A Field Test of Cut-Off Importance Sampling for Bole Volume

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Abstract.—Cut-off importance sampling has recently been introduced as a technique for estimating bole volume to some point below the tree tip, termed the cut-off point. A field test of this technique was conducted on a small population of eastern white pine trees using dendrometry as the standard for volume estimation. Results showed that the differences in volume estimates between the techniques were generally within acceptable limits.

Importance sampling, since its first appearance in the forestry literature over a decade ago, has been used to estimate numerous dimensions of trees. Originally introduced in conjunction with randomized branch sampling by Valentine *et al.* (1984) to estimate total, aboveground, woody volume and dry matter, it has proven to be an adaptable technique. For example, importance sampling provides unbiased estimates of bole volumes in conjunction with tree-selection schemes like 3-P (Gregoire *et al.* 1986). It also can be used to estimate log volume (Furnival *et al.* 1986) and bole increment (Gregoire *et al.* 1987).

In the above contexts, importance sampling is generally applied to the entire unit to be sampled, whether a log or the entire bole. Cut-off importance sampling (CIS) is a variant of the usual importance sampling where an intermediate point is used as a cut-off for the sampling process. For example, in the estimation of tree bole volume, one might use merchantable height or height to the base of the live crown as logical cut-off points, depending upon the application. One or more heights along the tree bole from base (or stump) to the cut-off point are chosen by a probability mechanism for measurement of diameter. The measurement converts to an essentially unbiased estimate of bole volume for that bole section in which sampling occurs. Estimates for total bole volume are also possible. Since its introduction by Robinson *et al.* (1997), however, there has been little else published on cut-off importance sampling for bole volume. These authors presented a field test in which CIS performed well in the estimation of bole volume as compared with importance sampling. In this paper, we present a field test of CIS in the northeastern U.S. CIS is compared with detailed dendrometry of the population of trees of interest.

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METHODS

A population of 74 eastern white pine (*Pinus strobus)* trees in a well-managed, even-aged, research stand on University of New Hampshire woodlands, Durham, NH, was used in this study. All of the trees in the stand are numbered, and the stand has been subjected to various intensive silvicultural methods in the past to try to promote advanced regeneration, including several controlled burns. Average diameter at breast height (dbh) for the stand was 17.8 in., average total height was 83.8 ft, and the average live crown ratio was found to be approximately 46 percent. Each tree in the stand was measured using a Barr and Stroud (B&S) FP-12 optical dendrometer and subsequently resampled using CIS. Measurements required for dendrometry began at the base of the tree—both stump diameter at 1.5 ft and dbh were measured with diameter tapes. From dbh, the first actual B&S measurement was always taken at a height of 4 ft above dbh, located with a measuring pole. In general, other B&S measurements were taken at points that reflected the taper of the tree rather than at set regular intervals. The only other exception to this rule was that a B&S measurement was always taken at the base of the live crown to facilitate comparison with the CIS estimates. Sixty-eight trees were actually used after excluding those trees with forks, broken tops, or other abnormalities that did not lend themselve to the techniques described below.

The dendrometry measurements were recorded in a format compatible with STX (Grosenbaugh 1974, Rennie 1977, Space 1974). A separate FORTRAN-90 program that also read STX input file formats was written to compute cubic foot volumes from the dendrometry cross-sectional areas and heights via three different spline algorithms: natural cubic splines (Press *et al.* 1986), hermite polynomials, and the method of Akima (de Boor 1974, IMSL 1994). This latter program also computed cubic foot volumes by conic sections as in STX. The Akima method was judged to be the best of the three splining algorithms used and was therefore retained in this analysis along with the conic section method. These methods allowed for interpolation of cross-sectional areas (and hence diameters) at any height from the fitted splines, or by linear interpolation between the measured diameters. Calculation of volume was also possible to any height by direct integration of the Akima spline functions or again, by linear interpolation from the measured conic section volume. Specifically, diameter and volume were desired to the base of the live crown to compare with those taken in the CIS procedure described subsequently. All volumes and diameters in this analysis included bark.

In all comparisons of volume and height measurements between the two techniques, the dendrometry measurements are taken as the "true" values for each tree with full regard to the measurement errors and biases associated with this technique. Accuracy of the CIS estimates can therefore be determined using a chi-square approach first put forth by Freese (1960), who suggested several variations on this statistic. The version used here is based on the error sums of squares from a simple linear regression of the CIS estimates on the dendrometry estimates. In this statistic, the standard method (dendrometry) is the independent variable in the regression. This may seem backwards because one might naturally want to think in terms of using the CIS estimates to predict the standard. However, a little reflection on OLS theory reminds us that the independent variable is assumed fixed (constant and measured without error) while the dependent variable is the random variable; therefore, the standard method is taken as the independent variable in this regression. According to Freese (1960), this is an approximate test that corrects for linearly increasing or decreasing bias with respect to the dendrometry data; such linear trends were common in these data (fig. 1). The form of the test is given as

$$\chi^2_{(n-2)df} = \frac{sse}{\sigma^2} \qquad (1)$$

where *sse* is the regression error sum of squares and σ^2 is the hypothesized variance; it is defined as

$$\sigma^2 = \frac{E^2}{z^2}$$

Figure 1.—*Difference in volume to the base of the live crown:* $V_S(H_L,H_C)-\tilde{V}(H_L,H_C)$.

Here, E is the allowable error in the same units as the variable to be tested and z is the standard normal deviate at the required significance level. The significance level was set to 0.05 for all tests. In each accuracy test done, the allowable error was specified to be 5 percent of the mean for the dendrometry-base variable of interest; these values are shown in table 1. Using a simple re-arrangement of (1), it is possible to determine what the "target" allowable error ($\tilde{E}$) would have to be in order for the desired accuracy to be met in each test (Bell and Groman 1971); these are also presented in table 1.

Cut-off importance sampling has been described in detail by Robinson *et al.* (1997). In this study, the portions of the tree bole of interest lay between a 1.5-ft stump height (H_L), height to the base of the live crown (H_C), and total height (H_U). CIS was conducted on each tree between H_L and H_C. A Finnish caliper (Schmid and Werner 1970) was used to measure the diameter at the randomly chosen cut-off height within this section. Heights of CIS sample points were measured with a height pole; a clinometer was used to measure total height and height to the base of the live crown. Volumes of interest include volume from stump to live crown base ($V(H_L,H_C)$), and total volume from stump to tip ($V(H_L,H_U)$). CIS theory provides estimators for both of these volumes; they are given in Robinson *et al.* (1997). Denote the CIS estimator of $V(H_L,H_C)$, as $\hat{V}(H_L,H_C)$, and the CIS ratio estimator of total volume as $\tilde{V}(H_L,H_U)$. The proxy taper function used here is the same as that used by Robinson *et al.* (1997) with $C = 4$; viz.,

$$A_p(h)=A(H_B)\left[\frac{H_U-h}{H_U-H_B}\right]^{4/c}$$

Here, A(•) denotes the bole cross-sectional area, H_B is breast height, h is any height in the range $H_L \le h \le H_U$. Furthermore, let the volumes from dendrometry computed via the spline and conic sections be denoted $V_S(H_L,\cdots)$ and $V_C(H_L,\cdots)$, respectively to either the base of the live crown or the tree tip.

RESULTS

Height Comparisons

The dendrometry and CIS were conducted independently by the same crews, approximately 2 weeks apart. No attempt was made to mark the base of the live crown on each stem. Therefore, this point differed somewhat between the two samples. As such, it is expedient to first see if these points coincide for the two different inventories on average. Because this point was a judgment call in both instances, neither can be considered the "true" value and, therefore, Freese's technique is inappropriate. A paired t-test was used instead to test the null hypothesis that the mean difference between the two points was equal to zero. This could not be rejected at the $\alpha = 0.05$ level; thus, the points can be considered the same between the two methods, even though there were rather large differences on some trees (table 1). This assumption facilitated the calculation of volumes to the base of the live crown (BLC) between the two methods because no interpolation was required initially for the computation of either spline or conic section volumes.

Total height comparisons were between the clinometer measurement from the CIS sample and the dendrometry measurement. Remarkably, there was little overall bias in the clinometer estimate of total tree height (table 1). However, the MSE shows that there was a fairly large

Table 1.—*Comparison of dendrometry and CIS estimates for tree heights and volumes for N=68 eastern white pines*

	Mean B&S	Mean CIS	E	$\tilde{E}$	Percent $\tilde{E}$	Average Difference	MSE
Height to BLC (ft)	39.0	40.5	—	—	—	1.5	48.5
Total Height (ft)	83.8	86.6	4.2	9.0	11	2.8	41.3
Volume to BLC (ft³)							
Spline	51.3	48.9	2.6	12.3	24	-2.3	68.7
Conic Section	51.2	48.9	2.6	12.1	24	-2.3	65.3
Total Volume (ft³)							
Spline	69.4	72.0	3.5	15.6	22	2.6	86.7
Conic Section	68.6	72.0	3.4	15.2	22	3.4	87.8

amount of variability associated with these readings. Indeed, the accuracy test inference is that the two methods are different, unless one is willing to accept an 11 percent, or 9 foot, allowable error.

Volume Comparisons

Inasmuch as the heights to the BLC between the two measurement periods were not found to be different on average, the two (possibly) different heights were used for the calculation of volume to the BLC via CIS and dendrometry (table 1). The first thing to notice is that both the spline and conic section volumes to BLC were very close; hence, these two methods could be used interchangeably to compare against the calculated CIS volumes. CIS estimates are slightly less than the spline and conic section volumes by approximately 4 percent on average. Likewise, the total volume estimate by CIS to BLC for all 68 trees is 4.6 percent less that the total spline and conic section volume. However, regarding the last two columns of table 1, notice that the standard deviation of these differences is almost four times the mean. This large degree of variability is found to influence the chi-square test so that one would have to accept a 24 percent allowable error before the two methods could be considered comparable.

The results on total bole volume were much the same as for volume to the base of the live crown (table 1). In this case, the difference was slightly greater and in the opposite direction from the results on volume to BLC. But again, the large variance dominates the mean squared error with a standard deviation of 9 cubic feet on the difference between $V_S(H_L,H_U)$ and $\tilde{V}(H_L,H_U)$. As a consequence, one would have to accept an allowable error of approximately 15.5 cubic feet, or 22 percent, for the CIS estimate of total bole volume to be considered as accurate as the dendrometry. In addition, the sum of the volumes for all 68 trees produced a positive difference of approximately 4 percent. For total volume, the spline and conic section volumes were not quite as close as for volume to BLC; however, a simple linear regression of $V_C(H_L,H_U)$ on $V_S(H_L,H_U)$ still had $R^2 = 0.99$.

DISCUSSION

Because the cut-off point at the base of the live crown was estimated independently for CIS and dendrometry, the spline and conic section volumes were recalculated to the estimated height to BLC,H_C, from CIS. Even though the paired *t*-test led to no difference in the respective heights to BLC on average, using one height for both methods should make for more consistency in the results. Indeed, the averages for spline and conic sections in this case were 52.2 and 52.1 ft^3, respectively; this led to average differences of -3.3 and -3.2 ft^3. Note that the average difference has actually increased by using the common BLC cut-off height for volume comparison. However, the MSE for CIS comparisons with spline and conic section volumes were 44.9 and 42.9, respectively. This decrease in MSE is attributable to a substantial decrease in variance as would be expected. Instead of being approximately four times the average difference, the standard deviation of the differences has decreased to only 1.5 times the average difference. The chi-square test reflected the increase in precision even with the larger difference. The results showed that instead of requiring an allowable error of 24 percent using the independent heights to BLC, this has now dropped to an allowable error of 18 percent. As Freese (1960) notes,

> Inaccuracy may be due to lack of precision, to bias, or to a combination of these. The chi-square technique will reject regardless of the source of inaccuracy. However, the test can be modified to evaluate accuracy after elimination of bias.

The results of the bias-adjusted approximate test used here shows that even though the difference increased using the same height to BLC, the decrease in the variance component led to better accuracy on average.

In cut-off importance sampling, heights at which sampling takes place are selected in the lower portion of the bole between H_L and H_C, whereas in traditional importance sampling as described by Gregoire *et al.* (1986), the height to be sampled can fall anywhere within the bole. In this study, it was found that the clinometer estimates of total height had a significant bias compared to the B&S heights and that they varied greatly from tree to tree. If a clinometer were used to determine total height in traditional importance sampling, this height-measurement bias would also affect the final estimate of volume since it enters into the estimator for total bole volume through the proxy volume and associated probability density function. One important strength that CIS has over traditional IS is the fact that the total height does not enter into the calculation for bole volume below the cut-off point. If this point is chosen wisely, such as the base of the live crown, then one should be able to measure any height within the region $[H_L,H_C]$ with good accuracy as in this study. If total bole volume is desired, however, then the estimate of total height does again enter into the calculation through the ratio estimator, and one should strive for accuracy in the total height estimates.

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