developed hypotheses concerning the mathematical functions describing the processes, and collectively expressed them as a system of equations capable of predicting the growth of the stand. This approach has been used in a number of studies (e.g., Moser and Hall 1969). The following notations are for per hectare stand variables by species class $k \in \{1,2,..,7\}$.

N^i_k, N^m_k, N^s_k : Sums of in-growth, mortality, and survivor trees

D^i_k, D^m_k, D^s_k : Sums of in-growth, mortality, and survivor tree diameters

B^i_k, B^m_k, B^s_k : Sums of in-growth, mortality, and survivor tree basal area

N_k, D_k, B_k : Sums of trees, diameters, and basal area (initial conditions)

$\delta N^i_k/\delta t, \delta N^m_k/\delta t, \delta N^s_k/\delta t$: Annual rates for in-growth, mortality, and survivor trees

$\delta D^i_k/\delta t, \delta D^m_k/\delta t, \delta D^s_k/\delta t$: Annual rates of change for sum of in-growth, mortality, and survivor tree diameters

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$\delta B_k^i / \delta t, \delta B_k^m / \delta t, \delta B_k^s / \delta t$:	Annual rates of change for sum of in-growth, mortality, and survivor tree basal area
$\delta N_k / \delta t, \delta D_k / \delta t, \delta B_k / \delta t$:	Net change rate for sums of trees, diameters, and basal area
N^P, D^P, B^P :	Sums of trees, diameters, and basal areas (initial conditions)

Quantifying the Number of Mortality Trees and Stand Density Relationships

Mortality may differ in scale and intensity for various ecological reasons, and the underlying stochastic processes may not be comprehensible given the myriad of factors, including disease. This study predicted the number of mortality tree by the numerical integration of the density-related function:

$$\delta N_k^m / \delta t = a_{1,k} + a_{2,k} X_f(N_k) + a_{3,k} X_f(D_k, B_k) \qquad (1)$$

Where $a_{1,k}$ and $a_{2,k}$ are species class $k' \in \{1,2,\ldots,7\}$ parameters, and X_f is an expression involving the stand attribute satisfying the relationship. Equations (2) and (3) were used to force compatibility between predicted numbers, sum of diameters, and basal areas of mortality trees:

$$\delta D_k^m / \delta t = \alpha S_k / N_k \, \delta N_k^m / \delta t \qquad (2)$$

$$\delta B_k^m / \delta t = \beta \, B_k / N_k \, \delta N_k^m / \delta t \qquad (3)$$

Quantification of Size Distribution of Mortality Trees

To ensure a realistic number of mortality trees N_k^m, mortality rate equations were constrained by the limits specified by the probability distribution of mortality trees occurring within the period. Specifically,

$$f\,(x) = \frac{x^{\alpha_1 - 1}(1 - x)^{\alpha_2 - 1}}{\beta(\alpha_1, \alpha_2, x)} \qquad (4)$$

Where

$$\beta(\alpha_1, \alpha_2, x) = \int_0^1 t^{\alpha_1 - 1}(1-t)^{\alpha_2 - 1} \delta t \qquad (5)$$

Components of In-growth

In-growth refers to the process of trees reaching some specified lower size limit. This study predicted the number of in-growth trees by the solution of the following differential equation relating in-growth, number, and relative basal area of trees:

$$\delta N_k^i / \delta t = \alpha_{1,k} e^{(\alpha_{2,k} B_k / B^P + \alpha_{3,k} N_k)} \qquad (6)$$

Where $\alpha_{1,k}, \alpha_{2k}, \alpha_{3,k}$ are species class parameters. Basal area and sum of diameter growth rates of in-growth trees were constrained by tree in-growth rate using the equations:

$$\delta D_k^i / \delta t = \alpha_{1,k} + \alpha_{2,k} \, \delta N_k^i / \delta t \qquad (7)$$

$$\delta B_k^i / \delta t = \alpha_{1,k} + \alpha_{2,k} \, \delta N_k^i / \delta t \qquad (8)$$

Survivor Growth Functions

The choice of mathematical models to describe biological growth process has been the subject of many studies (Prodan 1953). This study fitted the derivative form of the Chapman-Richards function (Richards 1959, Chapman 1961) to model basal area and diameter growth of survivor trees.

$$\delta B_k^s / \delta t = \alpha_k \, B_k^{\beta_k} - \gamma_k \, B_k \qquad (9)$$

$$\delta D_K^s / \delta t = \alpha_K \, D_K^{\beta_K} - \gamma_K \, D_K \qquad (10)$$

Specification for Net Rates of Change

The following net growth in number of trees, sum of diameters, and basal area were used to predict future stand component values for trees of dbh ≥ 10 cm:

$$\delta N_k / \delta t = \delta N'_k / \delta t - \delta N_k^m / \delta t, \ \forall \ k \in \{1,2,\ldots,7\} \qquad (11)$$

$$\delta D_k / \delta t = \delta D'_k / \delta t - \delta D_k^m / \delta t + \delta D_k^s / \delta t, \quad \forall \ k \in \{1,2,\ldots,7\} \qquad (12)$$

$$\delta B_k / \delta t = \delta B'_k / \delta t - \delta B_k^m / \delta t + \delta B_k^s / \delta t, \quad \forall \ k \in \{1,2,\ldots,7\} \qquad (13)$$

Where $\delta N_k^i / \delta t$, $\delta N_k^m / \delta t$, $\delta D_k^i / \delta t$, $\delta D_k^m / \delta t$ t, $\delta B_k^i / \delta t$, $\delta B_k^m / \delta t$ and $\delta B_k^s / \delta t$ are as defined above.

Parameterization of Diameter Distribution Function

The system of equations provided the inputs for developing the following prediction equations for the parameters of the Beta:

$$\gamma_k = a_{11} \alpha_k + a_{12} \ln\left[(B_k / N_k)^{-1}\right] \qquad (14)$$

$$\alpha_k = a_{21} \gamma_k + a_{22} \ln\left[(B_k / N_k)^{-1}\right] \qquad (15)$$

The values for α_k and γ_k were obtained by simultaneous solution of equations in (14) to (15). The exponent K_k was obtained from the beta function:

$$\kappa_k = \frac{N_k}{\int_a^b (x - a)^{\alpha_k} (b - x)^{\gamma_k} \, dx} \qquad (16)$$

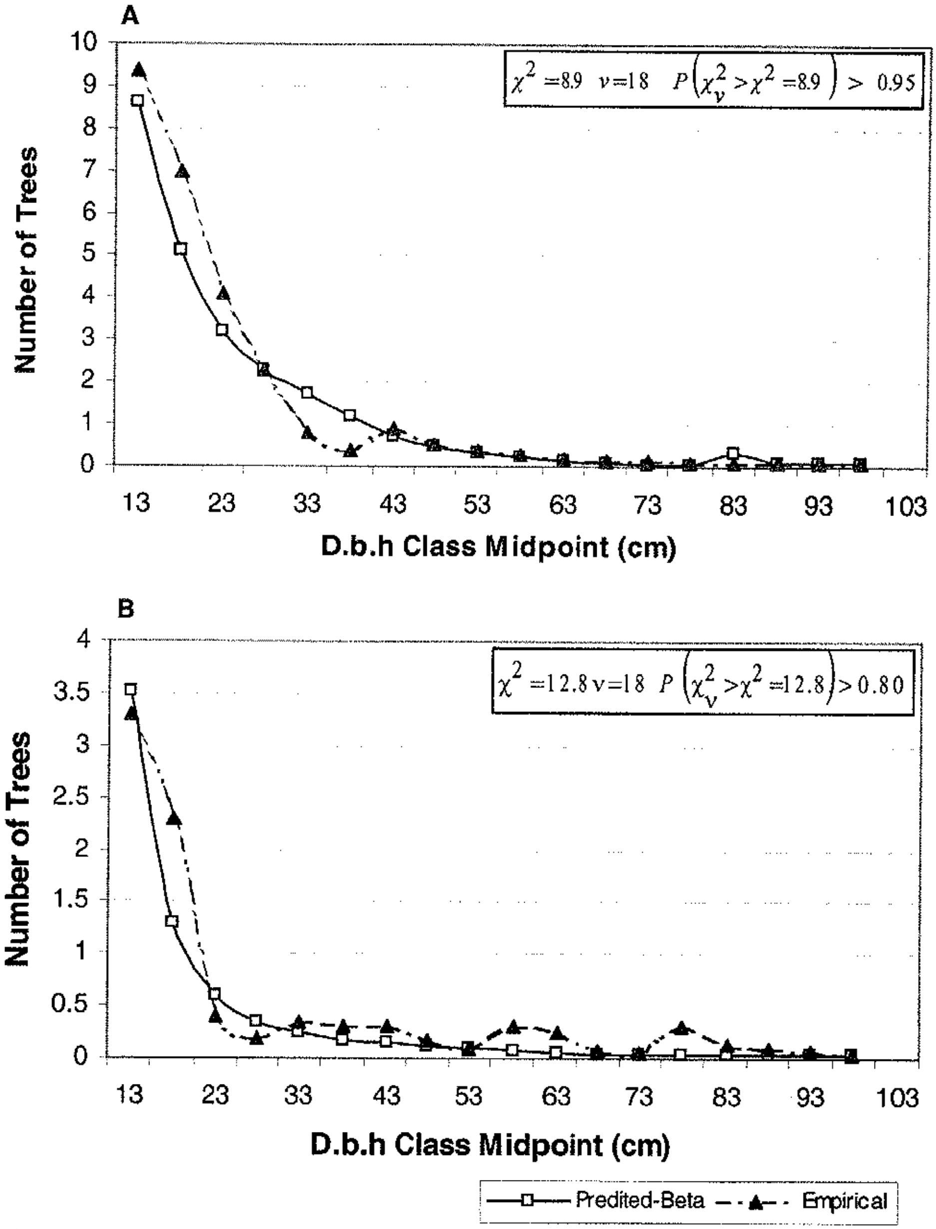

Figure 1.—*Comparison of empirical and predicted beta distributions for species classes within management unit 17. A. Class 1. B. Class 2.*

Figure 1 compares the empirical distribution, nonlinear optimized best-fit beta distribution, and the beta distribution obtained by equations (14) to (16).

The future predictions of stand components were used in models (14) to (15) to obtain estimates for the exponents of the beta distribution and subsequently generate the diameter distributions from which other forest management activities will be assessed.

RESULTS

Growth and Yield Consideration in Forest Management

Logging and timber stand improvement activities needed to maintain stand structures and regulate densities could be specified as harvesting rules. Rules may specify a percentage reduction in stocking in a size class. These considerations are implemented in the yield model to evaluate cutting limits, allowable cuts, and felling cycle lengths. Cubic volume cuttings that are sustainable under

Table 1.—*Stock table at year 5, 71-cm min. cut and 67 percent allowable cut regime*[1]

	Residual stand (km^2)			Harvest (km^2)		
Dbh (cm)	Trees	BA(m^2)	Volume(m^3)	Trees	BArea	Volume(m^3)
11-15	21,696	288.0	2,745.5	0	0.0	0.0
16-20	7,524	191.5	1,910.5	0	0.0	0.0
21-25	4,328	179.8	1,857.0	0	0.0	0.0
26-30	2,897	178.4	1,893.3	0	0.0	0.0
31-35	2,080	177.9	1,932.9	0	0.0	0.0
36-40	1,554	176.2	1,952.2	0	0.0	0.0
41-45	1,187	172.4	1,943.7	0	0.0	0.0
46-50	920	166.5	1,905.7	0	0.0	0.0
51-55	718	158.5	1,839.3	0	0.0	0.0
56-60	563	148.6	1,747.1	0	0.0	0.0
61-65	440	137.3	1,632.7	0	0.0	0.0
66-70	344	124.8	1,500.0	0	0.0	0.0
71-75	252	105.4	1,279.4	3	1.2	14.8
76-80	193	92.2	1,129.6	2	1.1	13.6
81-85	146	79.0	976.5	2	1.0	12.1
86-90	109	66.1	824.0	1	0.8	10.3
91-95	79	53.8	675.4	1	0.7	8.2
96-100	56	42.1	532.2	1	0.5	6.1
101-105	36	30.1	384.0	0	0.3	3.9
106 +	24	22.4	287.6	0	0.3	2.9
Total	45,146	2,591.0	28,948.7	11	5.8	72.0

[1] Stand defined by the Beta Distribution: K_k =0.0095 for α_k = 0.9417 and γ_k = 2.3463

Cumulative net basal area culminates at a simulation period of 60 years (fig. 4), with different final net basal areas. The magnitude of net basal area varies directly with minimum cut limit and inversely with allowable cut percent. Consequently, by combining minimum cut limit with an allowable cut percent a desired final net basal area can be obtained. Rate of accumulation of survivor basal area growth is lower than the other components of net growth. Most of the contribution to survivor growth comes from seedlings and poles and these have relatively smaller contributions of stand basal area.

Tables 2, 3, and 4 indicate that in-growth contributes significantly to the development of the stand. The rate of in-growth basal area accumulation contributes significantly to survivor basal area growth in subsequent periods of the simulation.

CONCLUSION

As the tropical timber industry enters the 21st century, it will search for competitive advantage in harvesting. Forest management will require intensive management and high yields. Forest managers will have to find ways to adhere to new environmental regulations and address the public's concerns about forest health. Better information and data analysis are the key to this new brand of intensive management.

This study is unique in the development of a window-based growth and yield system that incorporated a novel technique to pool sparsely distributed tropical forest tree species with similar growth increment characteristics into groups. The technique is a landmark in the construction of growth and yield models for mixed-species uneven-aged forest. It presents an objective approach to minimize the variations within each species group and hence the bias in the estimation of growth model parameters of the species groups.

The compatible growth and yield models developed were based on well-established premises.

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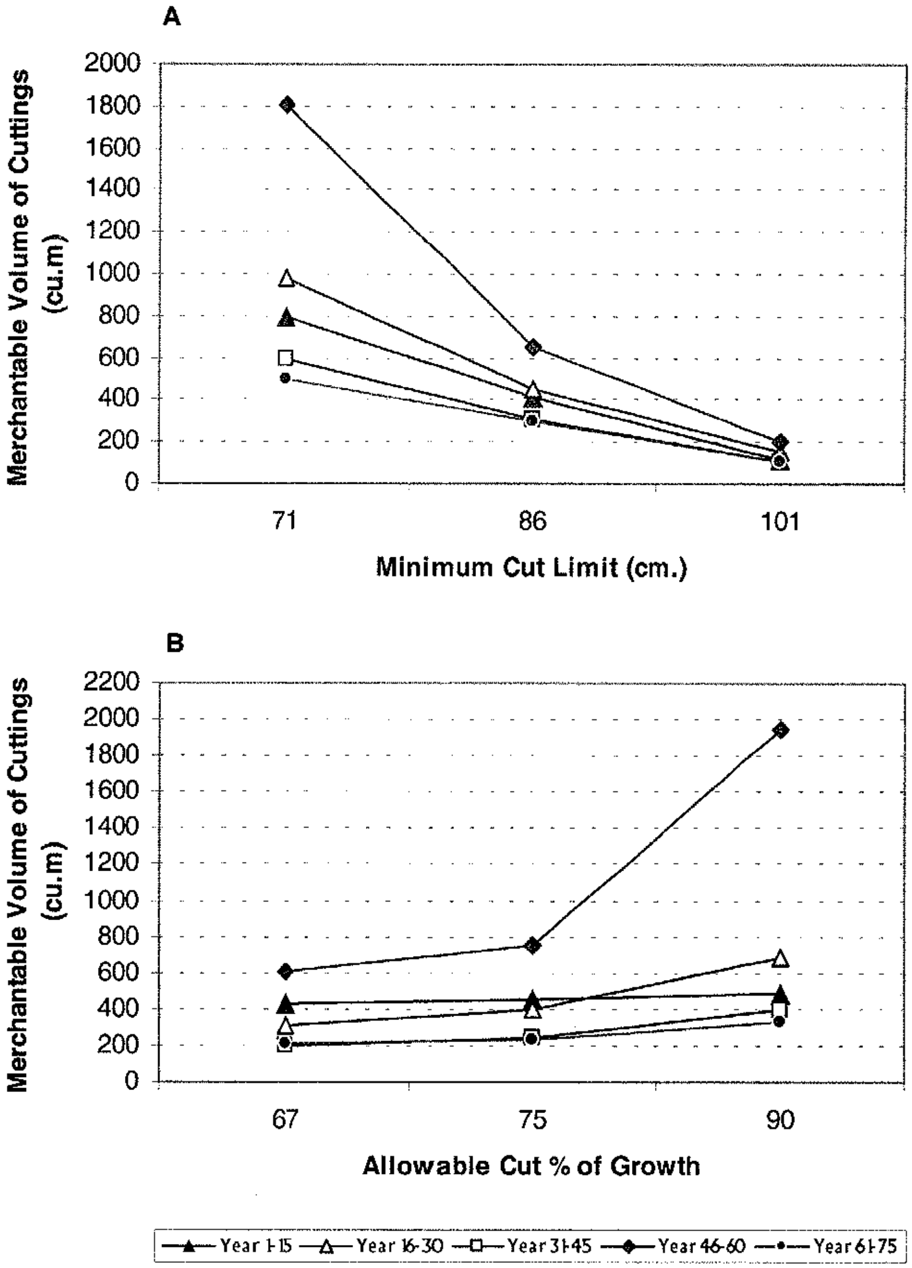

Figure 2.—*Merchantable volume of cuttings by allowable cut percent of growth, minimum cut limit, and planning period interactions for management unit 17.*

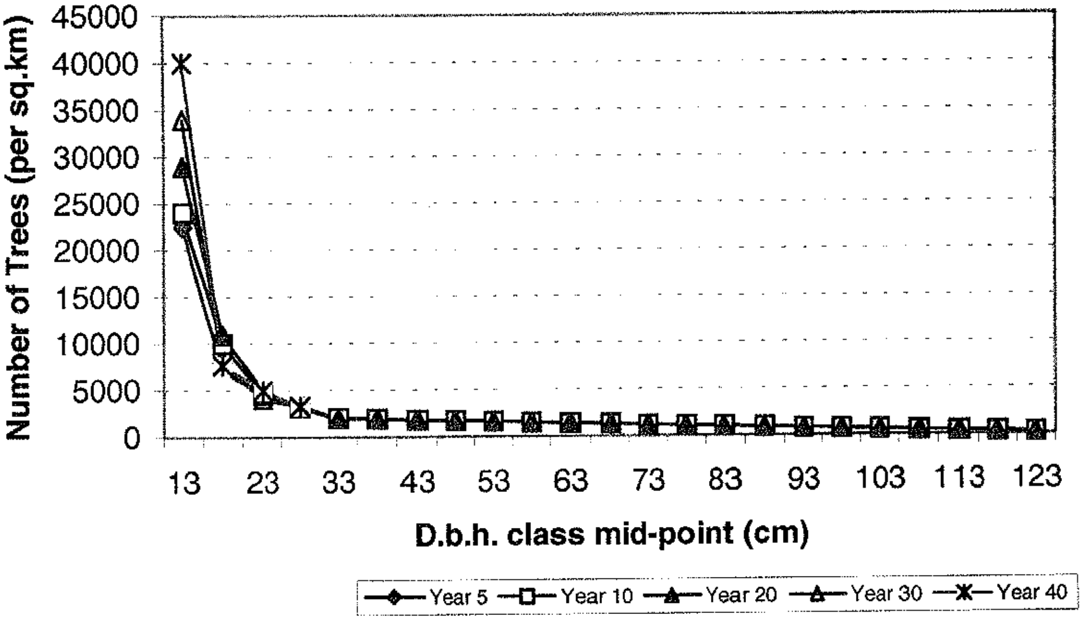

Figure 3.—*Projection of residual stand structures over a 40-year planning horizon.*

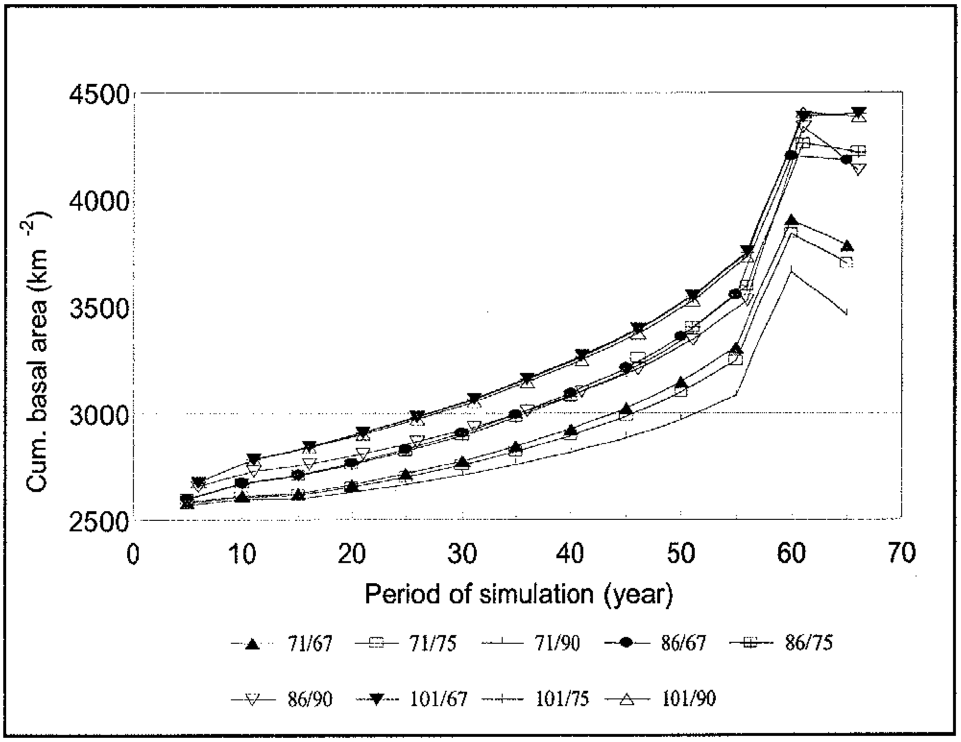

Figure 4.—*Cumulative residual basal area (km²) by cut limit (first number) and allowable cut percent of increment (second number).*

Table 2.—*Cumulative in-growth basal area* ($m^2\ km^{-2}$) *by minimum cut limit and allowable cut percentage*

Min. cut limit (cm)	Allowable cut %	Simulation period (years)												
		5	10	15	20	25	30	35	40	45	50	55	60	65
71	67	83.5	165.2	250.0	340.0	435.8	538.5	650.2	773.3	912.7	1077.3	1287.3	1417.2	1790.4
71	75	83.5	165.2	250.2	340.2	436.2	538.8	651.0	774.5	914.5	1080.3	1292.3	1422.0	1797.0
71	90	83.5	165.2	250.2	340.7	436.7	540.2	652.7	777.2	918.7	1086.5	1302.8	1432.2	1811.0
86	67	83.5	165.3	250.0	339.7	435.2	537.5	648.7	771.7	910.5	1074.7	1285.0	1413.8	1864.0
86	75	83.5	165.3	250.0	339.7	435.3	538.0	649.3	772.7	912.3	1078.2	1290.8	1419.8	1874.4
86	90	83.5	165.3	250.0	340.0	435.7	538.8	651.3	775.7	917.3	1086.7	1307.0	1435.2	1905.2
101	67	83.5	165.2	249.7	339.0	434.0	536.0	646.3	767.7	905.0	1066.5	1271.2	1399.2	1840.2
101	75	83.5	165.2	249.7	339.0	434.2	536.0	646.5	768.5	905.8	1068.3	1274.5	1402.2	1846.6
101	90	83.5	165.2	249.7	339.0	434.5	536.7	647.5	770.2	908.7	1072.8	1283.5	1411.0	1865.2

Table 3.—*Cumulative survivor growth basal area* ($m^2\ km^{-2}$) *by minimum cut limit and allowable cut percentage*

Min. cut limit (cm)	Allowable cut %	Simulation period (years)												
		5	10	15	20	25	30	35	40	45	50	55	60	65
71	67	173.0	344.2	515.7	688.3	864.8	1045.8	1232.7	1426.8	1630.5	1846.0	2078.3	2557.0	3256.7
71	75	173.0	344.0	515.0	687.5	863.3	1043.8	1230.2	1423.8	1627.0	1842.0	2073.7	2551.5	3251.2
71	90	173.0	343.3	514.0	686.0	860.7	1040.0	1225.2	1417.2	1618.7	1832.0	2061.5	2536.2	3285.5
86	67	173.0	345.0	517.7	691.5	869.0	1051.3	1239.7	1435.5	1640.7	1858.3	2093.0	2579.3	3368.7
86	75	173.0	345.0	517.3	691.3	868.3	1050.5	1238.7	1434.2	1639.3	1857.2	2091.7	2580.5	3376.5
86	90	173.0	344.8	516.8	690.2	867.0	1048.7	1236.2	1431.3	1635.8	1853.3	2088.7	2647.3	3419.2
101	67	173.0	346.0	519.0	694.0	872.2	1055.2	1244.5	1440.8	1646.3	1864.2	2098.5	2581.8	3362.8
101	75	173.0	346.0	519.0	693.8	872.0	1054.8	1244.0	1440.3	1646.0	1863.8	2098.7	2583.5	3369.2
101	90	173.0	345.8	519.0	693.7	871.3	1054.5	1243.2	1439.5	1645.2	1863.0	2098.3	2588.2	3386.8

Table 4.—*Cumulative mortality basal area* ($m^2\ km^{-2}$) *by minimum cut limit and allowable cut percentage*

Min. cut limit (cm)	Allowable cut %	Simulation period (years)												
		5	10	15	20	25	30	35	40	45	50	55	60	65
71	67	85.2	217.8	403.0	591.3	780.8	973.7	1171.0	1373.8	1583.5	1800.7	2026.7	2271.2	2604.8
71	75	85.2	218.0	402.5	590.3	778.7	970.2	1166.2	1367.8	1575.3	1790.8	2015.0	2257.4	2588.2
71	90	85.2	218.0	401.8	587.5	774.0	962.2	1153.8	1350.7	1553.8	1764.2	1983.0	2220.0	2541.6
86	67	85.2	217.8	404.2	594.8	787.2	983.2	1183.3	1389.5	1602.8	1824.0	2054.8	2303.4	2726.8
86	75	85.2	217.5	404.0	594.0	785.8	980.8	1180.8	1386.3	1599.0	1819.5	2049.5	2297.2	2719.8
86	90	85.2	217.8	404.0	593.2	783.3	976.7	1174.5	1378.3	1589.0	1807.5	2035.5	2281.2	2701.2
101	67	85.2	217.7	405.0	597.3	791.7	989.5	1191.8	1399.7	1614.7	1837.7	2070.0	2320.2	2525.0
101	75	85.2	217.7	405.2	597.2	791.2	988.8	1191.0	1398.8	1613.7	1836.3	2068.8	2318.8	2745.2
101	90	85.2	217.7	405.2	596.8	790.5	987.2	1188.8	1396.2	1610.5	1832.7	2064.	2314.6	2741.4

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